Process Control - Design, Analysis and Assessment
Doctoral Research Scholar Parameswaran S
Department of Chemical Engineering
Indian Institute of Technology, Madras
Stability Analysis–Various methods -Part 2

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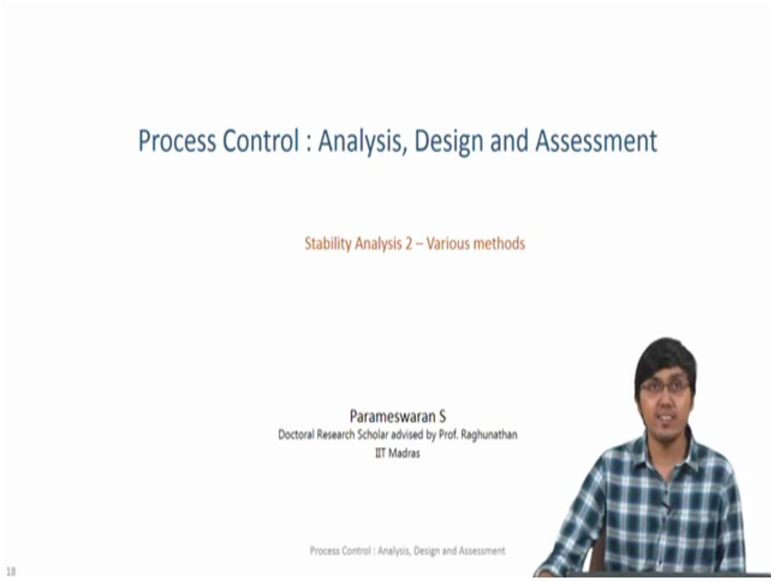

So we will continue with our stability analysis in this lecture.

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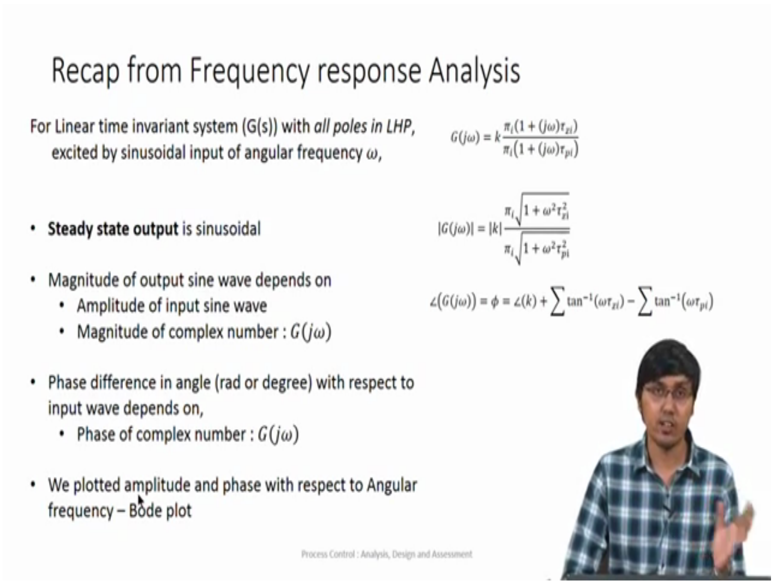

So like from the previous lecture we saw a root locus and then we say we said that root locus is plotting all the poles are changing which when we increase the gain value and then we said we are not interested in the exact K P value but we are interested in the like qualitative understanding and also like not so rigid quantitative value but some range of K P where in like between system will be stable etcetera and that is what we are going to use the concept of frequency analysis into this and then we are going to see how the frequency analysis helps in the concept of stability.

And so if you can recollect from the frequency analysis presentations, so this is what we had as a somebody slide from that, so when we exert his term it is assigns all input, so the output is also sinusoidal at the steady state and then the magnitude of output depends upon the sign of the input the amplitude of it is input and also the magnitude of complex numbers G of G Omega where G is a system, G of a system and then phase that is also your phase lag that we have observed in the output.

So we are we are we add the (ampli) magnitude change and also the phase change, so this what we observed and in these formulas will vary we gave for the amplitude and the phase. So we also plotted the amplitude and phase with respect to the angular frequency and we said this plot is called the bode plot, so we are going to see how this bode plot is going to help us with stability analysis so that is what that topic for this lecture.

So we will also try to understand intuitively like what happened in the single case and also you will towards end of it we will try to see where it fails that is another kind of analysis fails.

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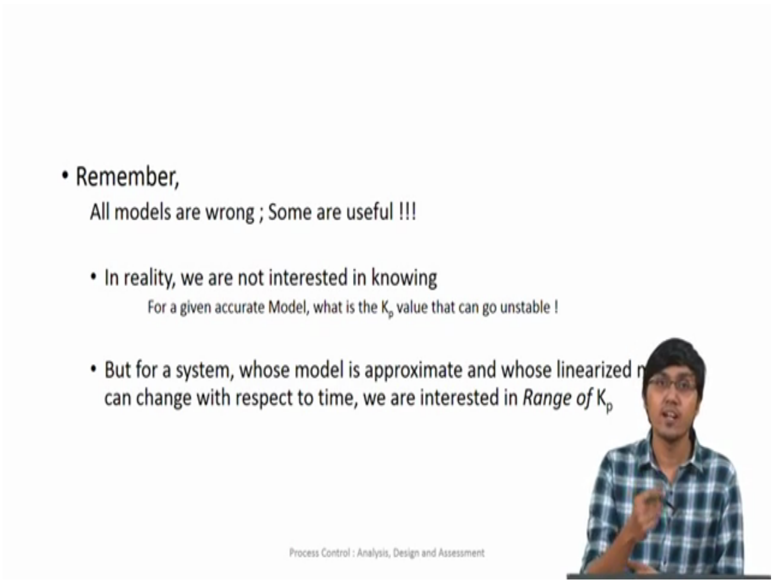

