Process Control - Design, Analysis and Assessment
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Stability Analysis–Various methods -Part 1

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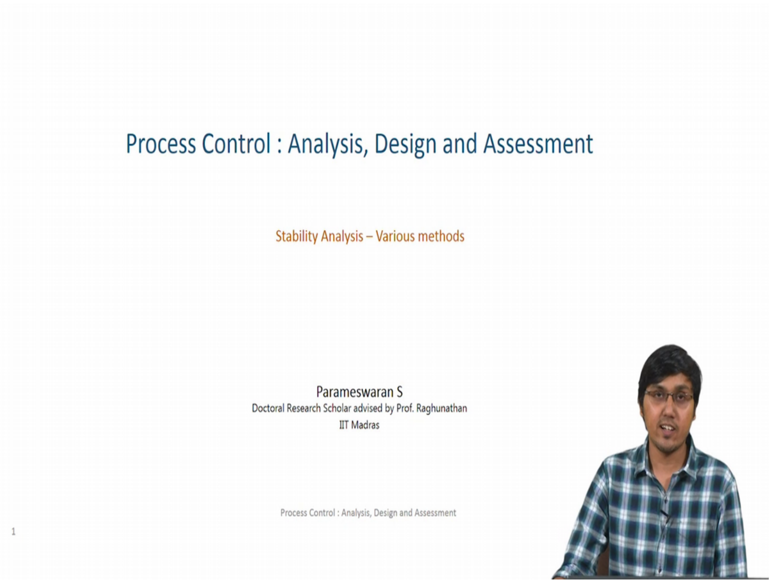

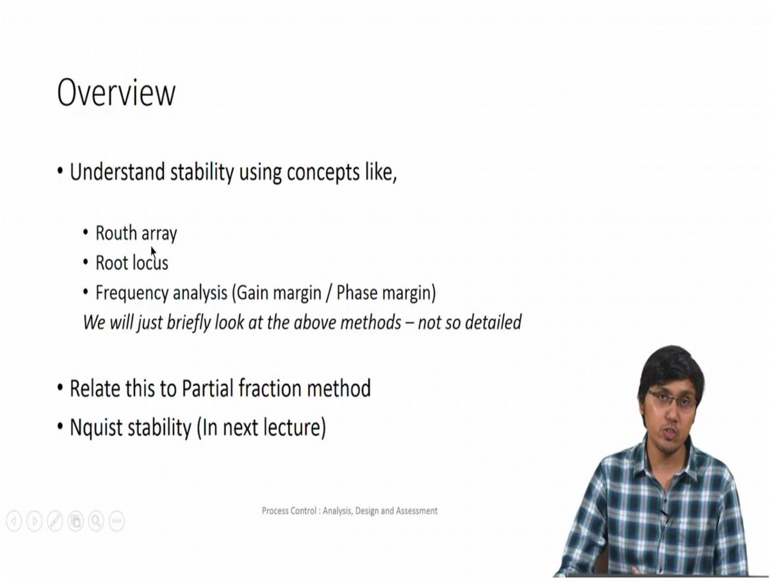

Welcome to the next lecture on Process Control - Design, Analysis and Assessment, so today we are going to look at various methods that are used for stability analysis, so here are some of the methods that are already used for stability analysis one is Routh array other is a root locus and frequency analysis (())(0:31) and then we will briefly look at these methods will not go into much detail into these methods and then we will also understand… see everything is like stability analysis is of the same system, so all these things are connected to each other so that is what we are trying to look at it in this and then in the next lecture we will also see some examples of Nquist stability.

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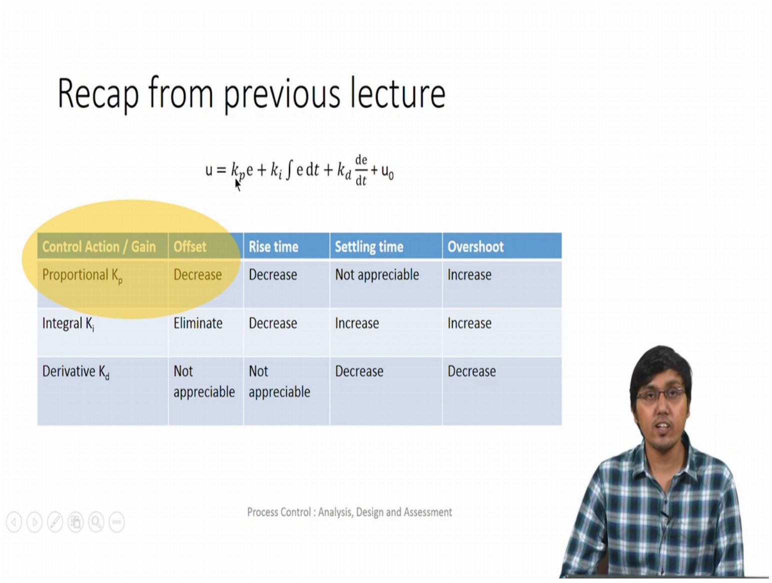

So from the lecture on like what are the efforts of PID gains on the controllers response we were able to observe this table, so if you look at this stable one thing that is of interest is like from proportional controller there is an offset that is (())(1:18) and if there is a change in set point and then as an when we increase the value with a proportional gain kp the offset decreases, so on natural question we might ask is like why cannot I increase this kp? Why cannot we keep on increasing in kp and is there any problem associated with this.

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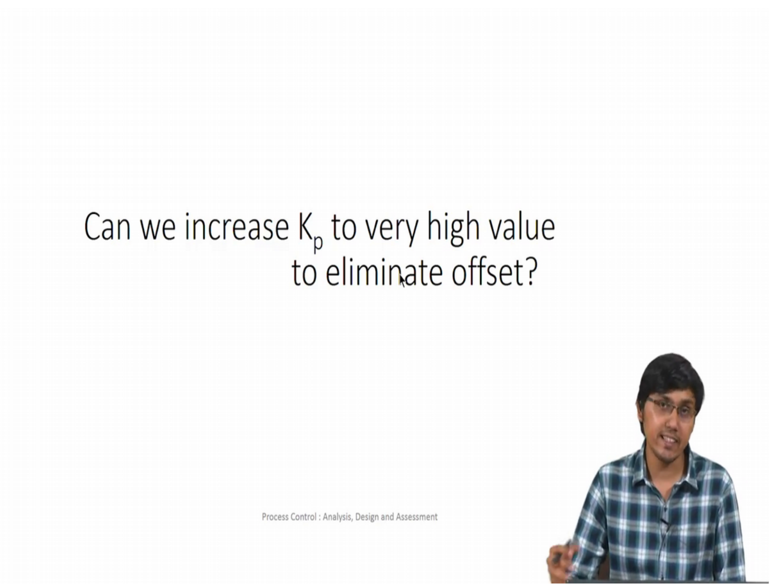

