

Process Control – Design, Analysis and Assessment
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MATLAB Tutorial Controller Tuning – Part 1

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Recap

- System modelling
- Laplace transformation and its application
- Controllers for SISO systems
 - P, PI, PID, PD controllers
- Tuning controller parameters
 - Finding best controller configuration
 - Stability based
 - Performance based



Welcome everyone to MATLAB tutorial and process control analysis design and assessment let us look take a brief recap of what we have seen till now we have seen little bit of modeling can model the system and Laplace transformation and application in both time domain and frequency domain and the controllers for single input single output system mainly P,PI and PID controllers tuning controller parameters is the topic which we are looking at now the tuning is finding the best controller configuration that can be implemented into the event system this can be done in two way one is stability base and the other one performance base we have looked at this stability base controller using zychlor equals tuning.

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Performance based controller Tuning



Direct synthesis method of tuning

- In this method we specify the desired closed response transfer function

$$G^{cl} = \frac{G_p G_c}{1 + G_p G_c}$$

- If G^{cl} is specified as G^{des} then

$$G_c = \frac{G^{des}}{1 - G^{des}} \times \frac{1}{G_m}$$

- While designing the controller we use the best known model of the process G_m .



Now we will look at the other method which is performance base method controller during in this method we have direct synthesis method of tuning where we specify the required closed loop performance in terms of GCL as can be seen in the close loop transfer function we can specify GCL and ultimately back of GC, now one small distinction which has to be noted is that the controller will be implemented on a true process where as we can design the controller based on a notion of the process which is model of the process that has been denoted as GM here, so while designing the controller the use the best known model of the process which is GM this

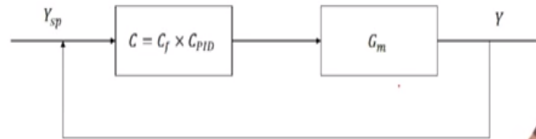
distinction becomes a important as we complicate the structure and we will look at mean more systems.

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Direct synthesis method of tuning

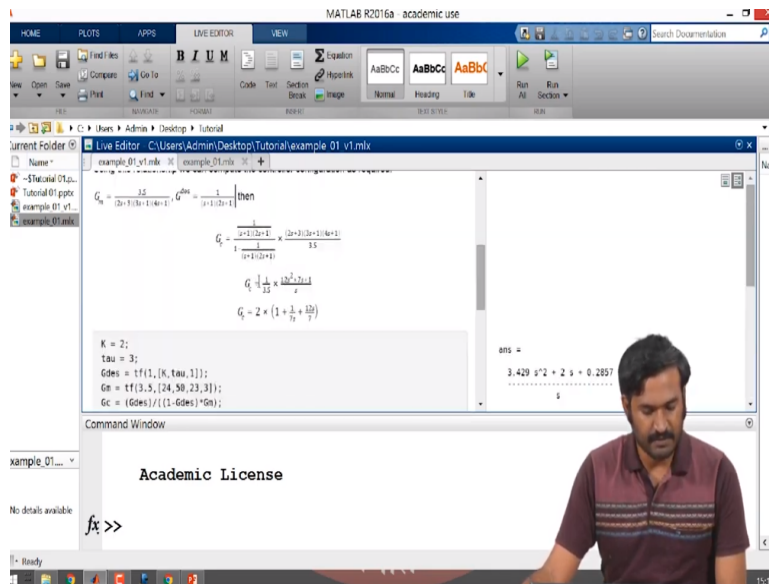
- In some cases we may not readily get the PID controller configuration
 - Then we may have to use a filter in addition to PID controller to get proper tuning parameters
- The controller transfer function