So again to summarize to (cal) to tell this from the previous (les) lecture, so we said that ok why we are inter in the range of K P because like the model itself the transfer function itself is same approximation, so we are not interested in a particular value of gain at which it goes unstable by the quality terms assigning of what value of gain system goes unstable etcetera but so that is what this will give this analysis will give us in a better way.

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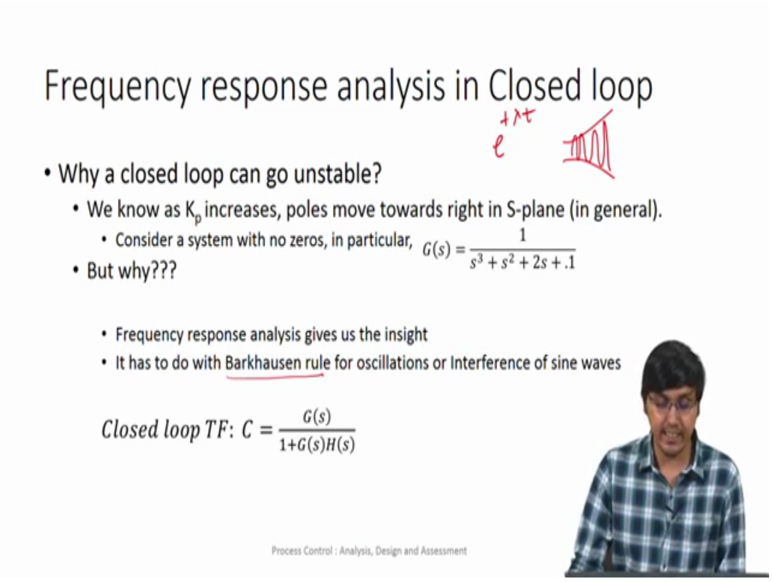

So now so why now I again ask ok this is good like for when we say that when we increase the gain the poles are moving towards the right of S plane is ok but why everything happens can we ask this question deeper sense is something important to say. So this frequency analysis is helping us to give some insight but this insight is not a very strong inside basically because this is very this intuition is not a straightforward intuition and failure at some times but it is ok to have this intuition to begin with, ok.

So remember this see like what point we are trying to emphasize is this intuition is ok to help to begin with but this is not a strong intuition and it can fail at times ok, so that is what we are going to like keep this value keep this point in mind, so that we will go and we will see this varied file etcetera but we will just have this. So why it is goes unstable? So one thing we can say is like ok whenever we have a poll on the right hand side of S plane we are going to have some positive lambda and the t and that is actually causing an increase in the amplitude.

So of sine wave or E or something, so that that is what is going to unstable but why is this increase so now we ask this question why this increasing amplitude happens, so what so we if you ask this then there is a rule called Barkhausens rule ok, if you in electronic socketing and oscillator etcetera we use this Barkhausens rule.

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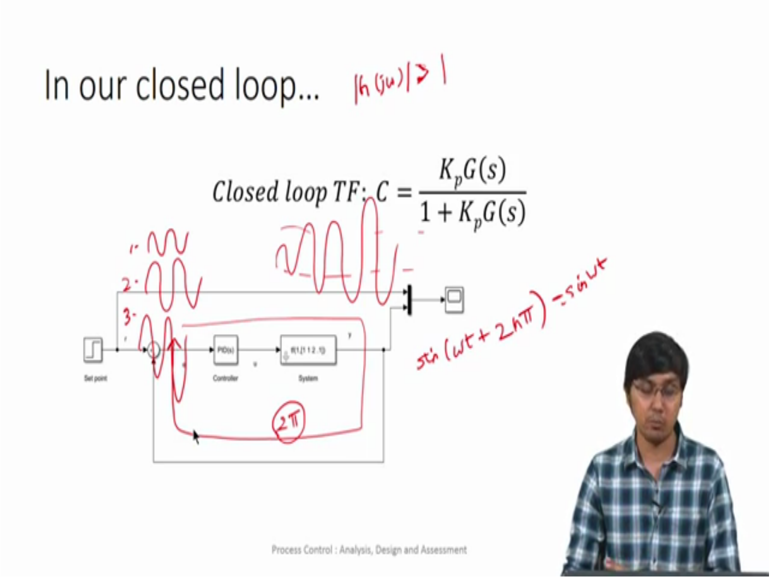

So that is what it basically says ok what it basically says is Barkhausen rule what it basically says is if in a system ok if you have a frequency for a particular frequency if this frequency when it goes around the loop and comes back at the same point here if it is in phase with this particular sine wave, so this is the input and let us say I got an output like this this which is in phase with this and the if the net gain of this particular wave like how much the magnitude has changed ok if it is greater than or equal to 1 either that it has increased in the amplitude or that it is having the same amplitude then basically this is going to this sine wave is going to go through the loop again and again right.