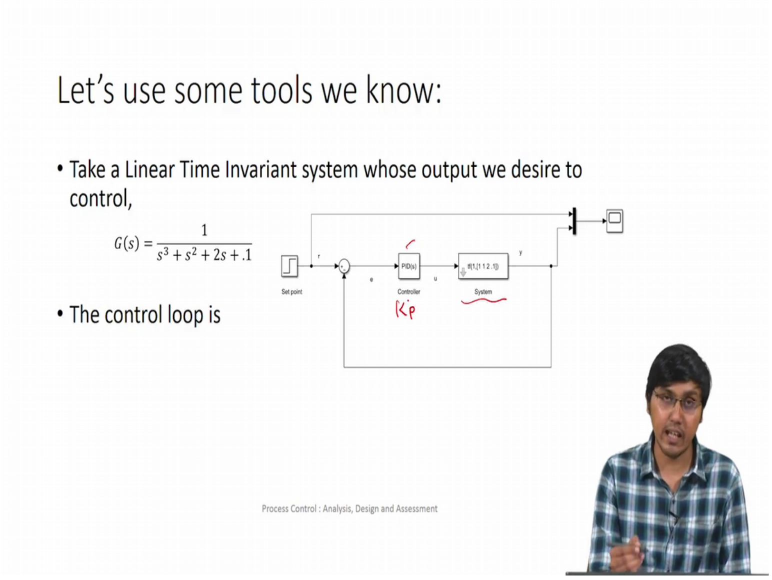

So that is what we are trying to ask the question and let us actually do this experiment again like we did in previous lecture and then we will see what exactly happens and what problems it is introducing. So now we take a linear time invariant system which is of this form Gs equal to 1 by s cube plus s where plus 2s plus 0.1.

You can take any other system is just for illustration purposes I am taking this particular system and this is the Matlab (())(2:09) this is the control loop basically we are having a set point here and we are trying to change set point by step change and then this is the control loop where this is the controller and this is the system. System is nothing but 1 by s cube plus s square plus 2s plus 0.1 and we are going to see the value of the control variable in the scope and what we are trying to do is basically we are going to do server response by changing the set point of the control variable is changing whether it is following the set points change or how it is behaving is what we are going to see.

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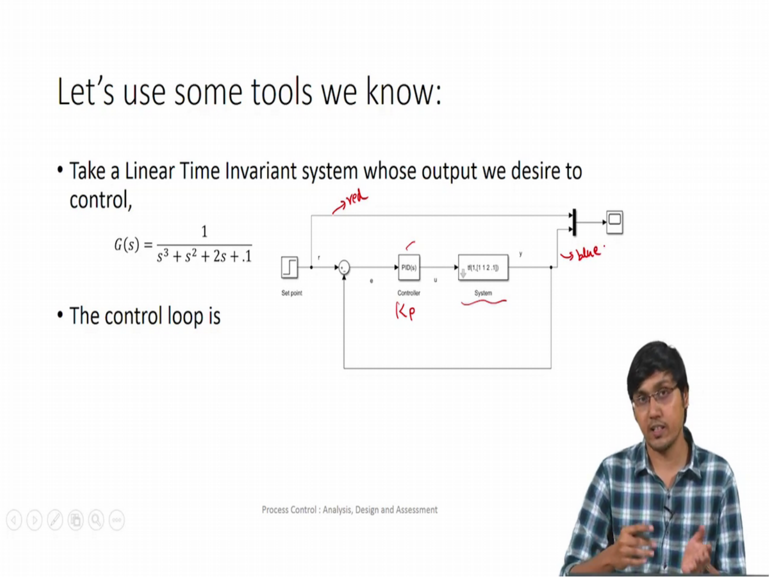