$$C = C_f \times C_{PID}$$



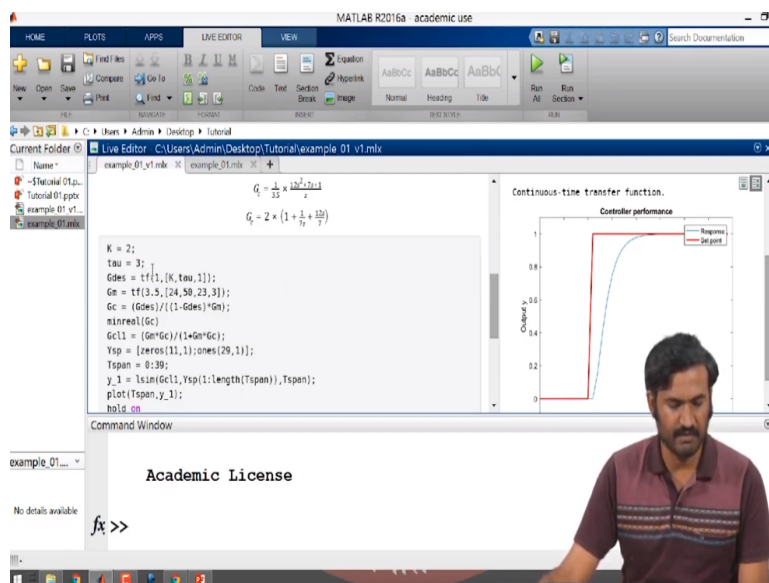
Now let us look at how we can do and calculate the direct synthesis controller transfer function using and MATLAB example as I had already told you we specify the close loop required closed loop transfer function GCL and G des and we back calculate the controller transfer function C.

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This example where I have taken GM to be 3.5 by 2S plus 3 into 3S plus 1 plus 4 into 4S plus 1 and the G des as into mention as here base then this the controller transfer function the GC can be calculated like this the same system has been a simulator in the code given here.

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For that I have use the couple at MATLAB in build function first is TF, TF enable as to specify the transfer function of the given process and we can perform any multiplication division addition something any mathematical operation using this transfer function models.

So we can calculate the GC as directly the G_{des} by one minus G_{des} time model of the system, the decided has been sorry the controller configure the G to see has been printed out here as you can see the calculation turns out to be controller a gain is to which can be seen here and the integral constant value is seven that also can be seen in the in this constant and the differential value is 12 by seven that also can be seen here we can run and check whether this calculation work out.

So as you can see the process reaches the set point very fast we can change the values of the designed transfer function here and the check whether the performance looks similar so let us change the value tau and run the example again, so as you can see there is a little bit of over shoot in this graph and similarly you can do the tuning by specifying different G_{des} but keep it in mind that we are expecting a PID controller configurations, so only if the controller transfer function see which is a second order then we can compute PID value so in some cases it cannot be directly computed.

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Direct synthesis method of tuning

- In some cases we may not readily get the PID controller configuration
 - Then we may have to use a filter in addition to PID controller to get proper tuning parameters
- The controller transfer function

$$C = C_f \times C_{PID}$$

So we may have to define something called the filter and used both the filter and the PID transfer function together as a controller configuration.

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Direct synthesis method of tuning

Example 1

$$\begin{aligned}
 G_m &= \frac{1}{s^3 + 5s^2 + 7s + 3} \\
 G^{des} &= \frac{1}{2s^3 + 3s^2 + s + 1} \\
 G_c &= \frac{1}{G_m} \times \frac{G^{des}}{1 - G^{des}} \\
 &= \frac{s^3 + 5s^2 + 7s + 3}{2s^3 + 3s^2 + s} \\
 &= \frac{(s+1)^2(s+3)}{s(s+1)(2s+1)} \\
 &= \frac{s^2 + 4s + 3}{s} \times \frac{1}{2s+1} \\
 &= \frac{1}{2s+1} \times 4 \times \left(1 + \frac{3}{4s} + \frac{1}{4}s\right)
 \end{aligned}$$

So the solution is $K_C = 4$, $\tau_I = 1.3333$, $\tau_D = 0.25$



This is an example where we specify both GM and G des as both as having a same order of numerator denominator and based on the definition GC equal to 1 by GM times G des by 1 minus G des we compute the controller calculations and the we find that KCSO tau I is 1.3 tau D is 0.25 but still we have this one by 2S plus one which called it as a filter transfer function and use this in addition to the controller we specify.