So basically if it is having let us just like run through it ok once and then see how it goes, so basically the first time let us have a sine wave is like this here I mean it goes around the loop and then let us say now the amplitude has decreased let us say the amplitude is the modulus of G of j Omega at that particular angular frequency ok at that particular angular frequency say point 5, so basically when it comes back it will be like and let us say this is the frequency at which it is going through the loop and coming back I in phase ok.

So anyway if this comes in phase and it is point 5 in amplitude and then becomes half of this ok and in the third if it goes around again and comes back it becomes like again smaller, so basically you can see here when it goes around loop and comes on it is oscillation is going to the amplitude is going to die down, so basically this frequency will die down as time progresses and becomes it is a stable system.

But what happens when you gain down to and it becomes equal to 1 when it becomes equal to 1 this again is like same sine wave and this is also same sine wave everything like (()) (06:19) in phase, so basically you know what happens the output is having a same constant amplitude, ok. So it is going to keep on oscillating in it is amplitude but still it is not going to unstable it just keeps oscillating, the oscillation is nothing died on.

And what happens if the gain is greater than 1? So if the gain is greater than 1 what happens next time it comes around loop this is going to increase and an extra time it is going to increase more. So basically if you can plot the output basically the amplitude gets increasing so this is the intuition that the Barkhausen it is criteria gives but ok and this intuition we are using it for (())(07:01) first stability, ok.

So Barkhausen says that it should be in phase or a multiple the phase shift caused by this loop must be multiples of 2 pi ok, it is in phase like, so after every 2 pi interval the sine wave is going to get phase, right so what does that mean by omega t plus some multiple of 2 pi is again going to be sine omega t, so any multiple of 2 pi is going to become the same wave again and again.

So if this particular wave is going to be like when it goes around loop and comes back if it is going to be multiple of 2 pi the phase lag introduced by the loop is going to be multiple of 2 pi the sine wave is going to be in phase and if at that particular frequency if the gain is going to be greater than or equal to 1 then it going it is going to oscillate if it is equal to 1 it is going to be oscillating if it is greater than 1 it is going to be oscillating with the increasing amplitude, if it is not if it less then 1 then it is going to die down, it is going translation are going die down, so it is a stable systems that is what the Barkhausen says and then with the Barkhausen finite criteria if you go and look at the bode plot let us see what happens.

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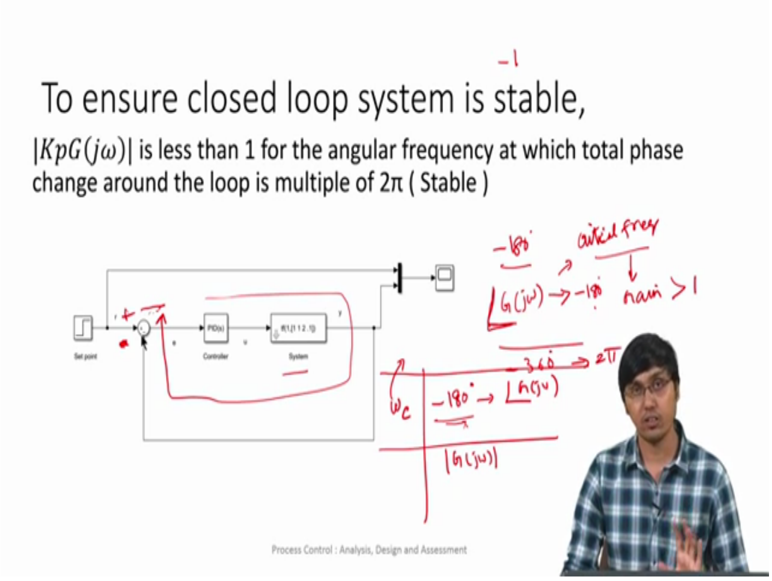

So this is what we saw already, so this is the sort summary of it, so if it is in phase or multiple of 2 pi and then the (())(08:19) is less than or equal to 1 then it is when it is going to go to unstable is domain basically actually equal to 1 it is still like stable keeps oscillating greater than 1 it is going to be unstable that is what we are going to see. So in every term what happens is here we have an minus sign ok, here we are minus sign so we saw in our frequency response analysis like every minus sign is going to contribute to the minus 180 degree phase shift right.

So we be about it we did minus 1 and we saw the broad part etcetera and then we saw it is minus 180 degree so every minus n is going to contribute to 180 degree, so this minus sign is target we are subtracting, subtracting the output from the input, so 1 minus 180 degree phase shift is already there so far away that comes out through the loop and causes this an and then it is going to if you do if there is a wave here and it is coming throughout the loop and coming to the same point again one thing this is minus 180 degree is already there due to this minus thing and then there is also a phase lag it is contributed by the system and this phase lag is nothing but the magnitude of the amplitude of G of sorry the phase lag is nothing but the angle contributed by the G of j omega complex number G of j omega.

