Process Control – Design, Analysis and Assessment
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Multivariable Control – Part 1

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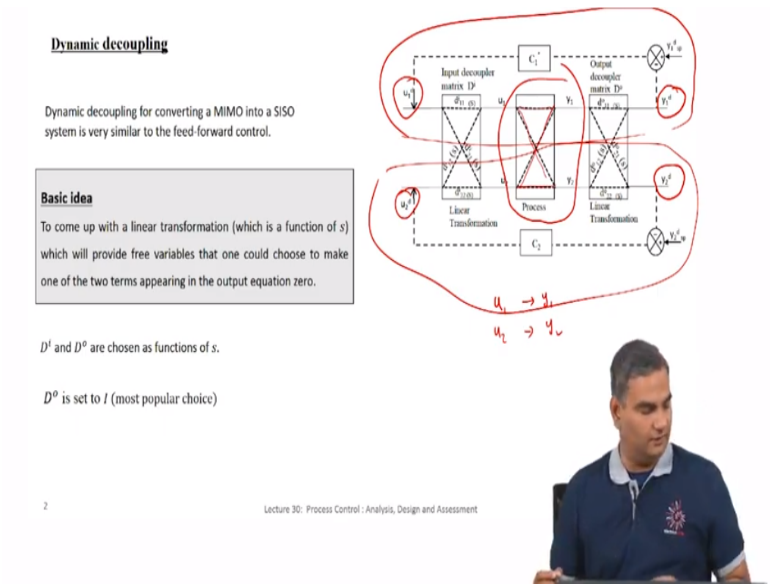

So let us look at this idea now let us assume that I have a process here which is like most process is truly multivariable, so the way I have shown that this process is truly multivariable is that there two input come in this plot and there are two output Y1 and Y2 and then you see there is a path from U1 to Y1 path from U1 to Y2 that means whenever you change U1 both Y1 and Y2 get effected and similarly whenever I change U2 both Y1 and Y2 get effected, so it is truly multivariable however typically what we have been doing is se have said ok, let us look one be manipulated to change Y1 and Y2 be manipulated to change Y2 and so on.

So that is clearly not possible it is not possible most of the time but we still have ignored this and said whatever is effect of the other loop I will handle that as disturbance and so on, which is what I have told you at the beginning of this course, but now let see whether they are any better ways of doing now that we are used to using a model of the process with the controller implementation starting from a controller design for time delay system and so on, so we are really looking at seeing if there is any better way of handling this while still retaining the basic single input single output structure for us tuning another things are considered.

So this the idea that we are looking at so basically let us assume that somehow we are going to get output which are decouple in some senses, so I have one output which is called Y1 decoupled another output which is called Y2 decouple and we are going to do some transformation so that when I look at the outside I am going to manipulate U1 decouple to control Y1 decouple and U2 decouple to control Y2 decouple, so basically what I want to think about is there are still see so loops if you kind of combine all of this together into let us say one process if you look at this, this will look like a single input single output feedback loop and this will look like a single input single output feedback loop.

So is there some way in which we can define U1 decouple U2 decouple, Y1 decouple and Y2 decouple is such a way that from the outside it looks like a single input single output controller, so that I design my controller for only this is what we are trying to do.

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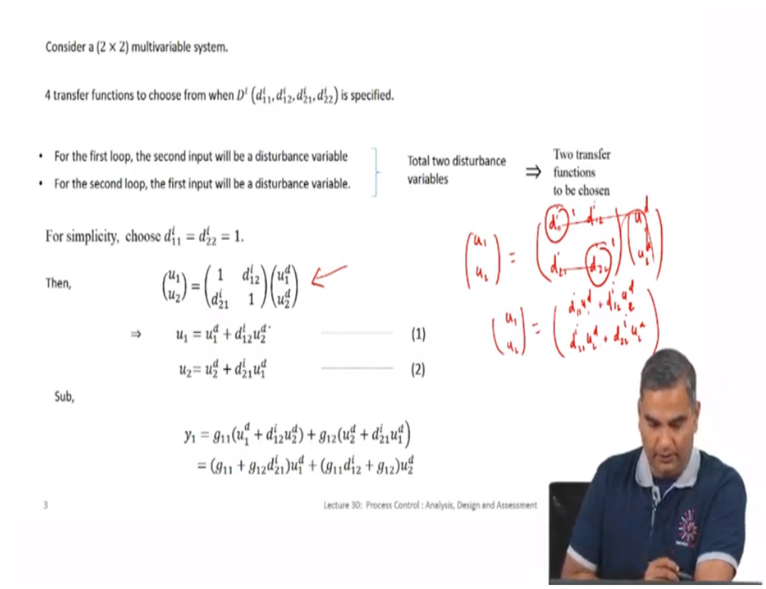