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Direct synthesis method of tuning

Given

$$\begin{aligned}
 G_M &= \frac{1}{(s+2)(s+3)(s+5)} \\
 G^{des} &= \frac{1}{(s+1)(2s+1)} \\
 C &= \frac{s+5}{2s+3} \times \frac{s^2+5s+6}{s} \\
 K_p &= 6 \\
 \tau_I &= 9/6 \\
 \tau_D &= 1/6
 \end{aligned}$$

$$\begin{aligned}
 C &= \frac{1}{G_m} \times \frac{G^{des}}{1 - G^{des}} \\
 &= \frac{1}{2s^2 + 3s + 1} \times \frac{(s+2)(s+3)(s+5)}{s(s+1)} \\
 &= \frac{1}{s(2s+3)} \times \frac{(s+2)(s+3)(s+5)}{s} \\
 &= \frac{s+5}{2s+3} \times \left(\frac{s^2+5s+6}{s} + \frac{1}{s} \right)
 \end{aligned}$$



This is another example which I can show by working out GM, so let us say C is one by GM times G des by one minus G des so that will be a one by 2S square plus 3S plus one minus one

time S plus two into S plus three into S plus five, so that will be one by S into $2S$ plus three times S plus two S plus three and S plus five now note this we have third order polynomial in the numerator whereas second order polynomial in the denominator, so what we can do is we can separate one of the terms in the numerator together with the additional terms which is $2S$ plus three and we can take it out as a filter.

So we will have S plus five safe are example $2S$ plus three as the filter then we have S square plus $5S$ plus six divided by S as the controller so based on that we can compute the controller tuning parameters, so that can be computed as five, five time one plus six by $5S$ one by five times S , now K here is five one tau I is five by six and tau D is one by five and the filter function is S plus five by $2S$ plus three.

So like this we can design any controller configuration based on the des transfer function des close loop performance which we as specify the main drawback is method of tuning is that we are specifying the required performance because of the we do not know how optimal the performance is the zychlon glass tuning which is stability based tuning there we are more concern about the stability then the performance because of that we back to large extend from unstable regions because of that the controller.

Will be more or less stable whereas in this method we are specifying the desired closed loop performance because of the that we have to reboot thus stability till now we had when looked a tiny complicated a systems and once such we will increase the complexity system further once such system inverse response system.

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Inverse Response system

The screenshot shows a software interface with a slide titled "Inverse Response". The slide contains the following text:

- Inverse response systems are the systems whose initial response is in the opposite direction to that of the ultimate response
- Example - boiler drum, distillation column bottom composition control and Crystallization process etc]

Below the text is a block diagram of a system. The input u splits into two parallel paths. The top path contains a block with transfer function $\frac{k_1}{\tau_1 s + 1}$. The bottom path contains a block with transfer function $\frac{k_2}{\tau_2 s + 1}$. The outputs of these two paths are subtracted at a summing junction to produce the final output y .

Figure 1. Inverse Response system

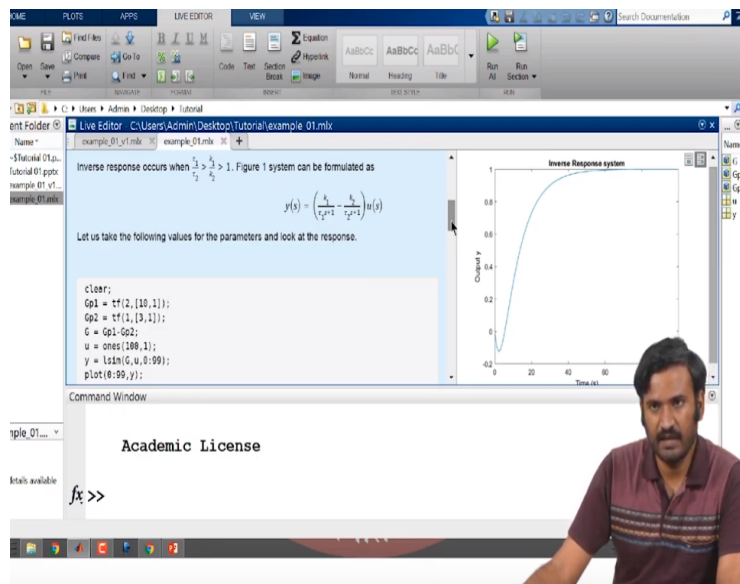
Command Window

Academic License

fx >>

Inverse response systems are the systems whose initial response is in the opposite direction of the ultimate response, so such responses can be found in boiler drum steam boiler drum operation, distillation column bottom composition control and crystallization and similar processes whenever there are two parallel first order processes combined together to give an output. Such processes display an inverse response, so the idea is whenever you give a step response, the system will initially respond in the opposite direction of the final value, so there let us take a configuration which is given in the pictures here, so we have K_1 by $\tau_1 s + 1$ and K_2 by $\tau_2 s + 1$, and these are subtracted together to give I .