So for some particular frequency if this angle of if this angle of G of j omega is going to be minus 180 degree again then basically this minus 180 degree due to this particular system and this minus 180 degree is going to get added to be minus 360 degree which is like a multiple of 2 pi right, so (multi) since this because multiple of 2 pi this particular frequency at which the phase angle is going to be minus 180 degree that is (())(10:05) that particular frequencies it is of some importance I mean that particular frequency we call it as a critical frequency, critical frequency in sense it is like an important figure of all the frequencies we look at this particular frequency and this is of the most importance stability perspective that is what I want to say.

So if at this particular frequency if I in this frequency on only at this particular frequency the system is going to be the sine wave is going to come in phase right, so at this so Barkhausen says for such frequencies if at that frequency the magnitude is greater than or equal to 1 then the system is going to probably go to either oscillator the system is going to be oscillatory or if it is greater than 1 it is going to be a unstable system.

So now at this particular frequency which look at the gain of system and if gain of the system is less than 1 at that point then basically it means systems is stable if the gain is equal to 1 then the system is going to oscillate at this particular frequency value if it is greater than 1 then system is going to oscillate with increasing amplitude so that is what we have we are seen from the analysis of how this frequency interference of the frequencies happen, ok.

So this is the idea ok this is the idea of the frequency response analysis coming into the stability perspective, so to summarize basically what we are going to say is the summary of all this thing is nothing but at the frequency where the phase shift is minus 180 degree contributed by angle of G of j omega, so this is at this frequency so that frequency we call it a as a critical frequency omega C and what we say is if at this frequency the magnitude which is that the gain contributed by modulus of G of j omega if it is greater than unity then the system is going to get unstable.

So this particular thing is called a critical frequency this particular frequency is critical frequency at which the phase lag is given by minus 180 degree and why minus 180 degree because the minus sign contributes to another minus 180 degree so totally we get these minus 360 degree which is multiple of 2 pi, so the Barkhausen condition is satisfied, so that is what we are going to remember now.

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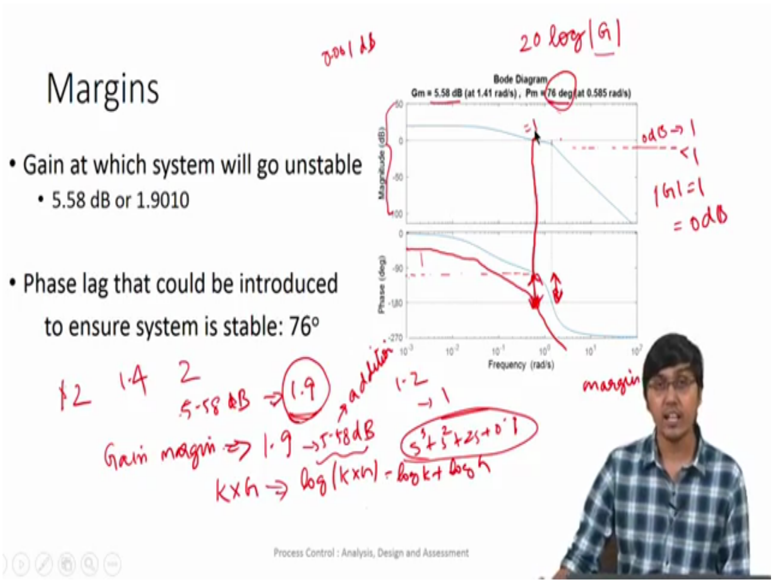

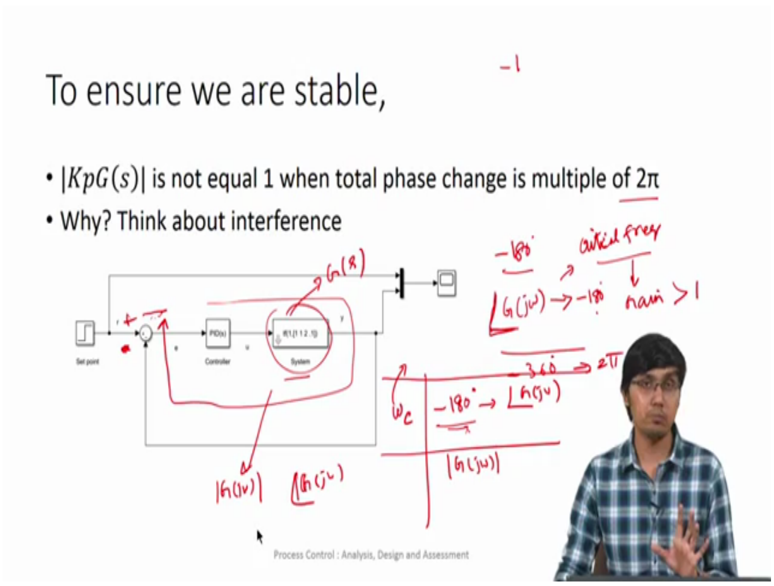

So basically here if you can do the bode plot and then we see at what phase at what frequency is this system going to (())(12:33) minus 180 degree phase shift, so minus 180 degree phase shift is at this particular frequency and this particular frequency is nothing but omega equal to 1 point 41 radians per second right. So if you can just plot here and this is 1 point 41 radians per seconds, so mega value, omega C value here.