So the way we do this is the following so what we do is ultimately the trough manipulated variable are U1 and U2 you might okay say I am computing U1 decouple and U2 decouple and so on you could go ahead and compute that but even then you still need to figure out what U1 and U2 are in actually in the controller implementation situation because those are the physical manipulated variable, so supposing let say I have come up with some U1 decouple and U2 decouple which I will show you how we come up with then I might say if I am going to recalculate U1 and U2 from this.

So then I could come with some linear transformation, so I might write this equation like this U1 U2 this is ultimately the final value I need to keep for the two manipulated variable I might say ok I have DI one-one, so this is input decouple that is there is a YI use super scrip I and one-one you will see what that is DI one two DI two one DI two-two and then I have U1 decouple input U2 decouple input, so these might be a standard form so basically what this say is I have computed this U1 decouple and U2 decouple somehow and I have really have to convert that to two input U1 and U2 because those are physical inputs.

So since I have this two I am going to take a linear combination of this two and I am going to take a linear combination in a particular way that makes it useful for us, typically U1 will be DI one-one U1D plus DI one two U2D and U2 simple matrix multiplication so you would write it like this U1 U2 equal to DI one-one U1 decouple plus DI one two, U1 decouple and DI two one U2 decouple plus D two decouple, so this is your matrix multiplication this way, so you get this so you notice U1 and U2 are written as linear combination of U1 decouple and U2 decouple okay now we have four thing to choose which is DI one-one DI two-two DI two one and DI one two.

What we are going to do is for the sake of convenience, let us assume that I choose this as simply one and this as one and you could have chosen something else but for ease of computation and you are going to do this and so that gives you these matrix form here U1 U2 one DI two one DI one two one U1D U2D, Now if you do the same matrix multiplication just this DI one-one and DI two-two it become one the other thing will be the same, so I will have the two physical input U1 as U1 decouple plus DI one two U2 decouple and U2 as U2 decouple plus DI two one U1 decouple, now from a process view point also we have decoupler.

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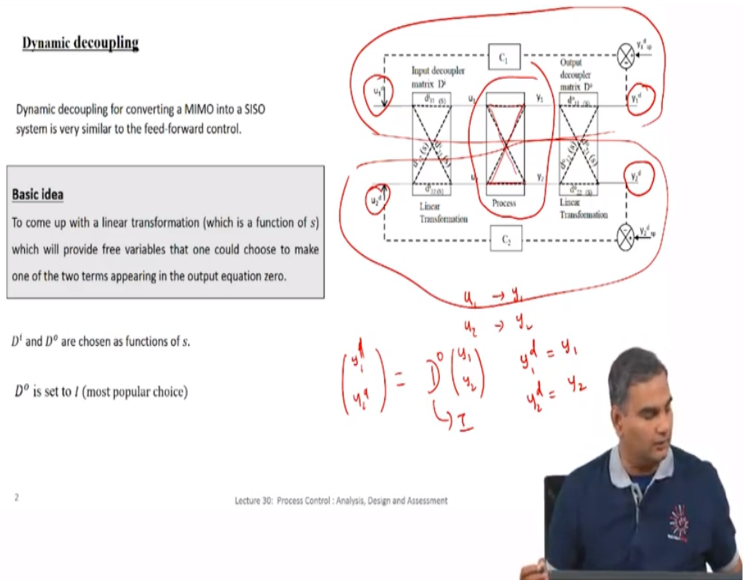