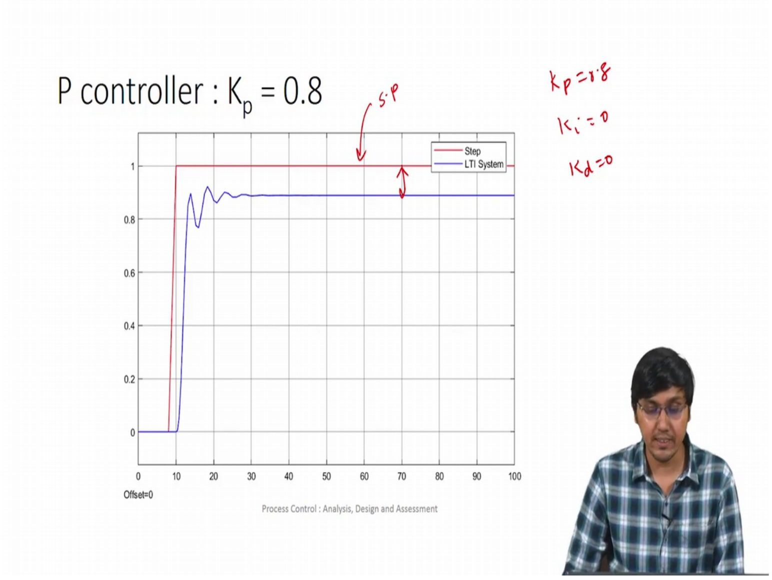

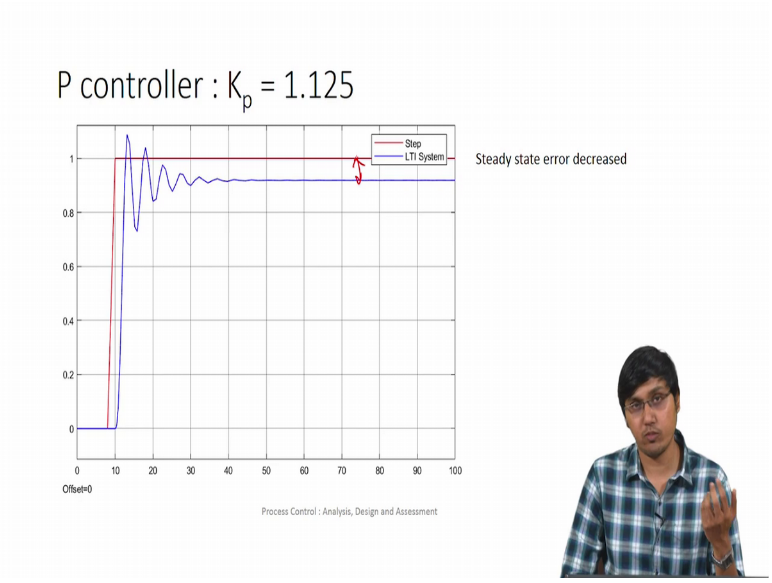

So when we have this controller p controller that is only Kp and Kn Kd equal to 0, the proportional gain is 0.8 here, integral gain is 0 here and the diameter gain is also 0 here so it is a p controller, so basically when you give step input so this red line is the input basically it is how we want the control variable to change it is set point and this is how the control variable is actually behaving, so basically we can go back the previous slide and see this is the red line and this is the blue line how the response is basically.

So if you want to related to your physical system basically if I say I want to change the flow for example I want to increase the flow by a step for example if I want to go from say a flow value of 3 meter cube per second to 5 meter cube per second that is how I want the change to happened that it is changing like this something like that lower temperature or anything that you can think of this.

Again we asked the question I want to decrease the offset so we can see clearly there is an offset here, so I wanted the flow to go to 1 but it has gone only to up to 0.9 or something so basically I want to decrease this and we are saying that by increasing the value of Kp we can actually decrease this. So evidently we can see this value as decreased when you compared to the previous thing and now again we have increased KP from 0.8 here and to 1.125 here and we are seeing the decrease in the offset, so again why do not we increase it little more.

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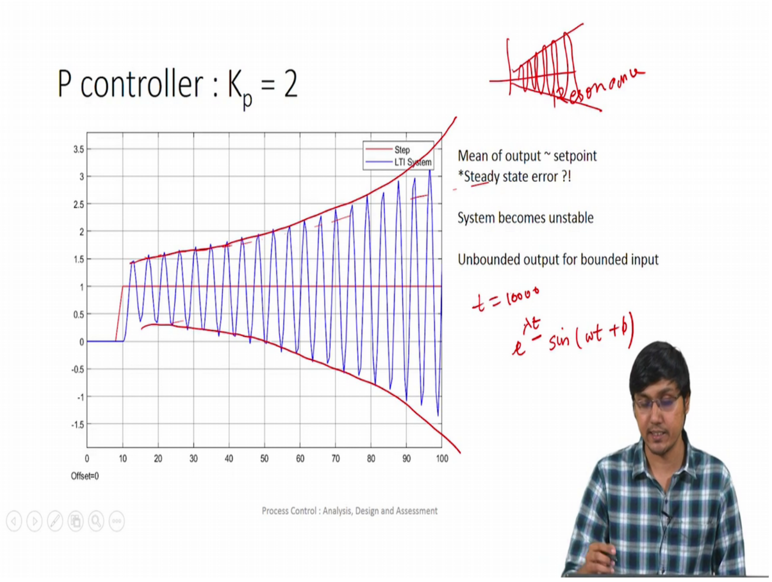

So let us make it as 2, Kp equal to 2 and then we see the response here, so interestingly what happens here this… the system goes unstable, why it goes unstable because as time increases it keeps oscillating and the amplitude of oscillation is going to increase, so basically if I can ask you time, time equal to say 10,000 what will be the attitude?

It may be definitely in terms of like some 100s or even some thousands and another thing we can see here is this is not straight line this is like an exponential increase, see if it was a linear increase if you could recollect the saw the linear increase in the amplitude when we have the concept of resonance and in that what happened we were actually having increasing amplitude even than when there were poles on the imaginary axis and when we (())(5:37) system…input of the same frequency then this was actually a straight line but now you can see that it is not a straight line which you can see here…we will just see another value of Kp and then you can confirm that it is exponentially, even here you can see clearly it is an exponential increase not (())(5:57) you cannot keep it as a straight line here.

If you have drawn a straight line you would have come like this but still it is exponential increase and another thing…now what happened to our (())(6:08) like okay increase in Kp decreases the offset what happened to the conservation. Here still we can observe that if you could see here the mean of oscillations, if you can take the mean of this oscillation it will be closer to 1 or even it could be like equal to 1.

So the mean of oscillations is still like getting closer and closer to set point value but what happens is output keeps on increasing the system goes unstable, so this if you could actually see looking at the positive fashion way of analysis basically what it means is we have introduced a pole on the right-hand side of the…so basically if you can observe from the partial fraction theme what you can say is we have introduced some term like e power lambda t into a sin wave into sin Omega t plus some 5 or something and then this lambda is actually like a positive number so amplitude of the sin wave is getting increasing with respect to the time, when time increases the amplitude also get increased so that is what we observed here.

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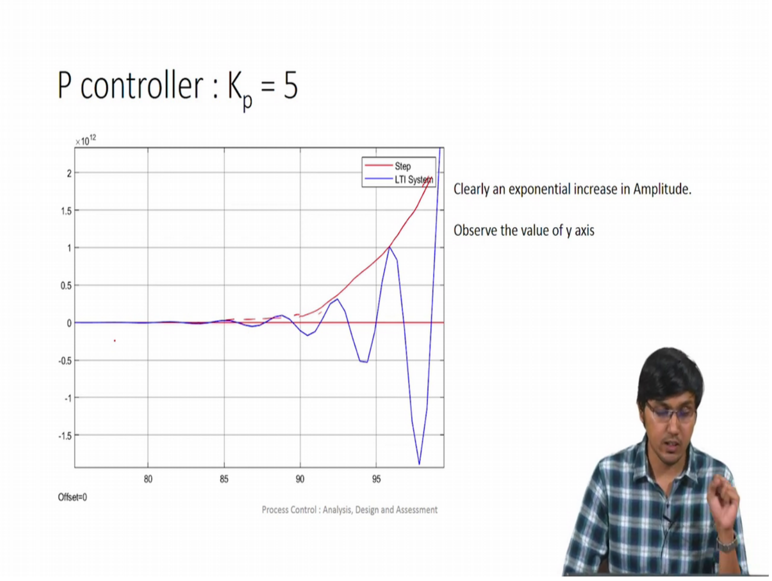

