Process Control-Design, Analysis and Assessment
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Model Predictive-Discrete Model

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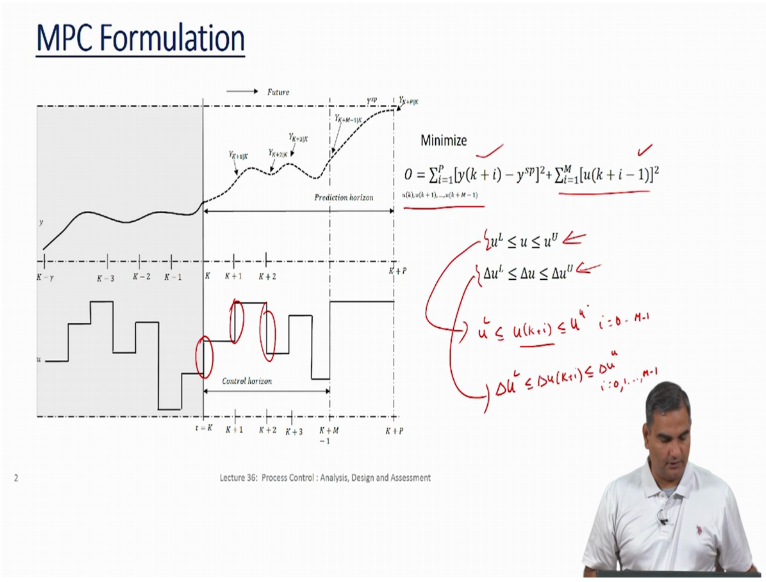

So, we will continue with our lectures on model predictive control. So, in the last lecture I showed you how you formulate the control problem as an optimisation problem. And I left with the question of where is the model in all of this. So, we will address that question here, just to recap, we talked about the optimisation formulation, where I have an objective function, which has 2 terms, so the 1st term has to do with the error in the output, when compared with the setpoint. And this is the term for the control effort that we take while we do this optimisation or the control of the systems, okay.

So, that gives you the objective function definition, the decision variables are UK, K +1, all the way up to K + M -1. And the constraints are on U and Delta U and I said that this constraint, if you expand this, so the way to write this would be to say U of K + i, less than equal to U upper, less than equal to U Lower, where i goes from 1 to M -1. So, this is the formal way in which you write this constraint. So, basically the idea is this constraint is generically written for you but since at every time instant we are talking about the same control input, so the variable that we have defined for UK, UK +1, sorry this is i equal to 0, UK, UK +1, all the way up to UK + M -1.

All of them have to follow the same upper and lower bounds. So, that is what is written here. When i is 0, this becomes UK, when i is 1, this becomes UK +1, when i is 2, this becomes

UK +2 and when i is M -1, it becomes UK + M -1. So, that is how, so this constant is applied on every control moved that we take. Similarly this general constraint on Delta U can also be expanded like this, Delta UL less than equal to Delta U, K + i lead than equal to Delta U upper and again i equal to 0, all the way up to N -1.

So, basically what this says is when i is 0, Delta UK which is the difference here should be between the range, the difference here also should be between the range, the difference here should be between the range and so on. So, this becomes let us say M -1 constraints, sorry, M constraints here and M constraints here. So, there are 2M constraints here that you have here. So, this is the standard optimisation formulation that we talked about.

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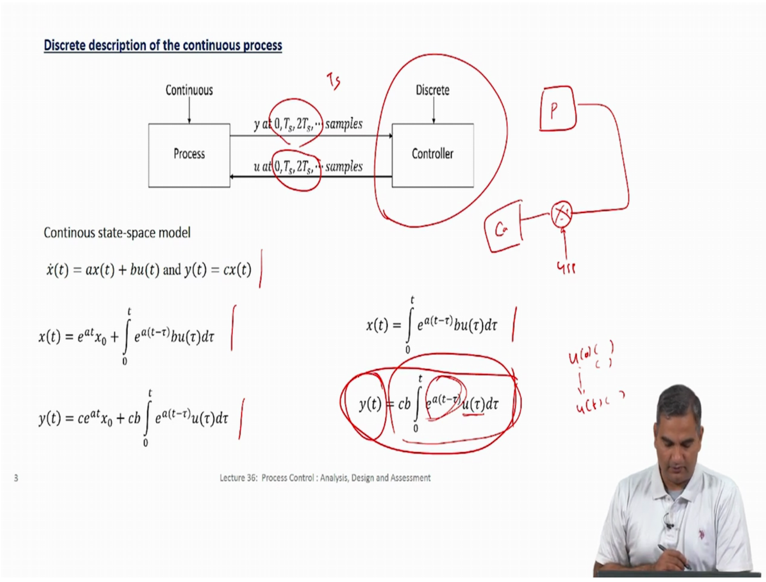