Let us assume that same way I can compute this view Y1 decouple and Y2 decouple as a function of Y1 and Y2 the simplest choice is to not do any decoupling here you do no really need to do that you can do other things but the simplest choice is what we call us an I matrix, so basically if you think of Y1 decouple Y2 decouple again as some output decoupler matrix the other matrix I showed in the previous slide it is an input decoupler matrix if you think of this is an output decoupler matrix multiplied by Y1 Y2 now you can set this as identity, identity meaning not doing any decoupling as far as the output is concern in which case you will get Y1 decouple equals Y1 and Y2 decouple equals to Y2.

So that is what you will get so basically what we are saying is there might be reason and there might be cases which are more advanced where you might actually choose a D output as so that it makes something better but as far as simple case are concern we can simply said D output equal to I basically that is equal to saying Y1 id Y1 decouple Y2 is Y2 decouple, now also notice there will be a transfer function between Y1 and U1 and U2 and Y1 and there will also be a transfer function between U1 Y2 and U2 and Y2.

So there are four transfer function which are the true transfer function which are already been identify so we will use that, so as far as this picture is concern for this decoupler module though there are four elements or four component I can choose I am going to choose two as one and the remaining two are yet to be compute at which is what happens which is D input side of the decoupling the D output I am simply going to assume it side identity so I will simply get Y1 decouple is Y1 and Y2 decouple is Y2.

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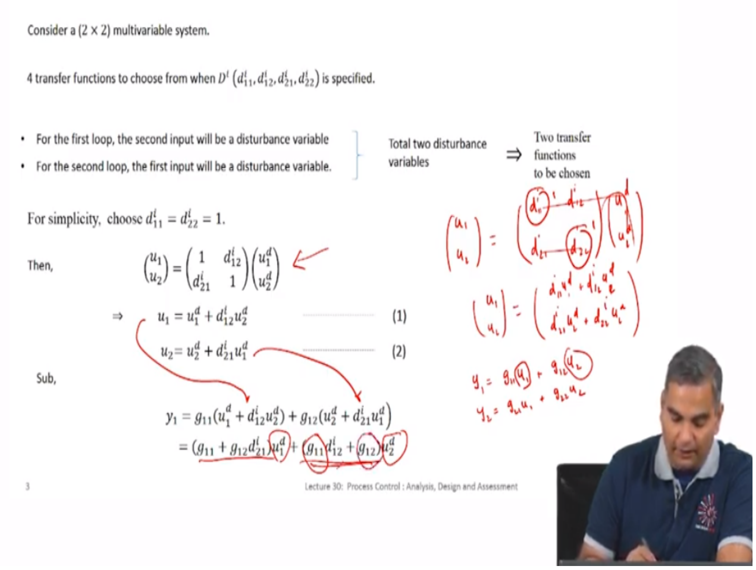