So that are the frequency at which the phase is minus 180 degree is called the critical frequency and that frequency in this case it is mile is 1 point 41 radians per second and at this frequency we look at the amplitude and remember this amplitude is now in worldwide it is in dB, so basically 0 dB means gain is 1 ok because remember dB is nothing but on t log of the game, so if gain is 1 then log 1 is 0, so that is when 0 dB comes.

So this anything which is less than 0 dB is minus, minus in logarithmic scale is nothing but less than 1, so in it if mod G is less than 1 then basically you get a minus value here, so here it is nothing but the gain is less than 1, so in this system in this system when you when the frequency when the system is at the critical frequency when the phase shift is minus 180 degree the gain is less than 1 so the system is stable, first conclusion we can make is ok the gain is less than 1 so that the system is stable.

Now not that is not the again that is not the question we are going to ask the question is like my model itself is an approximate thing, so with this that is understanding ok this model is stable you cannot conclude anything then we ask the question like ok what is the gain that I can multiply to the system so that it becomes unstable? So what to if I can multiply with a very high gain and then system to for making a system to go unstable then it means that I am very comfortable with the stability of system.

So if I just multiply system with for example let us take 4 here ok, so in this case the gain margin is (nothi) ok, we say the difference between this particular value the gain at omega C and 0 dB is nothing but 5 point 58 dB ok, so this particular value is 5 point 58 ok which is nothing but the 1 point 9 ok, remember followed the same system we have analyzed the root locus and also the we found that the ultimate gain was 1 point 9 that is the same thing here ok that has for the same thing it is all the methods will use the same values ok there is methods may differ but the values is the same, ok.

So you can use any method to whatever you feel comfortable to find this ultimate gain, so 5 point 58 dB is 1 point 9 why because for if 20 log 1 point 9 is 5 point 5 you can check this ok. So basically it went basically what we are going to say is I if I multiply if they multiplied by system with a gain of 1 point 9 then the system becomes unstable, so in when my modelling the system if I made some mistakes I instead of taking the gain as 1 point 2 ok due to some reason I took this gain as 1.

So instead of getting 1 point 2 pi s cube plus s square plus 2 s plus point 1 like I am getting a value of by value as 1 by I made a mistake in modelling so I took 1 by s cube plus s square plus 2 s plus 1, so that may be one reason or be it is a nonlinear system so I linearized it and then I made some approximations and then I got this or there is a lot of approximations in modelling so basically considering all the approximations I am saying I have a flexibility of making a mistake of 1 point 9 times the gain ok.

So only if my mistake if I multiply by a gain of 1 point 9 then a system will go unstable, so this is the (mom) this is the bandwidth higher ok, so for my system to go unstable I can (multi) can I can give a gain of 1 point 9 till like multiplied 1 point 9 times the gain, I am not going to go to unstable so that is what we are going to tell there. So this margin ok this bandwidth or this margin we call it as a gain margin how much gain I can multiply and still be stable?

So I have a stable system how much gain I can multiply and still be stable so that maximum gain it is what called gain margin so it is nothing but 1 point 9 and the in decibels is nothing but 5 point 58 dB. So it is actually a multiplicative gain but when we convert into decibels basically it is you it is something like if it is a multiplication K into G when you take the logarithm of it, it now it becomes like log k plus log g right.

So this decibels is nothing but how much gain you are going to add, so remember when you give the gain in decibels you are telling how much gain in decibels I am going I can add ok because when we convert into (la) multiplication become (sig) plus sign when we take them the log when we take the logarithm of it is, so logarithmic scale it becomes addition ok that is something you can remember but essentially all the things all logarithm, mathematics all these max left what we are asking is what is the gain I can multiply such that the system will still be stable? What is the maximum gain I can multiply still the system will be unstable system will be stable? That is what we going to ask.

So which means basically if we multiply by K p if you multiply by a gain of 1point 2 it is still stable, if you multiply by gain of 1 point 4 it is still stable but if you multiply with a gain of 2 it will go unstable, so that is what it is the gain margin what is there what is the margin what is the bandwidth of gain I have so that the system will be still stable that is what we are going to ask that is the question that is the answer for gain margin.

Now when I am modelling I not only can make mistake in gains I can also make mistakes in finding these poles and all approximations even in this particular term I can have some mistake, so even the phase is a problem so this minus 180 degree at this particular omega C may not be correct I can probably have a my actual model my maximum model may go my natural process may be having omega C of like say 1 point 3 I did some mistake in modelling because I did some approximation, I did something so maybe that so that this phase was something different, ok.

So now we ask the question like ok what is the phase difference? Ok I have to had so that the system (pha) what is a extra phase like that that I have to introduce so that the system will go unstable, so now forgetting about the gain let us say the gain is correct let us say that again plot is fixed. Now I will I asked this question, now I do not now I assume that ok I made some mistake in modelling, so even this phase plot is inaccurate so or like something else is going to happen so that is phase diagram is going to change.

So what is the phase change I can introduce phase lag I can introduce so that this system will still be stable similarly like the same question we ask for gain we are asking for phase now initially we fix the phase plot and then we ask the question for the gain for given phase plot what is the gain I can add, I can multiply so that system will be still be stable. So the multiplicative gain when the system is stable it is going to be called as a gain margin and the I now have fixed the gain plot and then I asked the question what is the phase lag I can introduce so that the system will still be stable so that is what we are going to ask now.