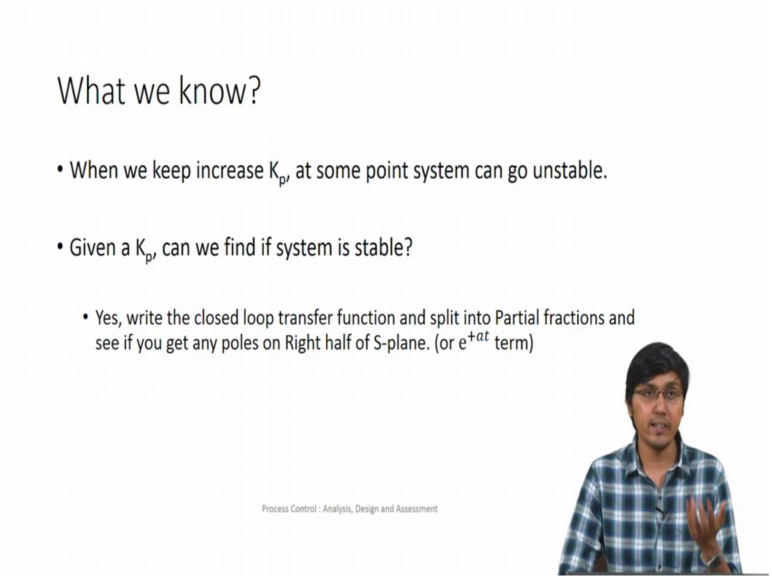

And now again like to (())(7:14) that it is an exponential increase let us take Kp equal to 5, so basically now you can see this…you see the value of y axis it is 10 power 12 so basically this is like in order of 10 power 12 and all these are like… so basically you can see as time increases the values not increasing by equal amount it is pleasing like exponentially that is what you can clearly observe from this particular graph and next we will ask the question like okay when we increase the Kp at some point it goes on stable so the question you can ask is like okay how do we find this value of Kp at which the system can go unstable.

So that is what we are going to ask and that is before that question if we are given a gain Kp can you determining whether the system is stable or not there is another question right so first question we will ask is you give me a Kp whether I can find whether the system is stable or not so if I move a Kp and what I am going to do? I am going to write the closed loop transfer function and then I can actually split in to partial fraction and then see if there is a pole on the right-hand side of the S-plane, so that if there is no pole on the right-hand side of the S-plane then it basically means that there is no problem of stability the system is stable system.

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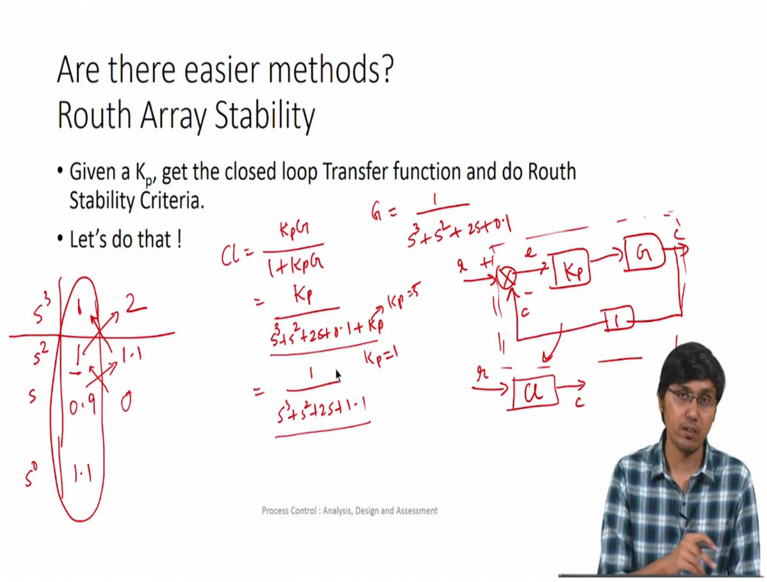

Instead of doing partial fraction, instead of finding all the poles for a given Kp is there any other easier way? So we can use this Routh Array Stability, so in routh array stability what we can do is we can take a system basically we will say G of s equal to some s cube plus s square plus 2s plus 0.1, so this is what we are going to take and then here what we are going to do is let us take the control loop as we are giving a set point here and then we are giving to the controller this is a P controller for now and then this is given to the system, control as output is given to the system and then this is how a sensor is measuring the value of G of s here so this is sensor value and let us take the sensor as unity transfer function, this sensor is actually just measuring it and sensor has no lag et cetera.

So if sensor has some lag we are going to introduce the sensor function also here but let us take one off sensor as unity function and then we will actually feedback the reading by the sensor back to the…and subtract it from the set point so basically this is c and this is the error, error is nothing but set point minus the value of the control variable as measured by the sensor and then now this is a proportional controller so we have only the Kp here, so now we can easily write the close looped transfer function, let us say C or okay let us say the closed loop CL equal to Kp into G by 1 plus Kp into G so this is what he closed loop transfer function is, so now if we can find the… What will be the close looped transfer function?

It turns out to be it will be like Kp by s cube plus s square plus 2s plus 0.1 plus Kp. So now let us take this Kp to be some value, so the question asked here is given a Kp can (())(10:38) resolving into partial fraction. Can we find (())(10:41) stable or not, so that is what the question we asked? So when we say system we can think like this is the full system, so now you have a system which is a black box you have a set point to system and the measure you are seeing the output which is the value of the controlled variable so basically I am having a black box with the controller.

So I do the input to the black box is set point and I observe like how the output of the black box is varying that is what we are going to…see this is what like if you can see if you can imagine once we implement all the (())(11:17) what will basically have is we want to change the input and then we want to see the output is following the input or not. Inside of it what happens to the control and how the control is performing all these things are…can be like we will not focus much on that if your (())(11:33) is out of control or et cetera so basically you can imagine this particular box has become this back box which is the control loop or something so this is what we are going to see now this is the close loop transfer function.

So this becomes cl, cl blog is nothing but (())(11:47) the input as the z mod and it gives output as the controlled variable itself so now let us take for now Kp equal to random (()) (11:55) we will take one value we will say one is a good number, so we will take one as Kp value and then now let us find out whether this system is stable or not, so to find whether the system is stable or not I am going to substitute the value of Kp here and I am more concerned about denominator, so basically becomes 1 by s cube plus s square plus 2s plus 1.1.