Now, coming back to how are we going to relate the y's to U's is the question that we have. Basically, now that we have talked about the whole controller itself as a discrete controller, the model predictive control, so this is a discrete block. Whereas true process is really a continuous process. However, what we are saying is we are going to basically take samples, different sampling time. So, 1st sample time is equal to 0, 2nd sample TS, 2 TS and so on. So, basically indiscreet, in a continuous controller I have this process which gives me Y values, which I compared with the setpoint, right and I sent it to a controller.

So, this continuous variable value is going into a controller, whereas here the process is a still continuous and what we are saying is we are only looking at the values are discrete time intervals so 0, TS, 2 TS, 3 TS, that is a sampling time is TS. And in the continuous case, then the controller went back and said it is going to manipulate some U and that is going to act on

P, that basically said that U is acting at all times. So, I keep changing U continuously, whereas in this case what we are saying is the way we formulated the model predictive control problem, if you had seen in the last lecture, we basically looked that making the control moves only at those sampling times.

So, the idea was if I were sitting at K, I will make a control move at K, then K +1, K +2, all the way up to K + M -1. So, the control moves are also made at those sampling times and the measurements also come in at those sampling times. However, since the process is continuous, we have done this before, if you go back and looked that the lecture where I talked about solving continuous state space models, you will see that I would have got the solution like this. And then you can get Y of T like this.

So, this is simply a repeat of what we have seen before for a state space solution. So, you can go back to these lectures and then look at the solution to the state space models and you will get this. And if we assume deviation variable X0 is 0, they are basically what you are going to get is XT is this and YT is this. Now, since we are talking about the output directly, so we are focusing on this equation here. Now if you notice, what this equation says is that the Y at anytime is going to be this integral right here.

And if you notice, this Tao goes from 0 to T, so U Tao, that means U0 to UT, all the values of U are taken into account and they are somehow weighted by some function to get me an output value. So, this is the key insight that we are going to derive from this equation. So, let me repeat this. So, what this says, if we look at this integral and if you think of kind of integration and then some, then you will see, says Tao goes from 0 to T, U Tao that it is going from 0 to T, this term.

And every time if you think of this as a sum, U of 0 is multiplied by some number here. U of, next U will be multiplied by some number here and so on. So, if you think of this is a continuous fashion, every U is multiplied by some number and then integration if you think of this as a summation, then all of these affects are added up to give you what the value of Y of T is. Okay, so this is an potted idea to start thinking about. Now, from a model predictive control viewpoint, what we are saying is, we are saying that this output now I can directly write as a function of these inputs, right.

So basically remember the question we asked, if I have a model plan, sorry, control input plan, how would that relate to the actual output values and that is related through a model like this. The only thing is this model is an continuous domain, whereas what we are interested in is deriving the model in the discrete domain because most of the formulation and the solution all in the discrete domain.

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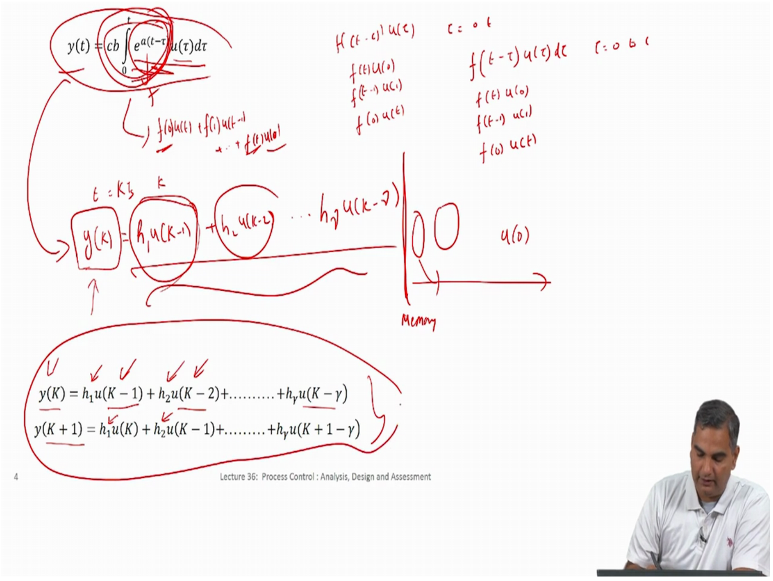