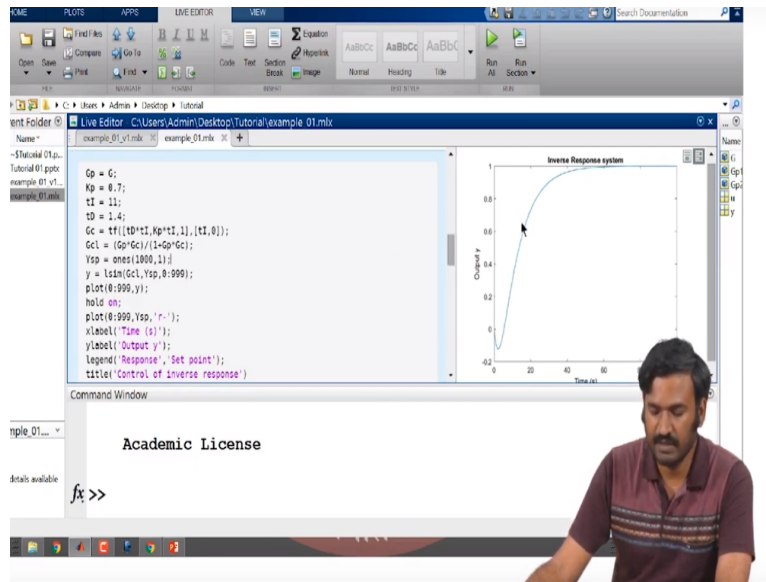
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Now if this is the transfer function of this system and we have $\tau_1 > \tau_2$ then $K_1 > K_2$ greater than one then the system displacement shows an inverse response. Now let us take an example, so let us take $2s + 10$ plus one and one by $3s + 1$ plus one as G_{P1} and G_{P2} and we will subtract to get the final assist model and we will simulate to see how this open transfer function response as you can see we have given step response of unit magnitude.

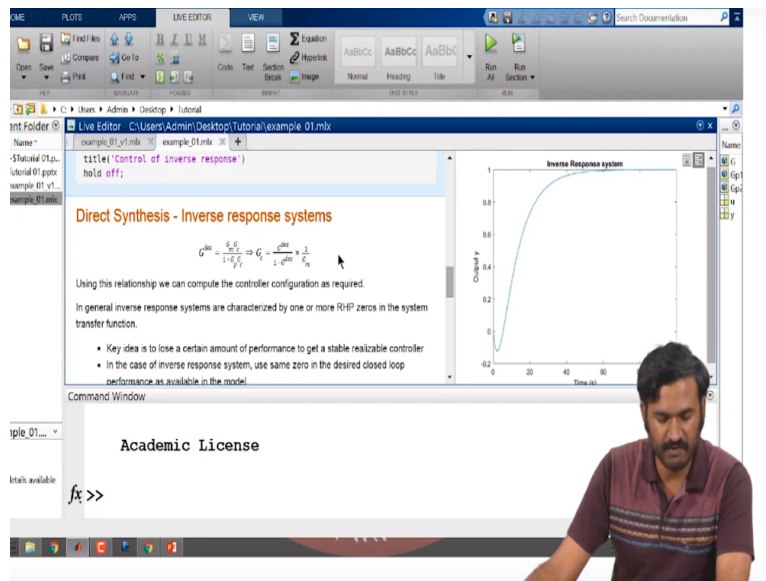
So the final value as to be close to one in the past two directions but the initial response goes in the negative direction first and finally settles to the past one which is expected, now this type of systems are characterized by zeros on the right of the plane so any if the system has one or more zeros on the right of the plane we can expect an inverse response.

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Now such in inverse response system have shown difficulty in control so went a implement PID control directly we may find some sort of performance last.

