

**Process Control – Design, Analysis and Assessment**  
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**MATLAB Tutorial – Controller Tuning – Part 2**

Welcome everyone to the second tutorial on advance traditional controls in this tutorial we will be seen the way with which we can design controller for time delay system.

(Refer Slide Time: 00:26)

**Advanced control concepts – Time delayed systems**

- The response of the system to an input is delayed
- Cause – Transportation time, Inherent process dynamics etc.
- General structure of a system with time delay is

$$y(s) = \frac{N(s)e^{-\tau_d s}}{D(s)} u(s)$$

- Controller can be designed 3 ways
  - Approximating  $e^{-\tau_d s}$  and direct synthesis
  - Smith predictor



The time delay system are the systems where the response to a input change is delayed, so the main task of delay in the system is transportation delay and there might be inherent process delay when we have huge distillation columns the effect of any change in the feet will take long time to reflect in the product, so such inherent process dynamics also lead to delayed process general structures of the delayed systems is as given here.

So the  $e^{-\tau_d s}$  denotes the time delay of the system the numerator by denominator is regular process model which  $G(s)$  the controller can be designed in two different face one is approximating  $e^{-\tau_d s}$  and direct synthesis and the other way is to use the predicted method we look at both these method and we will take an example and describe each method and the see the results.

(Refer Slide Time: 01:42)

## Advanced control concepts – Time delayed systems

- Approximating  $e^{-\tau_d s}$  - using Taylor series
  - First order approximation
  - Second order approximation
- Approximating  $e^{-\tau_d s}$  - using Pade's approximation

$$e^{-\tau_d s} = \frac{e^{-\tau_d s/2}}{e^{\tau_d s/2}} = \frac{1 - \frac{\tau_d s}{2}}{1 + \frac{\tau_d s}{2}}$$

- Smith predictor formulation



So approximating  $E$  power minus  $\tau D S$  can be done using two different ways one is directly to use the Taylor series expansion and the other is used to use Pade's approximations the main difference is that when we use the Taylor series expansion we use  $E$  power minus  $\tau D S$  as one minus  $\tau D$  times plus  $\tau D$  square by two factorial into square plus etcetera, but in these using this expansion leads to additional zeros and the poles are left as such when we use Pade approximation we can approximate both poles and zeros in the same order so the first order approximation and the second order approximation can be used for both pole and zero.

So these is the first order expansion of  $E$  power  $\tau D S$  the key point to notices the  $E$  power minus  $\tau D S$  has been split into two different parts one containing  $E$  power the numerator containing  $E$  power minus  $\tau D S$  by two and the denominator containing  $E$  power plus  $\tau D S$  by two when we divide both of them they will multiply and then add together to give  $E$  power minus  $\tau D S$  the other method of designing the controller is Smith predictor formulation.

(Refer Slide Time: 03:28)

### Advanced control concepts – Time delayed systems

Approximating the delay (Pade's approximation)

$$G_m = \frac{e^{-2s}}{(s+1)(2s+1)(3s+1)}$$

$$G^{des} = \frac{e^{-2s}}{(2s+1)(3s+1)}$$

$$C = \frac{1}{3s+7} \times 5 \left( 1 + \frac{1}{5s} + \frac{6}{5}s \right)$$

Handwritten notes and calculations:

$$C = \frac{5}{1-6} \left( \frac{1}{G_m} \right)$$

$$= \frac{5}{(2s+1)(3s+1) - e^{-2s}}$$

$$= \frac{5(1+s)(2s+1)(3s+1)}{(2s+1)(3s+1) - e^{-2s}}$$

Additional handwritten notes:  $e^{-2s} = \frac{1-s}{1+s}$ ,  $e^{-2s} = \frac{6}{6s+5s+1}$ ,  $e^{-2s} = \frac{6}{5s+1}$ ,  $C = \frac{6}{5}$ .



Now let us take the system where GM is the model of the system with time delay of two second, so and then we define a des transfer function we use the same technique which we have already use in inverse response system if the system has any terms which contribute to unstable controller or instability in the close loop system we will take the des close loop function such that the term which contains which results in which instability gets cancelled so we takes the same E power minus 2S here also and then define the G des function using this we use the same direct synthesis equation which is C equals to G des by one minus G des times one by GM.