In the Routh array table you write s cube, s square, s and s power 0 which is 1right, so basically we take all the terms of s cube one is here so coefficient of s cube is one and coefficient of s is 2 and coefficient of s square is 1 here and coefficient of s power 0…1 is like 1.1, constant term is 1.1, so now the 1st term here we are going to find this term by cross multiplying this 2 minus cross multiplying this, so it is 1.1 divided by this term so 1, so let us put this down here okay and so what is the value of this 2 minus 1.1 by 1 which is nothing but 0.9, so 0.9.

Basically again this is nothing here there is no term to fill because there is no other terms left out here, so we will take it as 0, so now basically we will cross multiply this and subtract with this, so subtraction is basically 0, so basically what we get is 0.9 into 1.1 divided 0.9, so that basically becomes 1.1, so this becomes 1.1, so all the terms here are positive there is no signs change basically and that is not even a 0 here, so basically all the terms here are positive so which means that system is stable okay, so when all the terms are…or if there is no sign change in this column then basically we say system is stable.

So definitely you can say system is stable, so now as an exercise for you, you take the same system but just substitute Kp equal to 5 and then see whether sign change here and you can see that basically you can try it for yourself and also like try to find out whether suggestion between the number of sign changes and the number of poles basically you can dissolve into partial fraction again and then check out this, so that will be an interesting phase for (()) (14:32) so maybe you can do it for yourself. So basically what you do, you count the number of sign changes here and then seeing whether… and resolve this particular term into partial fraction and see whether there is some relationship between the number of poles on the right-hand side of the plane and the number of sign changes here.

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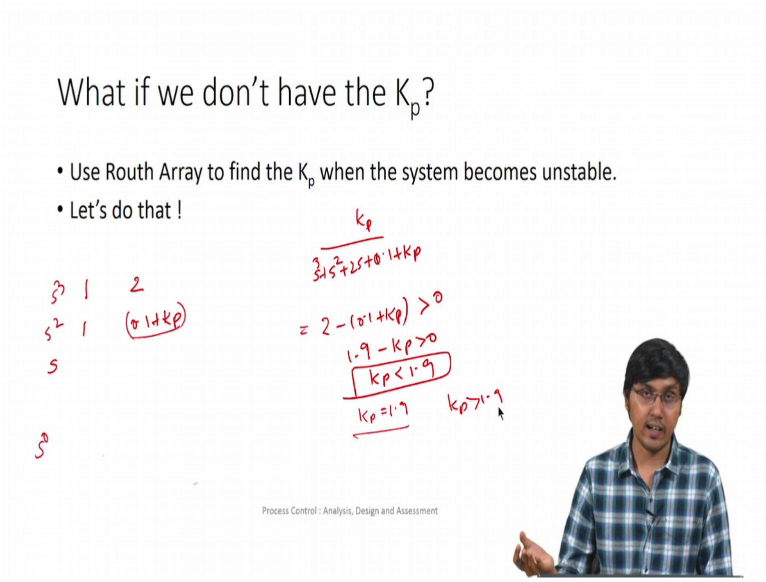

So what if we did not have the value of Kp? That is what you are going to ask now, so the question may be to put it in a different way I want to ask like what will be the value of Kp at which system will go and stable, so in the lecture of the (())(15:13) I told what will be the ultimate gain that is what you ask yourself that is same question you going to ask here, so the same exercise we are going to do basically we had Kp by s cube plus s square plus, 2s plus 0.1 plus Kp, so this was the closed loop transfer function that you got and then we write the s cube, s square, s and s power 0.

So this we add 1 and 2 here and this we add 1 and 0.1 plus Kp and now you put like 2 minus 0.1 plus Kp by 1, so we were calculating s power 0 itself we can say that if this is positive and it is definitely positive because Kp is again, so this term is obviously positive because what you are going to get here basically is 0.1 plus Kp into 2 minus 0.1 plus Kp by 2 minus 0.1 plus Kp that is what we are going to get here, so basically if you can see this gets cancelled whatever be the thing here this gets cancelled and this will be 0.1 plus Kp and provided 2 minus 0.1 plus Kp is not zero okay now you can cancel this.

So let us not bother much about this particular term for now, so the term which is of interest towards is basically 2 minus 0.1 plus Kp and what you can tell about this is if this is positive then system is stable right because we know if this is positive this also s naught term also becomes s power 0 term also is positive because this is a positive term (())(17:07) so this gets cancelled and 0.1 plus Kp is always positive because Kp is again it is a positive number and also one thing to remember is we are not trying to put the negative gain here since it is a p controller and let us (())(17:23) Kp only as positive values.

So which is not that you cannot put Kp as a negative value we can put but for this case or for p controller we do not normally put Kp as a negative value so basically we will put Kp as a positive number, so basically now we can take like this particular term, so this particular term is only term of interest, so for system to be stable this term has to be greater than 0 okay so this become like 1.9 minus Kp greater than 0 which means Kp is less than 1.9, so 1st conclusion we can draw is whenever Kp is less than 1.9 this term is going to be, this particular system is going to be stable that is a straightforward thing we are going to see.

If Kp is less than 1.9 this term is positive and this term is also positive okay so that is what we have seen in 1st case okay so when Kp is equal to 1.9 what basically happens is we will see that the poles are on the imaginary axis and we will see that later also, so basically now we have found value of Kp when the system will go from being stable to an unstable that is a point…it is like as the sir told in his lecture it is like the cliff (())(18:47), so Kp equal to 1 point is a gain at which system transition from stable system to an unstable system, so anything Kp greater than 1.9 it is going to go to unstable system.