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With direct synthesis we can specify which type of response we want in closed loop so because of that we will get a better controller configuration.

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The screenshot shows the MATLAB Live Editor interface. The main workspace contains the following text and code:

$$G^{cl} = \frac{G_c G}{1 + G_c G} \Rightarrow G_c = \frac{G^{cl}}{1 - \frac{G^{cl}}{G}}$$

Using this relationship we can compute the controller configuration as required.

In general inverse response systems are characterized by one or more RHP zeros in the system transfer function.

- Key idea is to lose a certain amount of performance to get a stable realizable controller
- In the case of inverse response system, use same zero in the desired closed loop performance as available in the model

```

K = 2;
Gdes = tf([-K, 1], [2, 3, 1]);
Gc = tf([-2, 1], [2, 11, 19, 10]);
    
```

The Command Window shows the text "Academic License" and "fx >>". On the right, a plot titled "Inverse Response system" shows the system's response over time, characterized by an initial undershoot followed by a rise to a steady-state value of 1.

So the key idea is to lose certain amount systems of is in order to get controller configurations so the key idea is to lose a certain amount of performance stable release controller.

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Inverse Response system

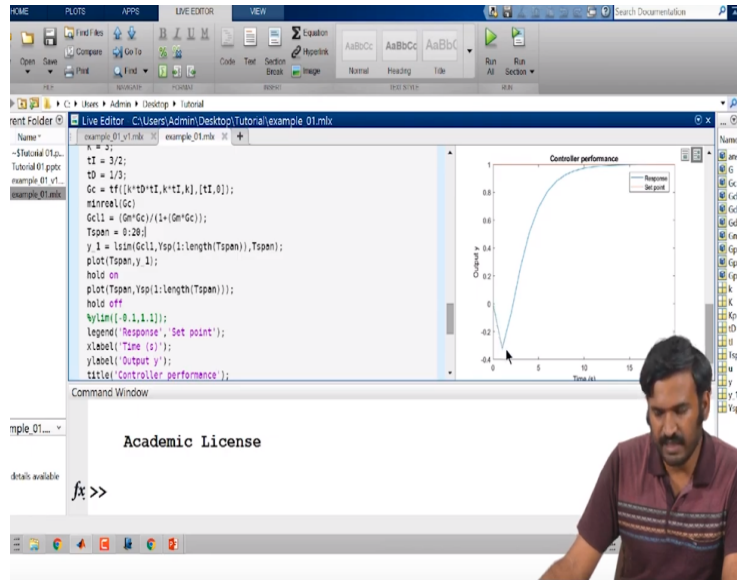
$$G(s) = \frac{1 - \alpha s}{D(s)}$$

$$G_c = \frac{G_c}{1 + G_c}$$

$$C = \text{PID} = K_p \left(1 + \frac{s}{\tau_i} + \tau_d s \right)$$

$$\text{Num}(G_c) = \frac{(1 - \alpha s) (K_p \tau_i \tau_d s^2 + K_p \tau_i s + K_p)}{\tau_i s}$$

The handwritten notes show the derivation of the controller transfer function. It starts with the plant transfer function $G(s) = \frac{1 - \alpha s}{D(s)}$. The closed-loop transfer function is given as $G_c = \frac{G_c}{1 + G_c}$. The controller is defined as $C = \text{PID} = K_p \left(1 + \frac{s}{\tau_i} + \tau_d s \right)$. Finally, the numerator of the controller is derived as $\text{Num}(G_c) = \frac{(1 - \alpha s) (K_p \tau_i \tau_d s^2 + K_p \tau_i s + K_p)}{\tau_i s}$.



The main things to remember is that the if the system transfer function is contains at least one RHP zero say like this with respect to some denominator then the close loop transfer function are any control configuration will have GC divided by one plus GC and let us assume C to be some PID transfer function, so that will be some K_P into one plus $\tau_D s$ plus one by $\tau_I s$, in this case there is a numerator of GC will be one minus αs into $K_P \tau_I \tau_D s^2$ plus K_P into $\tau_I s$ plus K_P divided by $\tau_I s$.