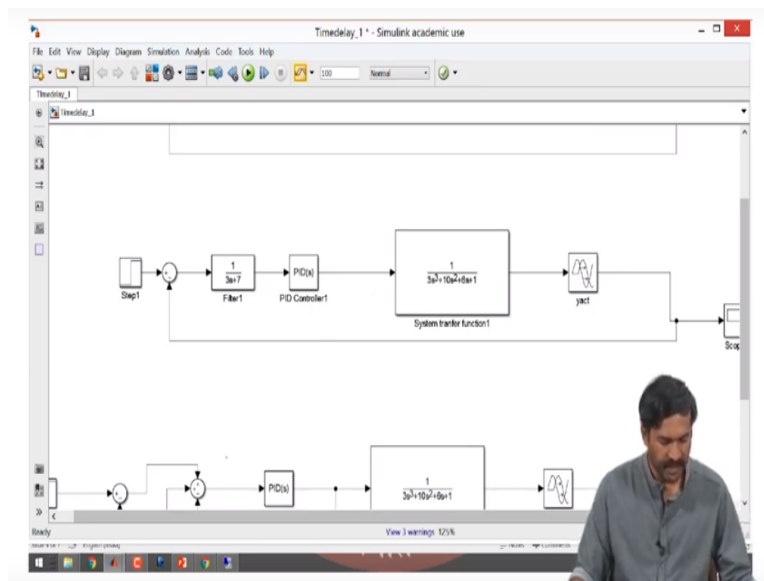
Now we substitute everything and then expanded but the key points to remember is that the E power minus 2S will get canceled from the numerator from G des and then the denominator of one by GM but there is a E power minus 2S in the denominator which is in addition to that the one term because of that we will have something like 2S plus one into 3S plus one minus E power minus 2S, so into S plus one into 2S plus one into 3S plus one divided by E power minus 2S now this two will get cancelled there is still some E power minus 2S left over here, so that we will approximate as first order Pade approximation and then do the computation.

So E power minus 2S is one minus S divided by one plus S so if we use that information and then do the design completely we will end up with this controller formulation, now the main point is to remember is the this controller will work and close loop transfer function will result in this, this dictate the performance of the controller, so when we look at the G des we have E

power minus 2S divided by 6S square plus 5S plus one, so now this six is the time constant of the process square of the time constant of the process.

So if tau square equals to six tau will be like root six now this is some value which is close to two so we are we will still get a process which is little bit slower than the optimal configuration that is because of the way how we have chosen G des one of the ways to choose G des is to have a tau are the time constant which vary small here it is considerably large, let us look at the Simulink example first and then the how the performance is are.

(Refer Slide Time: 08:04)



I have already configured all the parameters in this loop as you can see I have given a step input to the set point and I have configured the filter as given in here and I have taken a PID controller clock and I have given the parameter which had as calculated in this examples to K is five tau I is five and tau D is six by five, so here the integral parameter is given as one by tau I so because of that I have given as one by five K is five and D is six by five, now notice that PE is not multiplying every element in this PID, so because of that there is no need for these five one by five here, so we will take P be five and I to be one and D to be six.

If we this is where the total number of simulation is given now it is going to run for 100 seconds, this is the system transfer function which I have define assets so I have multiplied al the denominator element and taken the coefficient square is S cube S square S was zero so correspondingly I have given the coefficients the way with which we can simulates time delay in Simulink is using the delay block in the library browser I will just show how you can search for each block so you can just type delay and here you can see all locks which are available which contain the delay.

So the main thing with which we are interested in this rather transport delayed or this delayed uses this delay is uses in discrete time domain since we are working in continuous domain, so we will use this transportation domain, now this defines how the time delay constant and we have E power minus 2S there because of that I have given 2S time delay let us run this how the output works, we can see the output as a grasp from this scope if you double click it so as you can see there is initially some time delay but the process was able to reached the steady stay now if there is no control how will the open loop transfer function look like.

So that we can check by changing the connections here, now notice that I have actually directly connected this input to the system, so essentially we are stepping the input not the set point in this simulation, let us run this simulation and see and how the output work, so this is the output response so as you can see there is a considerable delay in initially and we can see that can process is taking as least like twelve seconds to reach tau which is the time constant of the process, if we were to.