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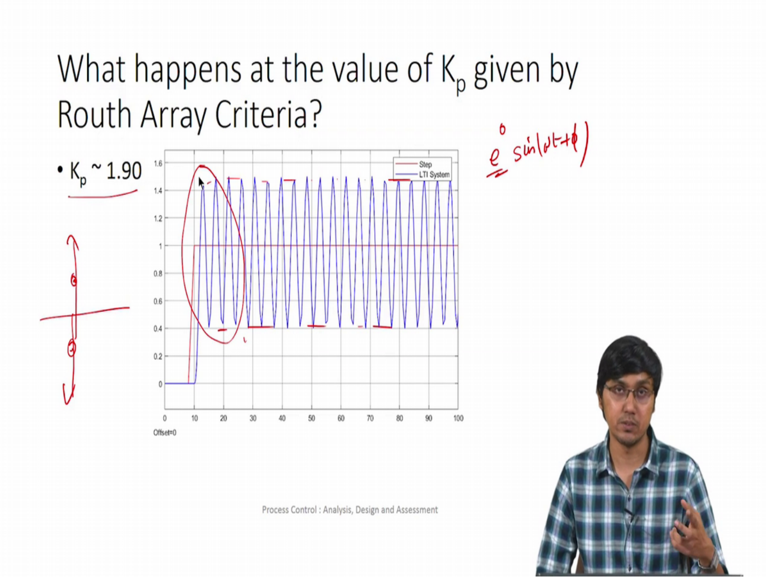

So now let us put this value of Kp inter system into the control loop and similarly in Matlab and see what is the response, so this is how you get the response, so basically you get a sustained oscillations so either an increasing oscillation or a decreasing oscillations so you get an sustained oscillations which means that when Kp is equal to 1.9 this particular pole is on the imaginary axis, so we have…so complex poles always have a current complex conjugate so basically you have 2 poles and this is on the imaginary axis.

So once it is on the imaginary axis if you take the partial fraction and then you take the inverse it becomes like basically e power 0 into some sin omega t plus phi so basically this is a constant 1 so basically the amplitude is the constant 1. Also remember this initial transient we neglect okay whenever we talk about frequency response the initial transient we neglect this may be due to other poles. This is not the only pole you have in system it could be due to other poles (())(20:12).

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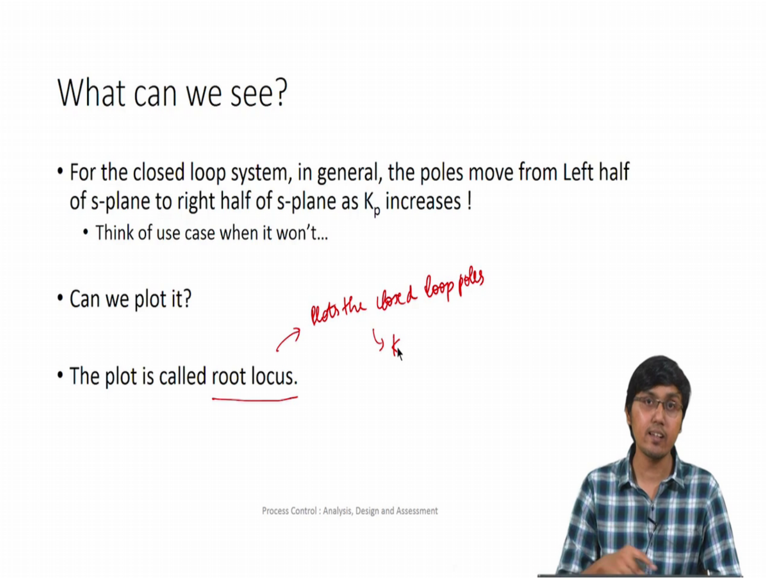

So what are the observations we can make from this when we increase the again a stable system became an unstable system which means that the pole has moved from left of s plane to right of s plane okay, so this makes us ask one question right so if increase the gain and the closed loop poles are moving from left to the right can be plot and then see like how these poles are moving, so that can be an interesting question to ask, so then we plot the poles which is moving from left of s plane to right of s plane that is called root locus.

So root locus what it basically does is it plots the closed loop poles okay because when all the closed loop poles was on the left of s plane the system was stable, when at least one pole moved to the right of s plane then what happens basically is like system become unstable, so it plots the moment of closed loop poles in some parameters, some parameters in the sense we are interested in the gain, when gain in the loop increases then all the (())(21:33) are moving as was the root locus is going to give us.

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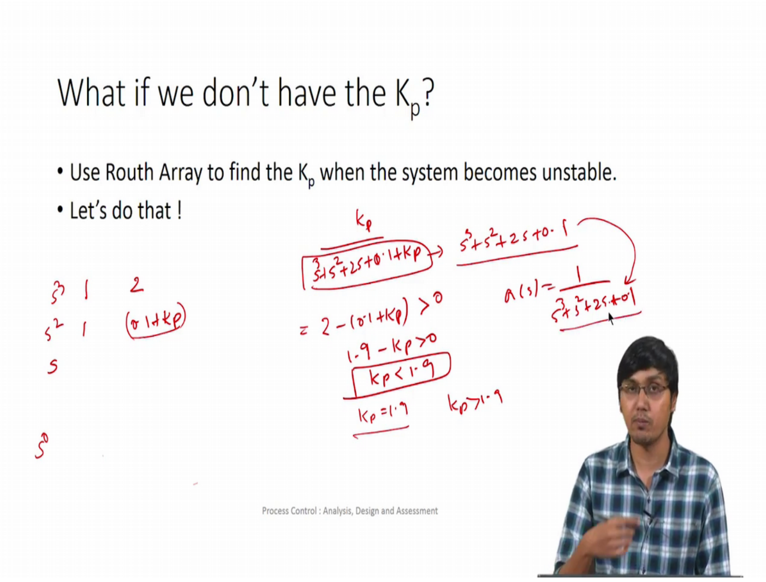

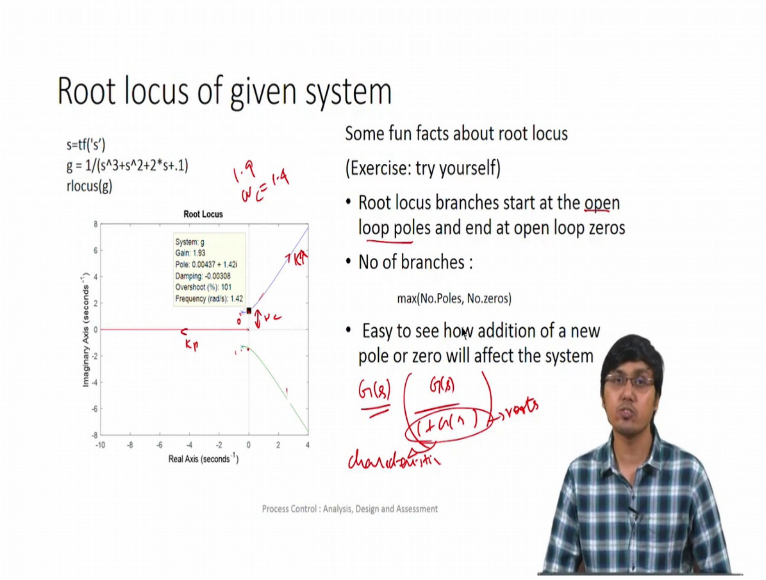

So will not go into detail of how to construct these root locus et cetera, you can use Matlab to plot this and then basically we will see how it is root locus looks for the system that we are actually interested for, so if you can see here this is the 3rd rod system, so basically it has 3 poles and one pole is here, one pole is here and here another pole is there and then if you can see what are these values, these are all nothing but the poles of the open loop system this is the open loop system.