Now this has one RHP whole sorry one RHP zero so this looking at the pole of the close loop the transfer function will introduce RHP pole on the close loop transfer function because of the controller will becomes unrealizable due to this we use direct synthesis method to compute the controller, so here I have taken a system where there is one RHP zero, so the system coefficients are given in the system description is given here so as you can see I have given one minus $2s$ as the zero and numerator and the denominator coefficient are two eleven nineteen and ten.

So this controller works out to be having parameter K to be τ_I by to the three by two and τ_D to be one by three so using this controller we are going to control the process mentioned in this model transfer function which is one minus $2s$ divided by $2s^3$ plus eleven s^2 plus nineteen plus ten, now let us simulate and check the response now even though there is a initial the direction the performance is still good because there are no over show as in the direct TAD tuning..

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$$G_m = \frac{(1-2s)}{2s^3+11s^2+19s+10}$$

$$G_{des} = \frac{(1-2s)}{(s+2)(s+3)}$$

$$G_c = \frac{G_{des}}{1 - G_{des} \times G_m}$$

$$= \frac{1-2s}{s^2+5s+6-1} \times \frac{2s^3+11s^2+19s+10}{1-2s}$$

$$= \frac{2s^3+11s^2+19s+10}{s^2+5s+5}$$

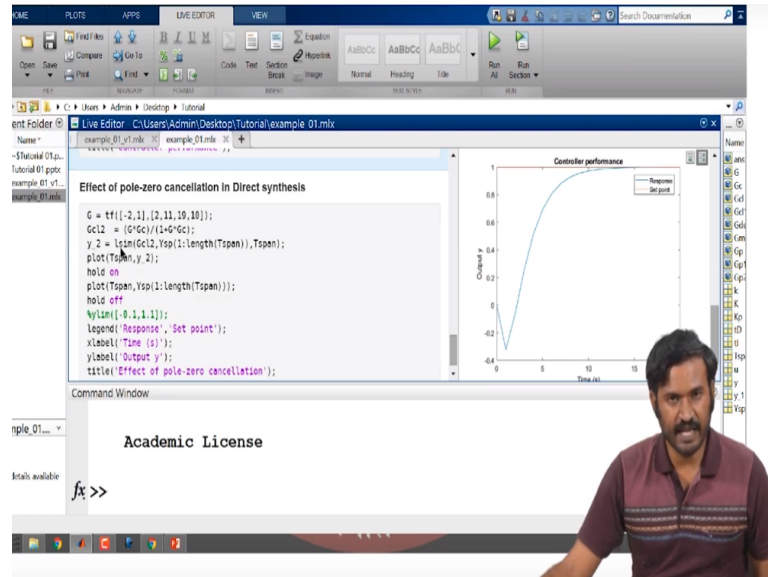
So if the process model which we have assume to be is like one minus 2S divided by 2S cube plus eleven S square plus nineteen S plus ten something like that then the desired close loop performance we also design same zero which we have in the model and then we specify other parameters like S plus two into S plus three something like, so based on this we calculate controller transfer function GC which is G des by one minus G des times one by GM now what will happen is that the numerator of G des which contains one minus two S.

So lets us do this so you will have one minus 2S divided by S square plus 5S plus six divided by S square plus 5S plus six minus one divided by S square plus 5S plus six into 2S cube plus eleven S square plus 19S plus ten divided by one minus 2S these two will get cancelled and these two will get cancelled, so we will end up with something like two S cube plus eleven S square plus nineteen S plus ten divided by S square plus 5S plus five, now we can specify any G des has we want.

So the main criteria with which we specify G des that the gain of the close loop transfer function has to be close loop one and the time constant above of the lose loop transfer function has to be as small as possible so remember there is pole and the zero cancelation here this introduces a problem when we have GM which is not singular to the process we are implementing the control around the process model can change over time it can also be very different from the model

which we have because of some phenomena which we might have missed while modeling so due to this reason we have to look at how this controller will work for a different process model.

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Let us take the look same example we have the process to be one minus 2S divided by 2S square plus 11S sorry 2S cube plus 11S square plus 19S plus ten and the we have the same controller configuration of a with K the KP value three tau I value to be three by two and tau D value to be one by three now how will this controller work and how this process is what we are trying to see now remember that we have design the controller using the same zero which is one minus 2S in the G des and we are using the same controller for a system which has same zero at two right of plane of two.