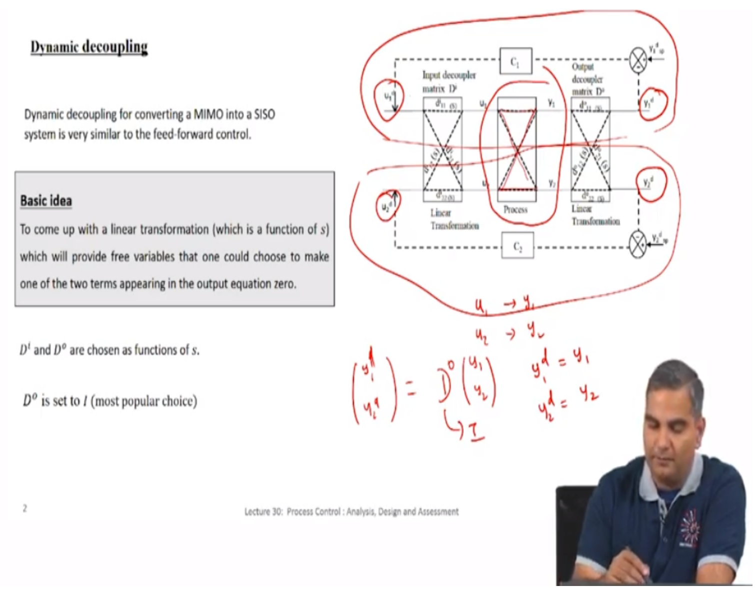

So we come back here now we write Y1 as a function of U1 and U2 and you will see that typically in a multivariable process in which we have seen before so if I have Y1 there is one transfer function G1 for U1 and then one transfer function G1 to for you Y2 similarly we have Y2 you will have one transfer function for U1 and one transfer function for U2 which is what we have written here, let us look at what we do with Y1 what we are going to do is U1 we are going to substitute from here, so Y1 is U one-one times U1 decouple plus D1 to I U2 decouple plus for this U2 we are going to substitute this here so this is going to be U2 decouple plus DI two one U1 decouple.

So very-very simple re-substitution, so this will U1 now I can collect all the term corresponding to U1 decouple U2 decouple and I will get a form like this where there is some transfer function U1 decouple plus another transfer function time U2 decouple, now if I ask you since we have the choice of G one two D two one and D1 two in our hands, so how do I choose this so that we get some benefit out of doing all of this is a question that we might ask, now if you look at this equation and if you look at the picture before.

Now we know this Y1 decouple is Y1 because we have not done any choices for this other than saying this is identity so clearly Y1 decouple is being control by manipulating U1 decouple, so in an ideal situation what we would like to have is make sure that this U2 decouple has no impact on this loop and similarly U1 decouple does not have any impact on this loop that basically means can we decouple this input in such a way that Y1 decouple is not a function of U2 decouple and Y2 decouple is not a function of U1 decouple, so that is a basic idea, so again let me repeat Y1 decouple is Y1 Y2 decouple is Y2.

If you want to really decouple this loop basically what we are saying is I want to come up with U1 decouple U2 decouple in such a way that U1 decouple has only impact on Y1 decouple no impact on Y2 decouple and U2 decouple has only impact Y2 and not on Y1 decouple, so that is basic idea now if you use that idea than we are saying Y1 only U1 D should impact U2D should nor impact, so if U2D should not impact then that means this has to go to zero now this is a fix part of the process transfer function, this is a transfer function which is known however we have introduce this here which is in our hands, so this something that we can choose.

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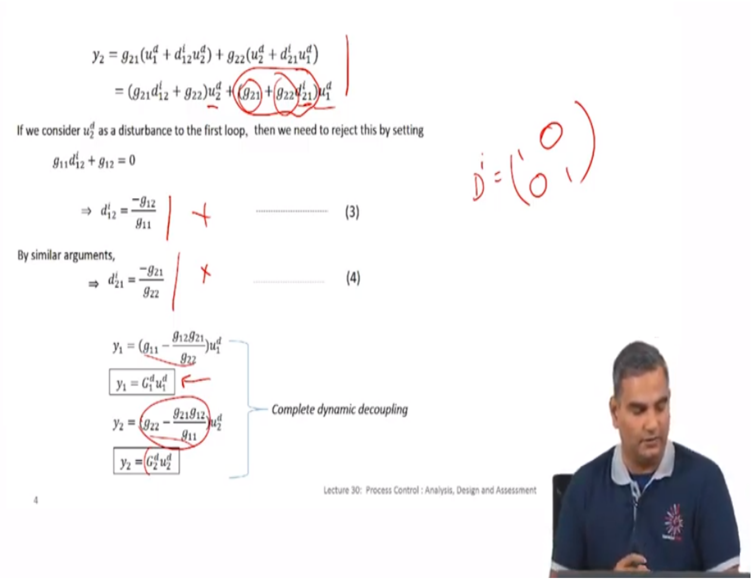

So if we choose that then you will get this equation here for one of the decouple matric and similarly if you look at Y2 and do the same algebra, now Y2 is a function of both U2 decouple and U1 decouple, now if you want Y2 to be effected only by U2 decouple then what you would say is you want this to go to zero, so that will give you one equation here again this is known and this is something that we can choose so once we decide that we want to make this to zero then choice for that is also obvious, so from this I can get this two decoupling elements which if I put into this overall DI.