So when we say open loop system it is nothing but the G of s and the closed loop system means when we close the loop and then we found that G of s by 1 plus G of s right so this is the closed loop system okay so now this trajectory of these lines mark how these close loop poles are changing so basically if you can resolve this denominator into factions and find the roots of the denominator, so only it is called the root locus how the root move. If you can find the roots of this denominator and recollect the denominator is also called characteristic polynomial.

So is called the characteristic polynomial so the roots of the characteristic polynomial that is what it is going to show and with respect to what? With respect to how the change in the gain, so basically how…if you change the gain how it is going to vary, so when gain is equal to 0 it starts and then it goes like how gain increases (())(23:21), so it is when gain increases and this also when gain increases, so clearly we can see that when gain increases and here it moves like this basically, so for these 2 particular poles what we can see is basically whenever the gain increases okay the poles are moving towards the right of a phase plain and that is a particular point, particular gain value whereas the particular point and the particular again value wherein a pole is actually meeting the imaginary axis and at that particular point if you can see here approximately the gain here is 1.9 okay.

So this is not exactly on imaginary axis as you can see it is somewhat tilted towards the like 0.00437 so basically if you can plot exactly at this imaginary axis if you can mark this pointer it would basically say the gain is 1.9 and the frequency and the omega value the y axis value is the frequency right and that y axis value is nothing but 1.4 something okay, so this is what the root locus say so basically it can show you how the poles are moving and so this gives us 2 things one is the frequency at which the poles are crossing the imaginary axis this is the y value at which (())(24:55) and then the gain at which it is going to go from the left of s plane to the right of s plane.

So this is a root locus thing and there are something that you can try for yourself this is mostly taught in like a standard control course but let us not focus on all these things much because we are…see when we read with whatever content that is been taken in this course without even root locus you can actually solve the problem, so this is just another method that you can just know that is what so we are not going to go into details and are we going to calculate all this or are we going to plot all these values but something like to just to have fun with root locus you can just observe certain things basically.

So root locus R is at open loop poles and (())(25:47) open loop 0. So why it starts at open loop poles basically if you go to the previous slide and then we can easily see right when Kp equals to 0 so this was denominator right, so when Kp equal to 0 this becomes same as the open loop denominator, so we have open loop system G of s is nothing but 1 by s cube plus s square plus 2s plus 0.1, so basically now what happens even Kp is becoming equal to 0 and this becomes same as closed loops transfer functions denominator or characteristic polynomial is becoming the same as open loop transfer function.

So when Kn equal to 0 the closed loops polynomial or closed loop poles is same as the open loop poles because the characteristic polynomial is the same, so the poles are going to be the same so it starts at the open loop poles and ends at the open loops zeros and then the maximum number of brands is of root locus is nothing but maximum number of poles as 0 because you can easily see from this thing because every root locus starts from open loop poles and ends at open loop 0, so basically like how many number of open loop poles and 0 should be there, how many balance should be there basically t its maximum either a maximum of poles are 0, so whichever is maximum that many (())(27:15) we are going to have okay and one thing we can do with root locus is what if we had a 0 and what if we had a pole that is what we are going to see.

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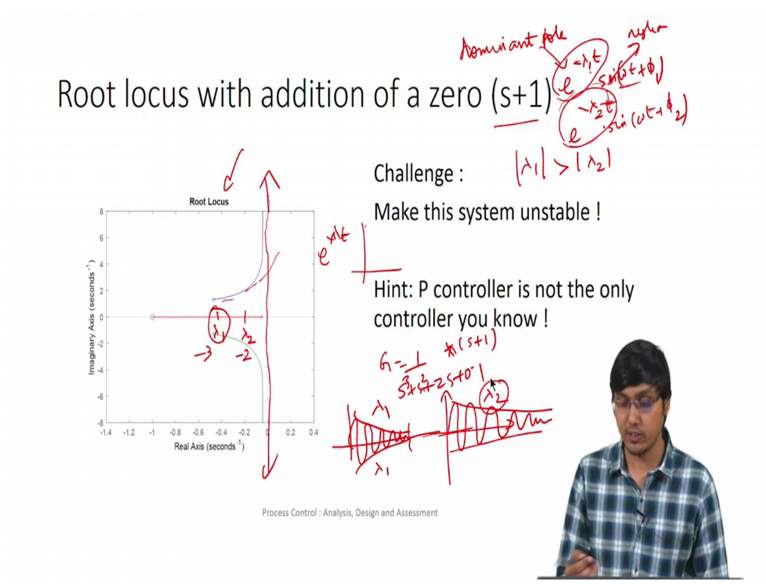

What we can basically do we had the G of s is nothing but 1 plus s cube plus s square plus 2s plus 0.1 is what we had, so what if we multiply this by s plus 1 so and then why does it effect of the addition of this particular pole is going down on the root locus thing, so now something interesting happens right so basically what happens this is the imaginary axis, so this is the imaginary axis, either multiply by a zero this root locus got shifted towards the like now we can see there are no poles on the right-hand side of the s plain so whatever be the gain value you increase this particular system is not going to go to unstable.

So now one challenge is like do something with the system so that you can make this, for some gain value you can make the system unstable. Now you can add a pole or you can add a 0 or you can modify instead of putting a P controller you can try to put some other controller you may know and then you can try to see like whether at some point some pole is going to cross the (())(28:37), so only when this particular branch is going to cross into the right-hand of his plane that is when it is going to go to unstable.

So basically what you want to do is you change the system by adding certain poles or certain zeros or change in the controller structure from P2PA or something and then see whether for something it goes into the right-hand side for some particular gain value, so that is what you can do it for yourself just to have fun with this but this slide what we are trying to say is this root locus can be used to analyse like what is the effect of addition of poles addition of zeros and one more thing that is normally like when we talk about this poles and zeros we talk something called a dominant pole, so what is this dominant pole?