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$$G_p = \frac{1-\alpha s}{2s^3 + 11s^2 + 19s + 10}$$

$$C = \frac{s^2 + 3s + 2}{s}$$

$$G^{\text{cl}} = \frac{G_p C}{1 + G_p C} = \frac{(1-\alpha s)(s^2 + 3s + 2)}{s(2s^3 + 11s^2 + 19s + 10) + (1-\alpha s)(s^2 + 3s + 2)}$$

$$= \frac{(s^2 + 3s + 2)(1-\alpha s)}{s(2s+5)(s+1)(s+2) + (1-\alpha s)(s+1)(s+2)}$$

$$= \frac{1-\alpha s}{s(2s+5)(s+1)(s+2) + (1-\alpha s)}$$

$$= \frac{1-\alpha s}{2s^2 + 5s - \alpha s + 1}$$

$$= \frac{1-\alpha s}{2s^2 + (5-\alpha)s + 1}$$

$5-\alpha > 0$
 $\alpha < 5$
 250%

Let us suppose the GP is assume to be one minus alpha S divided by 2S cube plus 11S square plus 19S plus ten then the controller which we design is S square plus 3S plus two divided by S, now because of the wave with which we have design the controller using G des as one minus 2S divided 2S cube plus 11S square plus 19S plus ten we may not be able to control the system with for all alpha values so we can find the limits of alpha till which the system can be control using a by computing close loop transfer function of this system so let us compute that so we will have close loop transfer function GCL equals to GPC divided by one plus GPC.

So which should be one minus alpha S into S square plus 3S plus two divided by S into 2S cube plus 11S square plus 19S plus ten multiplied by S into 2S cube plus 11S square plus 19S plus ten divided by S into 2S cube plus 11S square plus 19S plus ten plus one minus alpha S into S square plus 3S plus two , these two terms will get cancelled and we will end up with S square plus 3S plus two into one minus alpha S divided by S into 2S plus five into S plus one into S plus two plus one minus alpha S into S plus one into S plus two now what I have done is to just factorized just polynomial and this polynomial.

So we can see that S plus one into S plus two common here so we can take that out it and canceled it so we will have one minus alpha S divided by S into two S plus five plus one minus alpha S, so we have one minus alpha S divided by 2S square plus 5S minus alpha S plus one, so one minus alpha S divided by 2S square plus five minus alpha S plus one now if we perform row

through this denominator polynomial we have a five minus alpha being positive if this is the criteria alpha has to be less than five, so we assume the alpha to be two so any alpha which is less than five can be control by this controller.

So we can tolerate error up to 250 percent of the original model which we have assume which is one minus 2S divided by denominator polynomial that means even if the process transfer function is one minus say 5S divided by the same denominator unction we will still be able to control the system and get the same performance as specified this closed loop function G des.

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$$\begin{aligned}
 G_p &= \frac{1-2s}{2s^3+11s^2+19s+10} \\
 G_c &= \frac{s^2+3s+2}{s} \\
 G^{cl} &= \frac{G_p G_c}{1+G_p G_c} \\
 &= \frac{(1-2s) \times (s^2+3s+2)}{(2s^3+11s^2+19s+10)S} \\
 &= \frac{(1-2s)(s^2+3s+2)}{(2s^3+11s^2+19s+10)S + \frac{1-2s}{-10}} \\
 &= \frac{(1-2s)(s^2+3s+2)}{2s^3+11s^2+19s+10} \times \frac{1-10s}{2s^3+11s^2+19s+10} \\
 &= \frac{(1-2s)(s^2+3s+2)(1-10s)}{2s^3+11s^2+19s+10} \rightarrow
 \end{aligned}$$



So the GP is one minus 2S divided by 2S cube plus 11S square plus 19S plus ten and we have controller configuration which is S square plus 3S plus two divided by S, now using the this two we can compute the close loop transfer function which is GP GC divided by one plus GP GC that will be one minus 2S times 2S cube plus 11S square plus 19 S plus ten into S, so there is S square plus 3S plus two the numerator also divided by 2S cube plus 11S square plus 19S plus ten into S plus one plus 2S multiplied by 2S cube plus 11S square plus 19S plus ten multiplied by S, now these two get cancelled.