So I have one-one and this one computed this one also I have computer using these two equation, now I have an input decoupler now if you simplify this in term of transfer function only for U1 decoupler then you will get Y1 equal to G1 now it slightly different transfer function from before, so initially if it were U1 this will just be G one-one but because we are going to correct for the effect of U2 this transfer function becomes this transfer function similarly this transfer function becomes this transfer function, now if you notice this two equation it look at as if Y1 is only effected by U1 decouple and Y2 is only effected by U2 decouple.

So we have essentially got to single input single output control unit, so this is what is called complete dynamic decoupling and notice how we are starting to use a model more and more, if I had built the model for this four transfer function I am going to modify the transfer function between Y1 and U1 decouple through this formula here and similarly I am going to modify the other transfer function for Y2 and U2 decouple, so that way you now have written two single input and single output loops you can use standard techniques that we have been using to compute the controller equation and so on.

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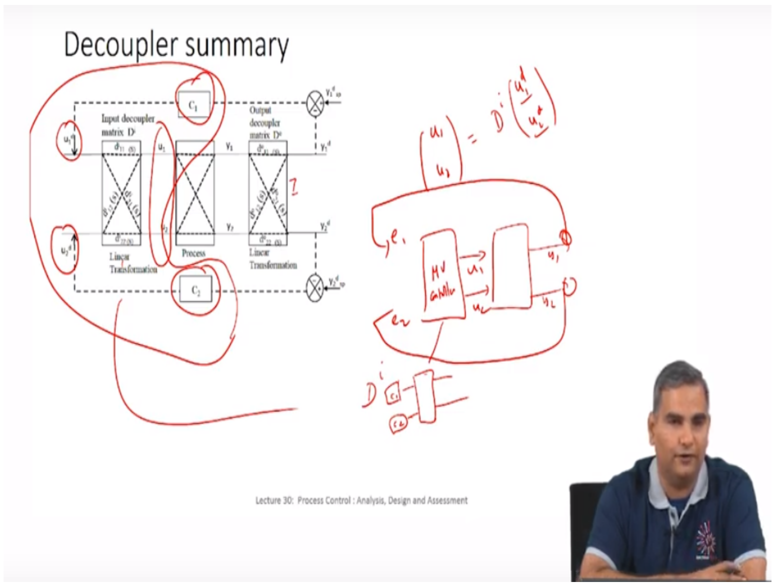