So basically now it means that so again I can (it boi) it comes down to the Barkhausen condition so we are interested in the gain of 1 right, so whenever the gain becomes greater than or equal to 1 that is when we are going to bother about the stability things. So now or in we have gain is 1 or we said (d) 0 DB right, so gain of 1 in multiplication becomes 0 dB when you take the logarithm 20 log 1 is 0.

So when we consider this 0 dB line, so this 0 dB line so this is when it interacts with the intersects 0 DB line, so I just mark this down here so this is the phase, so now this phase ok if I make a mistake of say this much phase like ok if I can introduce this much phase lag to make the system unstable the condition is at this particular value of gain equal to 1 I should have a minus 180 degrees phase shift.

So my but now with this approximate model I am not getting minus 180 degree phase shift at this particular game, so system is stable but (ye) for if I introduce what phase lag will this system go unstable so basically if I shift this deg by something like this ok then at this point it becomes (unsta) it becomes oscillated right because I now the critical frequency has changed here and at the critical frequency this becomes like equal to gain becomes equal to 1 and so it starts oscillating.

So that is a what is the phase lag that we can add to the system to make the system oscillatory or what is a phase lag that at resistant can withstand without going to without copy that becoming unstable that is what we are going to ask this is. So what is the value of this basically this is the value so this turns out to be around 76 degrees, so this is what matlab gives, so this is a 76 degree is, right so minus 180 degree and minus 8.

So if you made a mistake in modelling or if you introduce a some other component which is going to (())(22:42) phase lag without changing the amplitude etcetera then if you introduce a phase lag of minus 70 degree 76 degrees only then the system will become unstable so I can make a mistake off say 10 degree phase lag what system will still be stable or the when system is operating it can go like sound it can go to some other operating point and then at the operating point if you find the system is model will be something different because we take essentially what to do we take a nonlinear system and then we linearize it.

So all different operating part it will be different systems, so basically what we can say we can accommodate of 76 degrees, 76 degrees will be the phase lag that I have to had to make the system unstable so that is what is call the phase margin. So what is the phase I have to add ok to make the system unstable so that is what is called phase matching, so to summarize everything fixed the phase plot asked the question like at minus 180 degrees why a minus 180 degree? Because with a negative sign in the feedback loop it becomes minus 180 degree plus this minus 180 degree so minus 360, so it is multiples of 2 pi.

So with this so at the minus 180 degree whatever is the angular frequency that frequency is called the critical frequency that is the first point we observe then you ask the question like what is the gain at this particular critical frequency and if the gain is say less than 1 then the system is stable then we are even do not stop the question there we asked like ok what is the gain we are actually (multi) we have to multiply to make the system unstable.

So that is giving us the bandwidth of ok we can multiply this much gain so that the system will be (())(24:22) stable, so that a that we got is 1 point 9, so that is the ultimate that is nothing but the gain margin so that is 1 point 9 is nothing but by point 58 decibels, so 5 point 58 decibels, so that is the gain margin thing. And now we fix the gain plot and then we ask the question like what is the phase lag I have to introduce where the resistant goes unstable?

So that is nothing but this given by at 0 dB because like Barkhausen says like at equal to 1 rate (multi) the gain should be equal to 1 and so I have to at 1 when it becomes what should I do to shift this two become like minus 180 degree so what is the phase lag I have to introduced to make there the my make this particular phase has minus 180 degree at that particular value when gain becomes equal to 1, so that is called the phase margin.

So till what phase lag I have to introduce so that still system will still remain stable? What is the maximum phase lag that I out introduce so system will still remain stable so that is called a phase margin. So that is nothing but 76 degrees to 360 degrees here and as you can clearly see here higher gain margin and higher phase margin will tell me that my I am safe because I can multiply by a big number and still still safe I am still stable I can introduce a big phase lag I am still stable but a smaller gain margin will mean that for example if I have a gain margin of say very less value say oh I have a gain margin of say point 001 dB or something like that and it means that even a small mistake in modelling or even a small change in the gain of the system is going to push it to that instability.

So yeah higher gain margin and higher phase margin is going to tell us we are the stability the from a stability point of view we are safe, a smaller gain margin and phase margin is not disagreeable because which means that a small error in the modelling or a small change in the system parameters is going to cause a problem it is going to push a system push a system unstable and the one more point you can think it is not only about error in modelling it is also about say some system parameters are not constant basically they change with the spirit time for example if you take the (())(26:27) it is moving unnecessary desperate times, so all the time constant everything is going to change.

So in the real time everything is going to be changing with respect to time, so and this concept of like band with how much gain I have a band with an how much phase I have a bandwidth that is very important (cone) important notion not the exact K p value etcetera. So this is the concept of gain margin and phase merging so maybe you can use this command of in mat lab you can use a margins (basic) in mat lab you can use this command margins you can give a transfer function, you can give a transfer function here and then it is going to display this diagram and you can basically play with this and then give a different transfer function and see what happens.

So and remember this whatever we used to find this is nothing but that we plot the bode plot of open system ok for open loop system because remember again we took this, this is the this is bode plot going to give that magnitude of G of a G of j omega and then the angle unit is by G of j omega when this system is the G of is, so basically we are plotting for open loop and then we are saying ok in the open loop if there phase lag is minus 180 degree and if the gain is amplitude of the gain is equal to one extra they know what happens?