(Refer Slide Time: 12:58)

### Advanced control concepts – Time delayed systems

Approximating the delay (Pade's approximation)

$$G_m = \frac{e^{-2s}}{(s+1)(2s+1)(3s+1)}$$

$$G^{des} = \frac{e^{-2s}}{(2s+1)(3s+1)}$$

$$C = \frac{1}{3s+7} \times 5 \left( 1 + \frac{1}{5s} + \frac{6}{5}s \right)$$

Handwritten notes and calculations:

$$C = \frac{5}{(3s+7)} \left( 1 + \frac{1}{5s} + \frac{6}{5}s \right)$$

$$= \frac{5(1 + \frac{1}{5s} + \frac{6}{5}s)}{(3s+7)}$$

$$= \frac{5 + 1 + 6s}{(3s+7)}$$

$$= \frac{6s + 6}{(3s+7)}$$

$$= \frac{6(s+1)}{(3s+7)}$$

$$= \frac{6(s+1)(2s+1)(3s+1)}{(3s+7)(2s+1)(3s+1)}$$

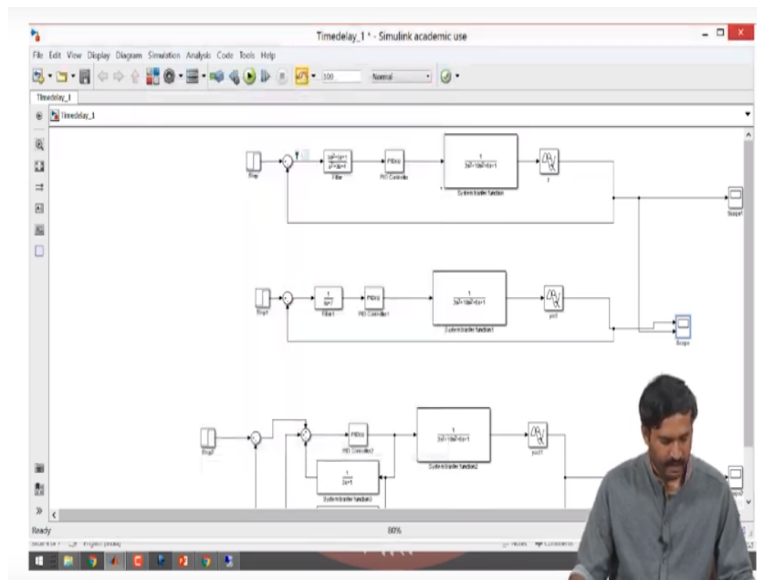
$$= \frac{6(s+1)(2s+1)(3s+1)}{(3s+7)(2s+1)(3s+1)}$$

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Now the next question we can ask is that if the model time delay with which we have the process time delay are not matching then how will this controller work the processes which have different time delay the three things to remember is that we have seen in this already response systems if the process time delay and the used for designing the controller are different then there would not be there will be a relation of terms in the denominator close loop control a close loop transfer function because of that there might be some are HP poles appearing in the denominator, so we have to analyze how far we can tolerate the error for between the model and the process.

(Refer Slide Time: 14:04)



So we can now remember the controller design is for the process which two seconds time delay let us change this two seconds and see how the control works the controller will work the process will start reaching instability so as we can see the starts oscillating here so any more model mismatch which is greater than ten seconds will result in unstable control so as we can see we can tolerate up to 500 percent error, now let us simulate some six seconds time delay and see the output, so it is way better than it is ten seconds and let us look at eleven also.

So that convince our self that after ten the process start so an so become unstable now you can see that there is a oscillation which is any increasing magnitude, so these is diverging because of that the process is unstable now, the other thing is the PID controller which we have design here is still not the optimal PID controller if we were to use some different PID controller which is with different desired close loop transfer function we can actually improve the performance that we can see from this simulation.

So here you can see the process reaches the set point faster than this, so we can actually see multiple let us go back to same configuration, so that we can compare the output we can compare two different signal using a signal graph so that will help us compare both parameters in a single pare and we can see how both of them look like so let us run these simulations, so the Y act is the controller without optimization, so why which we see here is optimized so as you can see there is

a little mismatch here, so there is some offset here which is eliminated in the optimal controller design.

(Refer Slide Time: 17:11)

## Advanced control concepts – Time delayed systems

### Smith predictor method

- Uses a particular controller configuration and computes the input by rearranging the closed loop transfer function

- If we assume the model is  $G_M = \frac{Ke^{-\tau D}s}{\tau s + 1}$  and

$$G^{des} = \frac{e^{-\tau D}s}{\tau_c s + 1}, \text{ then}$$

- $U(s) = \frac{\tau s + 1}{K\tau_c s} (E(s) - G_M^*(s)U(s) + G_M(s)U(s))$



Now let us go to the next topic which is Smith predictor method, so in this method we use particular  $G^{des}$  and particular controller configuration and compute the input by rearranging the closed loop transfer function, so we have if we assume the model of the function is  $K$  is that is first order time delay  $\tau D$  and  $G^{des}$  is also first order with time delay  $\tau D$  then  $U(s)$  can be computed as this following equation, so what we can do is we can actually configure each element in the Simulink and then see how the closed loop performance looks like.