Now let us think about this as a sum and then let us look at these terms in a little more detail. So, if you look at this here, right here, so if you look at this term here, so you see this term is the function of T - Tao, okay. And this T - Tao, whatever function that is is multiplying U of Tao as Tao goes from 0 to T. So, in other words, just if we substitute values of Tao as difference. So, if I substitute tao equal to 0, then basically this is a function of T times U of 0, right.

If I substitute tao equal to 1, I get some function T - 1, U of 1 and if I substitute tao equal to T, I will get F of 0, U of T, right. So, if you think of this as a function multiplying U of Tao, this is being done continuously in our regular continuous model. But just to understand this, all of this can be done very formerly mathematically but in the 1st control class I am going to show you why and how we go from a continuous model to a discrete model in a very simple manner, appealing to intuition, there is no hand waving here, one can actually show how this will come about with concepts such as 0 order hold and actually doing this integration and showing that this is actually the form that you will get.

But that is little too much for this course right here, so I am simply going to say how this happens and give you an intuitive feel for how these discrete models are derived. So, basically where we are going is from this continuous model, we basically want to go to the discrete form. Okay, so basically what this says is the following. It says, if let us say you are looking at this multiplication, I am calling this as F, so we are looking at F which seems to take as argument T - Tao and then U of Tao d Tao and clearly Tao goes from 0 to T.

So, let me repeat this, if I take Tao is 0, I get F of let us say T at multiplied by some 0, if I take Tao equal to 1, T - 1, U of 1 and if I take Tao equal to T, I get F of 0 U of T. So, in other words, as this keeps increasing, the argument, this keeps actually going down. So, in some sense if you think about what we are modelling, think of this as a simple sum, then you might want to think of this as an approximation if you want. But we can buy appropriately choosing this F, which is a slightly modified version of this, we will get it as an exact expression. But we will not worry about that right now.

So, basically what this is we think of this as a summation, it says Y of T will be some F of 0 times U of T + some F of 1 times U of T -1, all the way up to F of T U of 0 okay. So, you can think of this basically as the whole thing. Okay, including the CB if you want as F, you can bring the CB and think of that as a combined form of F. So, now you see that we have something which we can interpret, what we are basically saying is if I have made control moves starting from time T equal to 0, 1, 2, 3, all the way up to T.

And I want to know what is the effect of all of these control moves on Y of T, then basically every control moves is weighted by some function, right. And that function is let us say in this case F of 0, F of 1, all the way up to F of T. So, every control move here as an effect on the output. So, whatever basically you have done in the past, also has an effect on what is the current situation that you are in. Similarly all the control moves that you have made in the past have an impact on the current output and depending how far in the past you have made that move, there is a corresponding weighting that goes with that, okay.

So, for example you will expect that this weighting will be basically larger than this weighting, simply because this U of 0 is a control move I made quite a while in the past and the effect of that or the residual effects of that you would expect to go down over a period of time. Very simply again, very intuitive conceptually, if I do some actions today and think about the effect of these actions 5 years down the line, I would expect the actions I took you know one day before the 5 years will have a lot more respect on the output of the five-year that what I did today in general, right.

So, as these systems keep dynamically changing, the most recent changes will have a higher impact than the changes that happen quite a while in the past. However the current output is a collective sum of some weighted functions of all the inputs that have been taken before. So, all the actions I take now we will have some impact on what my future is and the way I compute that in a discrete model is I take each action and then I multiply it by a weighting function.

And if it is an action that has been taken long time back, then the weighting function for that input will be less when I think about the current output. And whatever I have done recently will have a much bigger impact, okay. So, if you think about this in the discrete form, now we will go back to the discrete form. Let us say if I look at T equal to K TS, which I am to simply write as K. So, if I look at K in a discrete form, so what I am going to say is that I am going to convert this F function that I talked about into H and this H is called the impulse response model and we will describe that.