So for a bode plot we use the open loop transfer function ok.

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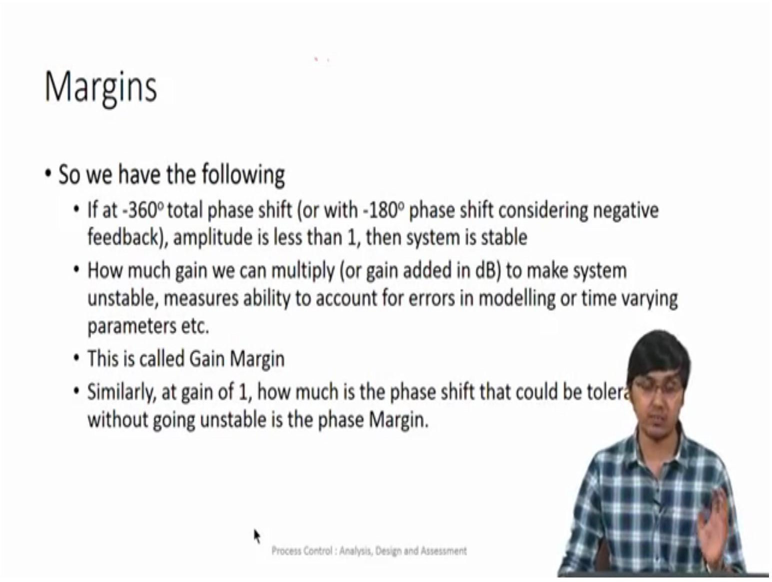

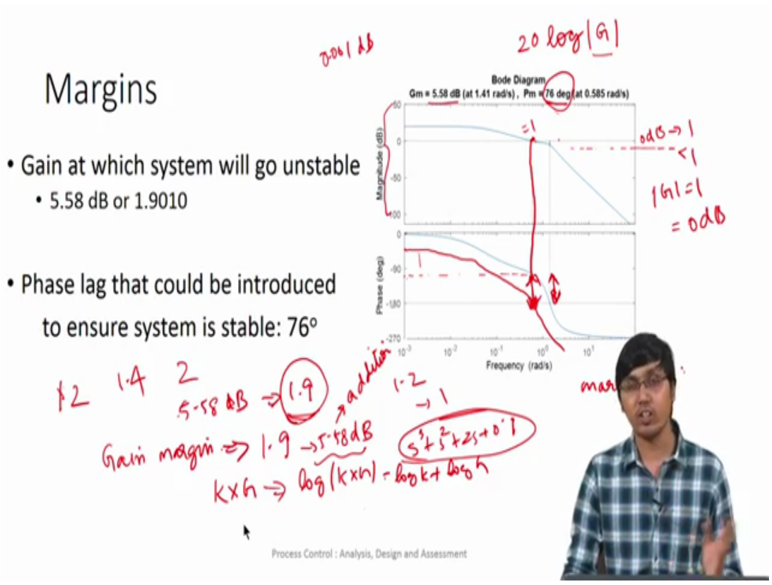

And for the this margin analysis also use open loop transfer function that is something you have remember, so it is not the course of transformation we are using it here. Then I said ok this method is not fool proof it can fail ok so we will see some examples that clearly shows that this method is not going to quality so that is what we will see you know.

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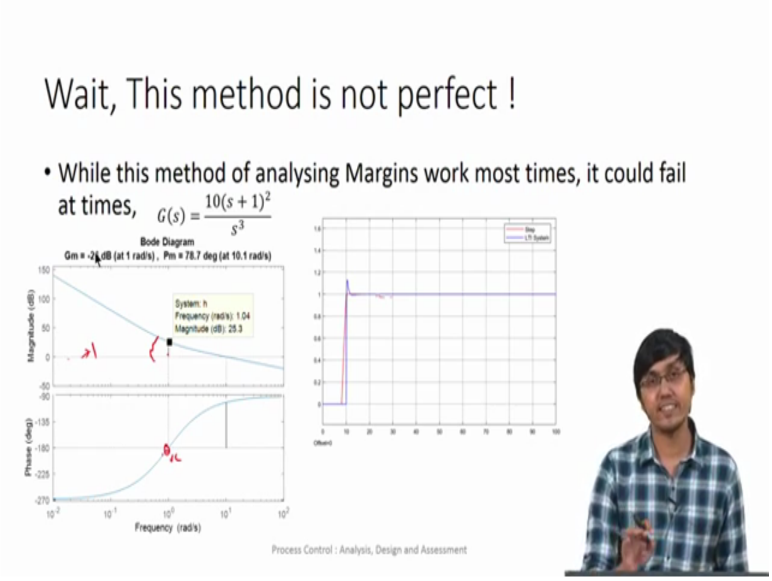

So you take the transfer function 10 s plus 1 the whole square by s cube, so this is the ope loop transfer function again and then you plot the bode diagram and then you see like I change the grain margin is minus 180 degree minus 180 degree dB so it means that like yeah we have already we are already greater than gain is greater than 1 so 0 dB is the line when the gain is 1 and this is the critical frequency, critical frequencies is when we have minus 180 degree phase shift.