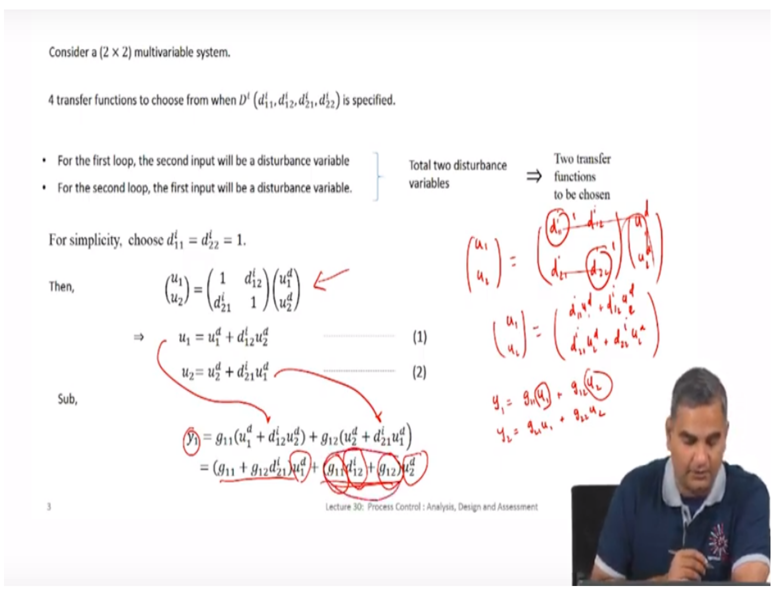

So there is nothing which is difficult to do here because all of these transfer function we know how to work with this transfer function so basically what will happens we have already identify this input decoupler, so in a real implementation let us say there is difference between Y1 and Y1 set point that will go to the controller, so the controller will decide what this U and Decouple value is similarly the difference between Y2 and Y2 set point will decide what this U2 decouple is, so that just figure out what this and this are but in a real implementation what you need to really do is actually go and manipulate U1 and U2 you cannot U1 and decouple because that is not a physical input that is input that is (())(14:02).

However we know that the two physical input U1 and U2 we can actually compute as this input decoupler matrix times U1 decouple and U2 decouple, now this and this are already given by the controllers here and here based on the error so once we have this we can actually have U1 and U2 which will be implemented on the process then you will get a Y1 and Y2 check it with respect to set point and the keep continuing this process, so if you think about really what the controller is there seem to be there are two single input single output controllers but if you really think about this is an identity matrix.

So basically if you think about combining all of this together into one block then you could say there is a process block from where Y1 and Y2 coming out this is being compared with this corresponding set point the error is going and then there is one controller block but this controller block has two block inside one is decoupler which give you this and before this decoupler there is a C1 and before this decoupler there is a C2, so we can think of this whole thing itself as one big block which is now the controller block because of the decoupler everything gets in a combine together.

So this is really a multivariable controller which has two single input and single output controller and the decoupler and then this really give U1 and U2 and from here you get by comparison by the set point with the two errors E1 and E2, so there compare it block here gives you the error and that come it to give you multivariable controller, however the beauty of this is because we have done this, this controller design like a single input and single output controller, so again the notion of including interaction is beautifully addressed here where we choose our decoupling matrix in such a way that we have minimal effect or zero effective the model are perfect of U1 decouple on Y2, U2 decouple on Y1.

Now you will also notice that this is very similar to how looked at feed forward control so you might think as some generalization of the feed forward idea remember in feed forward control also we had two term for the Y where there was one term which was coming from the disturbance and then we said if you want to fill the disturbance what should that be we said that term zero, now in this case for Y1 we notice that and if you assume that I want to manipulating U and decouple for Y1 then the effect of U2 decouple should be zero just like, how we said in feed forward the effect of disturbance should be go to zero, so that basically means that this transfer function had to set of zero.

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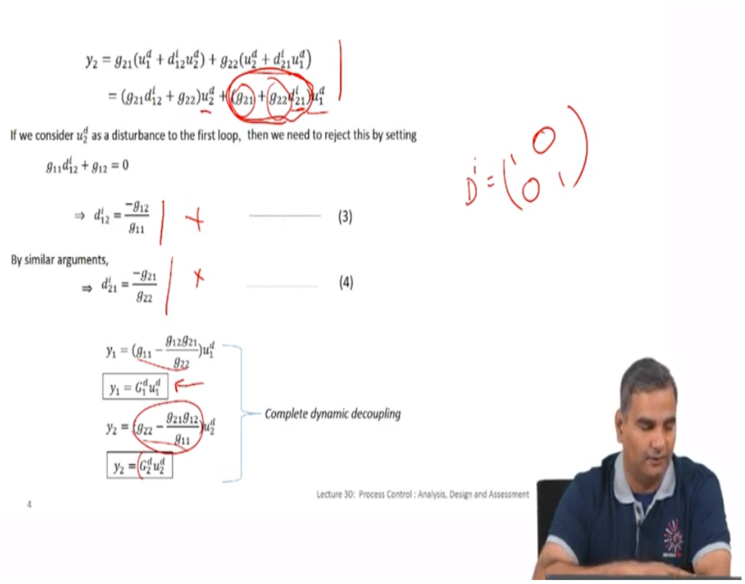

And similarly when you talked about Y2 again we want only Y2 decouple to have an impact on Y2, so the transfer function which is multiplying U and decouple set of zero much like how did this feed forward so what we initially at the very beginning at this close start about in term of being able to think of the input from the other loop of being disturbances this to the main loop and so on.