Just a slight change of notation, so I am going to say H1, I will finish this and then explain this H1 U1, H2 UK -2 all the way up to, maybe we will call this as H gamma U of K - gamma. So, what basically this says is the current output that I have is a function of all the input moves that have made, that have been made in the past. Okay, that is number 1 here. So, all of these input moves that are made in the past with respect to K, right, because this is K -1, K -2, K - gamma and so on, right.

So, the current input Y of K is a function of, current output Y of K is a function of all the inputs that I have had in the past and these inputs that I have had in the past have different weighting based on how recent the inputs is or how in the past the input is in general. So, you would typically expect that things that are recent will have more impact. In general this need not always be the case. But, anyway, just as far as this class is concerned, let us keep things simple and simply interpret this equation.

So, in the past moves or UK -1, UK -2, UK - gamma, in fact I will go all the way to U0. But in general when we look at this discrete model forms, what we will say is there is some memory in the system, right. So, while I could say what I am today is a collective impact of everything that are done from the time I was born, typically what I am today is largely dictated by what I have done in some recent past and before that do know whatever happened, that if it would have been minimised.

In other words, what I am today is whatever decision I have made in the last 2-3 years. So, that some memory are associated with it. So, if I am at YK, I can say the memory extends only up to gamma terms, not more than that. So, anything beyond that I will say the weighting 0 because that has been forgotten in terms of what the current output is, okay. So, there are many ideas that we are using here. Number-one is a very intuitive idea, that whatever is the current output will be a function of all the input moves that have been made in the past, that is the 1st idea.

And because it is a linear system, typically if I make some moves and ice and effect of that move on the output, this will be additive. So, the, let us say the impact of every move, if I add them up, then I will get the integrated impact on the output. So, that is the 2nd idea. So, output depends on all the previous control moves that I have made and if I can think of the effect of each of these control moves, then I can write the output as the sum of the effect of all of these moves because anyways that is what we have been doing for linear systems, the linear superposition works is the 2nd idea.

Now the 3rd idea is that the effect of each of the control move that I have made can be captured by a weighting function. So, in this case the weighting function is H1, H2, H gamma and so on. So, if it is just input passed, then the weighting corresponding to that is H1, if it is 2 inputs past, than the weighting corresponding to that is H2 and if it is gamma inputs past, the weighting is H gamma. So, the individual effects are actually captured through the weight and the collective efforts is actually captured through the summation and the summation is because of the linear superposition. So, that is the basic idea.

Then on top of it we are saying, we are going to assume that this H of gamma, as you know you keep going more and more into the past, at sometimes it is going to cut-off, the memory is only up to gamma. Beyond this whatever stuffs that are there, they are very close to 0, the weights are very close to 0. So, in other words if I have one more term in the past here, that will have a weight and this weight will be very small. So, at some point we are going to cut-off this expression, which is to say, look whatever is the current situation is the linear superposition of everything that has been done in the past but there is a limited memory in the system.

So, beyond a certain limit, whatever was done in the past do not affect the current situation. So, that is the idea of cutting this at gamma. So, this is the effect of each of the individual moves and the addition is the combined effort and beyond this I am cutting off because of memory, okay, beyond this there is no need to worry about this because those are such small effects that when you are looking at YK, it is enough to look at this, okay.

Now the interesting thing to note is once we understand it this way, by that this is K, K +1, does not matter, the same rule will follow which I will show here. But before that I just do not want you to think that we have done some hand waving and gone from a integration to summation. Actually under certain conditions, under certain assumptions, you can show that even if you integrate this equation with continuous U, where the U is changed in a stepwise fashion and then you actually do this integration and then actually compute YK at those times samples, you can show that this and this will be exactly the same under certain conditions, okay.

So that I can show using you know mathematics without any hand waving but that is not really important as far as this class is concerned. Because what we are really looking for is we are looking at trying to explain how this notion of model comes into the optimisation formulation so that model predictive control problems can be solved. So, that is our goal. So we will just understand it intuitively and iteratively the understanding is very simple.

One last time, the output at the current time depends on on the previous inputs we have made and because of linear superposition the individual effect of these input moves can be added up and the individual effect of each movie is computed by weighting function and this is where I appeal to intention and say if you think of this integration as summation, each of this has a weighting function here, that is what is being captured here. The last thing that I want to show is if I have Y of K, then it is H1 U of K -1 and so on, K is just a running variable.