So we will have one minus 2s time S square plus 3S plus two divided by, so let us just do the multiplication so that we can understand importance of doing this so 2S cube plus 11S square plus 19S plus ten S minus 2S plus one now you can see we have 2S cube plus 11S square plus nine sorry S power four S cube S square plus 8S plus one, now we can perform a Routh RA test

for the polynomial to check whether the R in HP pole as long as we do not have any RHP pole this transfer function will be controllable.

Now let us suppose the processes like one minus 10S divided by the same denominator 11S square plus 19S plus ten then we have minus ten instead of two here and this term will become zero, so we can analyze this by replacing this two with some coefficient alpha and checking whether this alpha will have any effect on the stability of this process.

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Handwritten notes on a whiteboard:

$$G_p = \frac{1-2s}{2s^2+11s+10}$$

$$G_c = \frac{s^2+3s+2}{s}$$

$$G^{\alpha} = \frac{(1-\alpha s)(s+1)(s+2)}{s \times \frac{2s^2+11s+10}{(1-\alpha s)(s+1)(s+2)}}$$

$$= \frac{(1-\alpha s)(s+1)(s+2)}{2s^2+11s+10 + (1-\alpha s)(s+1)(s+2)}$$

Annotations:

- $1-2s = G_m$
- $\alpha \leq 5$
- $\frac{5-2}{2} = 250\%$
- $5-\alpha > 0$
- $\alpha < 5$

MATLAB Live Editor screenshot:

```

Effect of pole-zero cancellation in Direct synthesis
G = tf([-8,1],[2,11,19,10]);
Gc1 = (0*Gc)/(1+0*Gc);
y,2 = lsim(Gc1,Ysp(1:length(Tspan)),Tspan);
plot(Tspan,y,2);
hold on
plot(Tspan,Ysp(1:length(Tspan)));
hold off
ylim([-0.1,1.1]);
legend('Response','Set point');
xlabel('Time (s)');
ylabel('Output y');
title('Effect of pole-zero cancellation');

```

Controller performance plot showing Output vs Time (s). The plot shows a step response that rises from 0 to 1.0 with a slight overshoot and then settles to the setpoint.

So let us do that so we have GP which is $\frac{-\alpha S}{2S^3 + 11S^2 + 19S + 10}$ plus $\frac{10}{2S^3 + 11S^2 + 19S + 10}$, now the GC is the same which we have used here $\frac{S^2 + 3S + 2}{S}$, now the close loop transfer function would be $\frac{1 - \alpha S}{S^2 + 3S + 2} \cdot \frac{10}{2S^3 + 11S^2 + 19S + 10}$, so divided by same thing so we have $\frac{1 - \alpha S}{S^2 + 3S + 2} \cdot \frac{10}{2S^3 + 11S^2 + 19S + 10}$, now these two will get cancelled and then we will end up in a polynomial which is like this.

So $\frac{1 - \alpha S}{S^2 + 3S + 2} \cdot \frac{10}{2S^3 + 11S^2 + 19S + 10}$, now we have to look at the stability of the polynomial this we can do it using Routh stability test based on that we will find the condition that as long as the five minus alpha is greater than zero we have stable holds now these implies alpha has to be less than five right so any alpha which greater five will result in one or more poles on the right of plane because of the we will have unstable controller and stable close loop commons, that we can see in this simulator example here.

So here I have simulated the same controller configuration used the same model here supposed let us assume this to be twelve or let us assume it is to be six and the run these code, so as you can see the system starts to become unstable here even if it is something like 5.1 you will find that the system starts to becomes unstable so the magnitude to become oscillation keeps on increasing, so we have used to one minus 2S divided by the same denominator as the model now we have found out that alpha can take any value between zero and five.

So because of that we can say the tolerance of this controller is like five minus two divided by two which is like 250 percent of the original value, so even if the model the which we have to identify is 250 percent who are then the actual process the process this controller with which we have found out using this model is still valid and it can be applied for controlling, so with this I will stop this tutorial we will look at time delayed systems and further examples in the next tutorial thank you.