See everything you can easily visualise when you do this partial fraction, so we are emphasizing the partial fraction thing to understand (())(29:34) of control because like mostly we can actually find out most of the concepts easier when we imagine that way, so basically which pole is going to have a bigger impact on the small (())(29:53) which pole is going to have bigger impact that is what we are going to see dominant pole, so basically let us say we have 2 poles let us say 1 pole is lambda 1 plus say J e power okay we will take this (()) (30:12) finally found this e power minus lambda 1t sin omega t plus phi 1 this was the response of first pole and I am assuming you took the partial fraction and then you took the inverse (())(30:29) and you got this particular thing and at the other pole you are going to have I power minus lambda 2 and sin omega t plus phi 2.

Now things we have assigned it is like complex poles okay so that is what we are going to do, so basically if you can see here if lambda 1 is higher, in the value of lambda 1 is higher than the value of lambda 2 and since I put minus here it is clear that these poles are on the left of s plane, so basically we have say lambda 1 somewhere here and magnitude of lambda 1 is greater than magnitude of lambda 2 basically it means that lambda 2 is here.

So you can see this minus 3 and this is minus 2, so 3 is greater than 2 that is what basically it is going to say, so now if you can see here (())(31:23) minus lambda 1 t and d power minus lambda 2t, so which of this term is going to have a bigger impact, so then when lambda since lambda one is higher what it does is this particular e power minus lambda 1t if you can plot here it is going to fall exponentially at a very higher rate, so at the same time e power minus lambda 1t would be like very less even compared to e power minus lambda 2t because lambda 2 is greater than lambda 1.

Lambda 1 is going to pull it down so fast as time increases it is going to pull the values so fast, so this is going to (())(32:02) fast so when compared to lambda 2, lambda 1 we can neglect so lambda 2 is the dominant pole. You can actually plot this you can take these 2 values you can plot for the same time and then seeing Matlab like how it is responding, so that is one way to do but we can easily see the max or intuition, this particular term is going to falls slower, decrease slower and this particular term going to fall faster because lambda 1 is greater than lambda 2 then what we say this term this pole is not causing a bigger impact in comparison to lambda 2, so lambda 2 is a dominant pole then compared to lambda 1.

So the poles that are closer to the imaginary axis is going to have a bigger impact so the poles that are closer to the imaginary axis is called dominant pole, you can just remember these terms, these terms can just…some (())(33:00) talking about these terms so you should not think like this is something new (())(33:03) it is something like very easy to observe when doing partial fraction. So if you want to understand just take some lambda one value, lambda 2 value and then see the (())(33:11) so that is what you will be able to see.

So again to summarise basically what happens both lambda 1 and lambda 2 are stable on left-hand side of the s plane so both are going to fall down and this is how the lambda 1 pole is going to fall down. For lambda 2 it is going to take much longer time to follow that is what it is going to say. So this lambda 2 will fall down, so lambda 1 is falling faster is getting 0 faster but lambda 2 is taking some time to go to 0, so basically lambda 2 is more significant than lambda 1, so lambda 2 is wrong input that is what we understand.

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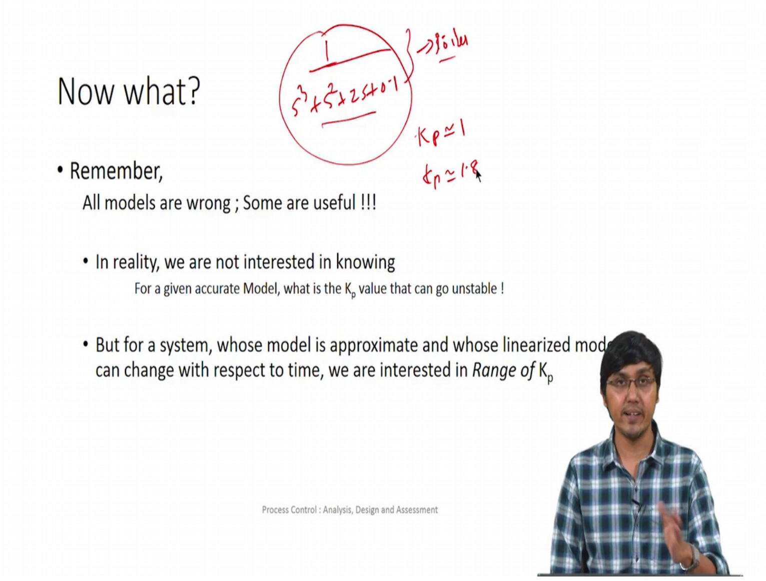

Okay fine now we are going to ask another question, so what we are going to tell is okay given a system, given a transfer function we are able to tell that what value of Kp the system is going to go to unstable but in reality how useful all these things are okay but it is something like given a model which is accurate you can find the exact value of Kp the system will go unstable but we also seen that all models are approximations okay like that is a one phrase that is actually very interesting which is like all models are wrong and some are useful so we are looking at some useful approximation of system as models okay.

So whatever we have as a transfer function like we have a transfer function s cube plus s square plus 2s plus 0.1 if this is going to represent say a boiler okay I am just taking some random word okay boiler okay. If this is going to be a process and this is going to be transfer function then this itself is an approximation, so with this approximation if I find a Kp value we do not know whether that particular value of Kp exactly at which it is going to go to unstable.

All these things does not make sense when this itself we say is an approximation right, so when Kp we can say is like okay Kp is somewhat closer to 1.9 it is going to go to unstable but we do not know exactly at what Kp it will go unstable what system will behave even this is a linear system, this (())(35:35) itself will not be a linear system, so all this that is a lot of approximation, lot of assumptions so in real life we are not interested exactly in finding the exact value of Kp where it goes unstable but we are trying to find out what range of Kp is going to be stable and what range…somewhat like if we can change like if Kp is approximately 1 then we say it is stable, if Kp is closer to say 1.8 then we say it may go to unstable.

So this is qualitative comparison that we are interested not in the so hard quantitative comparison, so this is when we go for the frequency response thing analysis which we can see in future lectures, so basically what we are going to do is like we are going to take the frequency (())(36:31) what we have done, what we have plot, et cetera and then we are going to see how this frequency response analysis is going to get useful in stability analysis that is what we will see the next lecture. Thank you.