So you can change K to K +1, in which case here also you have to do K +1, so K +1 -1 will be K, K -1 and this will be K +1 - gamma. Basically what this says is the same thing, so because K is just a dummy running variable, you do not have to really worry about it. The same idea is worst in both the equations. For example this equation says what the current value of K, depends on everything that is done in the past up to gamma. So, that would be the input I took at UK -1, UK -2, UK - gamma.

Similarly the output at K +1 will be depending on all the past, right. But now for K, K -1 is past, but for K +1, K is past and if I take terms up to K - gamma here, I have to take terms up to K +1 - gamma, that is it. So, the past gets moved 1 because the K has become K +1, that is the 1st idea. The 2nd thing you want to look at is when I do K, the immediate past input K -1 I weight H1, right. And the next past input I weight by H2 1 and so on.

And if you look at this equation, the same thing will happen, the immediate past input, which is UK in this case, because this is K +1 is being weighted by H1. The next past input, which is K -1 in this case, because this is K +1, this is weighted by H2 1 and so on. So, everything is consistent, except that the counter keeps increasing. So, at every point, so if I am here, I have to weight all the inputs to here, if I am here, I move this also 11, right, so that is what is what is happening here.

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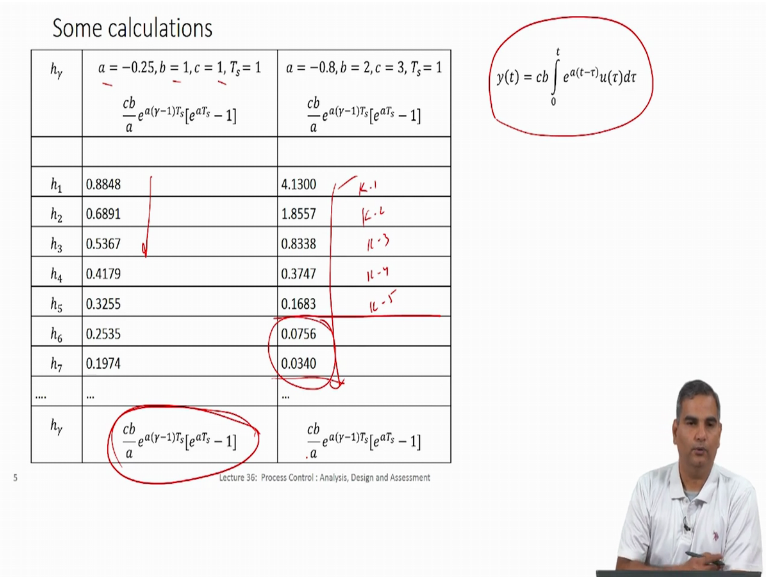

Now you see that you have a model between output and the inputs in the past. So, this is the critical model that we were looking for to complete this idea of model predictive control. So, let us take some examples to show you how this will work. Actually showed you this was the expression that we have derived mathematically. Now when you convert this to this H gamma, the coefficient form, actually mathematically you show that if you use this weighting, you will get an exact result.

We will not get into that but what I am more interested in showing is that as you compute this H1, H2, H3 and so on, as you keep going down computing more and more coefficients for the model from this continuous model, you will see the numbers keep increasing. At some point you can say that beyond this, this number is very small, not going to take the impact of this. So, for example here, the 1st number is 4.13 and if we keep going down and I come to 0.03 and so on.

So, you might say okay, for my model predictive control implementation, I am going to stop here because I am going to assume these are very small numbers and these are basically things that I can ignore. If you do that, then basically what you are saying is the current output is being affected by the previous, this is K -1, K -2, K -3, K -4 and K -5. So, I am going to only retain 1, 2, 3, 4, 5 terms here is what we are saying here. Now, if you say no, no, I want to be more accurate, I want to come till here, you will retain 7 terms and so on.

But this tells you that for typical systems where we had these States-based models and I have taken some values for A, B, C and then actually shown the computations of H, how this formula is derived, I have not told you but I have just told you that if you were to compute this, you keep going down, at some point the memory effect will kick in and these coefficients will become smaller and smaller, so that adding them in the model does not give you any purchase.

So this is a basic idea of how you derive discrete model and how you think about discrete model, which we are going to use in a model predictive control framework. So, the last lecture which will be the next one on model addictive control, now that we have looked at an optimisation formulation, we have looked at how to model an output as a function of previous inputs indiscreet framework, which is derived directly from this continuous solution.

We are going to put all of this together and then show you formulation for MPC and show you how we can get the controller implemented using MPC. So, I will see you again in the next lecture, thank you.