(Refer Slide Time: 17:59)

## Advanced control concepts – Time delayed systems

### Smith predictor – Another interpretation

$$U(s) = \frac{\tau s + 1}{K(\tau_c s + 1)} E(s) + \frac{1}{\tau_c s + 1} U^d(s)$$

- This can be configured as such in Simulink



We can also use another interpretation which you have already seen in the videos where we have an error signal a passing through the controller and the delayed version of input signal passing through another filter, now this also can be configured in as such in Simulink can we can compute the input values and then see the close loop response so the first way in which we can see a in which we have that U of S equals to.

(Refer Slide Time: 18:51)

## Advanced control concepts – Time delayed systems

### Smith predictor method

- Uses a particular controller configuration and computes the input by rearranging the closed loop transfer function

- If we assume the model is  $G_M = \frac{Ke^{-\tau D s}}{\tau s + 1}$  and

$$G^{des} = \frac{e^{-\tau D s}}{\tau_c s + 1}, \text{ then}$$

$$U(s) = \frac{\tau s + 1}{K\tau_c s} (E(s) - G_M^*(s)U(s) + G_M(s)U(s))$$

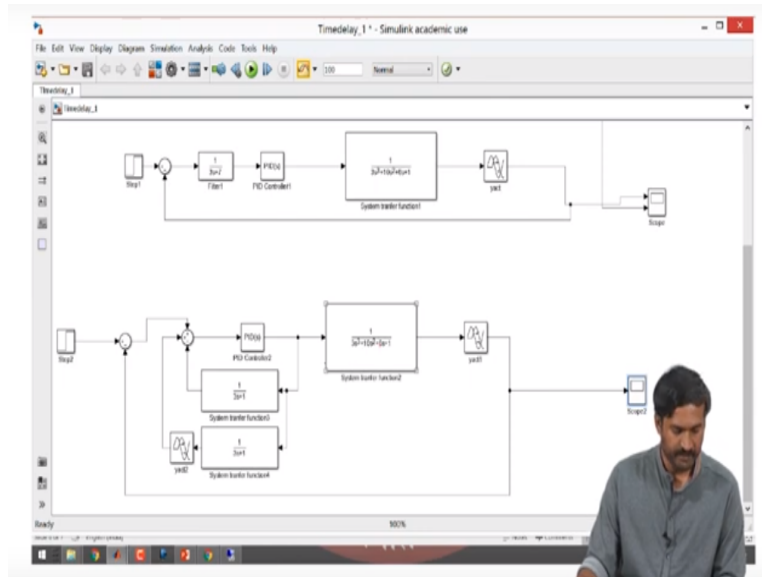
Simulink



Now this method is what I have used in this Simulink tutorial remember this equation so we have tau S plus one divided by K tau S K tau SC times E of S minus G star M of S in the U of S plus

GM of S into U of S, now GM of S in the model transfer function and GM star of S is the transfer function without delay.

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So here you can see the original system transfer function is  $3S^3 + 10S^2 + 6S + 1$  one I have assume the model to be one by  $3S + 1$  and the delays I have configured as two seconds here and two seconds here also, so there is one term without delay which is getting subtract at here as we can see here also so there is a subtraction and transfer function without delay time U of S.

So that has been configured here and similarly the same transfer function with time delay and then that is been added and we into which we give a step input to the set point and past through the PID controller the PID controller parameters I have chosen based on the previous analysis now we can just run it and then see the output, so as we can see it is able to control the system even when the model of the system is taken as one by  $3S + 1$ , so with Smith predictor method even if the model mismatches higher we can still control the system the other method where.

(Refer Slide Time: 21:14)

## Advanced control concepts – Time delayed systems

### Smith predictor – Another interpretation

$$U(s) = \frac{\tau s + 1}{K(\tau_c s + 1)} E(s) + \frac{1}{\tau_c s + 1} U^d(s)$$

- This can be configured as such in Simulink



We have this error signal passing on transfer function and delayed input system passing through another transfer function so this can be configured in the Simulink assets and that you can try the same for the system in by yourself with this I conclude the time delay system tutorial we will see an how to design for controllers for uncontrollable systems and other advanced topic in the next tutorial, thank you.