How we actually treat that mathematically and how we actually make sure that the effect at the other input goes to zero is seen now after several lecture and through a progression of ideas also notice that the model are been used more and more in this cases and we have started to see truly multivariable control however I also want to emphasis that once we have understood the single input and single output control quite well then you can see that lots of these are you know primarily extension of that idea.

So once you have that basic grasp and very good understanding of this single input single output controller design concept then you can keep adding on to that in term of how you think about multivariable design concept the other thing that I also want to explain here is now you will see the power of Laplace transform so if you want to do all of this in time domain then you have to really worry about how you are going to decouple this input from the other one you will have integration and complicated cancellation in integration.

So on because all of this is become algebraic when we start working with Laplace transform you see a transfer function and another variable multiplying transfer function and if you want to get rid of the effect of that variable the simply set the multiplying transfer function to zero which is a simple concept and the way we get this transfer function Y1 and Y2 them self are simple algebraic manipulation and so lots of other thing that we are doing here in term of decoupling and so on all get completely you know merge in some simple algebraic transformation which are very easy to understand, so I hope you got a good idea of how to do this for two by two.

Now if you are three by three it is the same idea there is nothing particularly complicated there if when it comes to three by three let us say then why one will have the effect of U1 decouple U2 decouple and U3 decouple so if you want only you want decouple to effect Y1 then the transfer function which are multiplying to U2 decouple and U3 decouple you will set them to zero so there will be two choices there and the with respect to Y2 similarly you will have which you will have to set to zero corresponding to U1 decouple and U3 decouple and for Y3 you will have to set again two transfer functions is zero which will be corresponding to U1 decouple U2 decouple.

Now you will make those six choices and remember if it is three by three matrix there will be nine elements in there three by three matrix you will again do all the diagonal elements to be one, so nine minus three six choices that will be those six choices that start about and then you will have a data playing matrix and you can continue this for four five variable and so on, this is one simple way of thinking about multivariable control which is very useful from the view of understanding how some of these are that, so in the next lecture I will continue these idea and till now we have always set a particular input will we use to control particular output.

Whenever we talked about see so and even we came to cascade control we still had a particular input that is use to control a particular output even in feed forward control we had a particular input which will be use to control particular output and surprisingly even in this multivariable case we had a notion of what input is going to control what output that is how we set the other decouple input zero, so in every case we always have some input which are use to manipulate some outputs and at the beginning of this course I talked about this choices so supposingly if I have let us say five control variable and five manipulated variable which manipulated variable do a match with which control variable is something that we have never talked about.

So what I am going to do in the next lecture we are going to talked about that and that will kind of end this whole sequence of transfer function base multivariable controller or more traditional advanced controller ideas I let hat to the end because now that we has understood all of this just that one part if I teach in term of how do we make this chances about which variable to pair then that will kind of complete the circle of arguments that we have been using till now and one thing that I have been mentioning is as we through multivariable control we use the model more and more.

So that something that you want to keep in mind now there is a notion of performance in direct synthesis controller there is really no notion of performance but still there are notion of stability and then back of in traditional stability based controller tuning and so on , however in both these case there is no explicit optimization of matrix online from a control view point so that is what is called as model predictive control so after I teach my next lecture on how you do the sparing we will move on to truly optimization based control where there is a performance matrix that is optimize in every controller calculation.

So that will be the first time you will see that you will not have direct analytical relationship but the input itself will be a solution optimization problem in some case we can actually write that effect direct analytical expression in other case we have to get to do things that are more complicated before you can write the controller manipulated input as a analytical expression so we will discuss all of these as we go through more lecture on this multivariable control thank you.