So basically after the at this critical frequency the gain is going to be greater than or equal to 1 so by Barkhausen condition basically it means that this term should be oscillating right but let us see what happens if you give a step input system is no (lon) it is not oscillating it, it actually goes and status here. So this method is not fool proof this method is actually will fail in certain special cases or and but this is still working for a lot of (sys) so what is the exact problem and if you can see here after this there are certain whether another example that I can show you but most of the times most other systems with this works fine this gain margin and phase margin constants works fine but not all the time.

So here you can see still it is gain is greater than 1 dB is (min) gain margin is negative remember this means that it is unstable but you can see the system is stable yeah.

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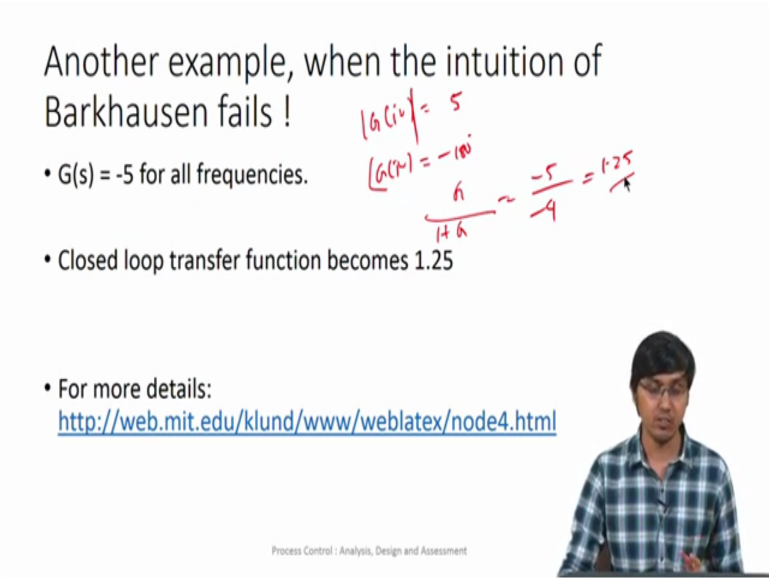

So another example is like if you take the G of s equal to minus 5 then what happens is basically the magnitude of G of j omega is equal to minus 5 for all the angle and then the angle of minus G of j omega is nothing but minus 180 degree right because minus sign contributes to the minus 180 degree phase shift phase lag. So basically this means that the modulus is first always so modulus is 5 and this also again kind of pushes has to intuitively feel ok this system is going to be unstable but if you can write the closed loop transfer function which I am G of s by 1 plus G of s what happens it is nothing but minus 5 by minus 4 is that 1 point 25, so this is a stable let us a closed loop.

Now if you give a step input it the output is going to be 1 point 25 times of the input that is what it is a stable system. So this is having some problem so this gain margin and phase margin thing and bode diagrams all these things help us to understand intuitively like what exactly happens, why these things why he can go unstable but it is not always true it is sometimes it sometimes it fails ok that is something that you have to keep in mind.

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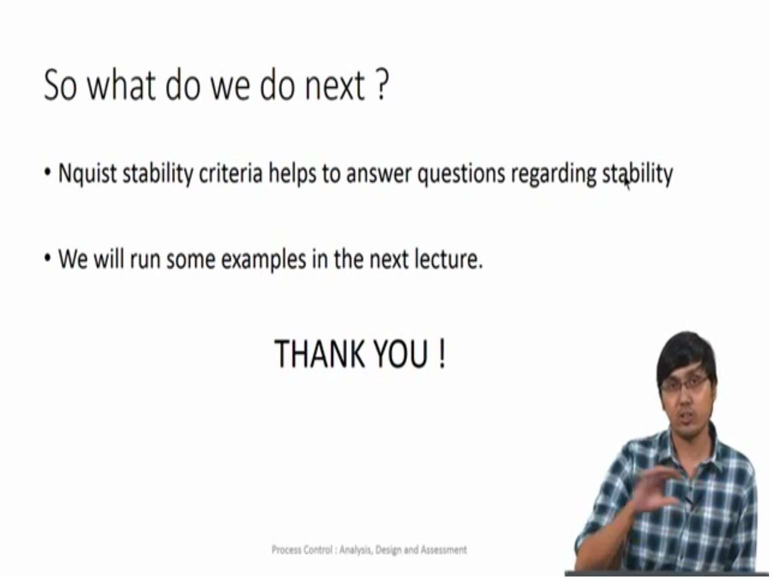

So now what we do? We go to the next concept called an Nquist stability criteria which we will do it in a separate index, so how that even in the lectures by professor you have a it is explain the Nquist ability in a detailed manner, so we will and learn through some examples of Nquist stability and then you will see whether it can answer all these questions of stability and it is can whether it can handle all these problems or phase pair, bode plot etcetera.

So that is what we are going to we are planning for the next lecture, thank you and I also like a request to play with you take some transfer function and then you play with it you draw the bode diagram of it and then you use mat lab to find gain margin, phase margin you do the root locus of it, you call the problem by like clicking close loop transformation function and seeing the poles of the closed loop transfer function.

So you play around all these things, all these three techniques should have the same answer, so that the answer should not change based on the technique you follow so and then you try it out yourself and you will actually start enjoying all these things so that is what I asked you to do it, is a small request and yeah, thank you.