Process Control – Design, Analysis and Assessment
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Nyquist Stability Criterion – Part 3

So let us continue with this 26th lecture in this course on process control analysis design and assessment and the last lecture on the miniseries of lectures on NYQIUST stability criterion that we have talking about.

So in the last two lecture I talked about the S plane the F of S plane and how you convert contours in S plane into an S plane and we also looked at a very interesting result which said if you draw contour in the S plane if you encircle amount of zeros and poles of F of S inside the contour and then as you move along the contour you keep tracking how F of S changes and that is a very simple thing because once you have a S value you can simply compute F of S and plot it in the F of S plane and if you track that and then you finally look at the F of S plane contour and the count the number of encirclements around the origin then the number of encirclements will give you Z minus P where Z is the number of zeros inside your closed contour in the S plane and P is the number of poles of F of S inside the closed contour in the S plane.

So that is very interesting and remarkable result which we can use for analyzing a stability of time delay systems, now there are lots of mathematical arguments and so on why this is true and there are things that you need to do to draw this contour such as excluding at the poles or zeros on the S plane contour itself and so on but those are complications which we will not really focus on in this course because there is a first introduction to this idea nonetheless what I want you to realize is while doing this while I showed you examples where we wrote the F of S as a numerator by denominator polynomial I showed you because we are just computing F first four different S and doing nothing else the numerator and denominator could have the E power minus tau DS done.

So that simply does not create any problem at all because for any S I can compute E power minus tau DS, so ultimately this E power minus tau DS term whether it comes in the numerator or denominator with other polynomial term for every S on the contour on the S plane you can simply compute this F of S and once I tell you that whatever result that we saw is also true even if you have E power minus tau DS terms then it clearly says now that we have a way to analyze the stability of time delay systems that is a connection that I want you to realize, now I also toward the end of the last lecture showed you how these results are useful for closed loop systems where I said we want to cover the whole right half plane to ensure that there no zeros of one plus GC in the right half plane.

So I said what we do is we start on the imaginary axis at minus J infinity and then keep traversing the imaginary axis all the way up to J infinity and then we take a long circular route covering all of our HP and then come back to minus J infinity and this you can imagine just in your head that this is a clockwise movement in the S plane you might think about how am I going to compute all of this right so there are programs that will do this and you do not have to do anything in terms of computation.

So in you labs that go with this you will see these NYQUIST plots what you really need to do is have the ability to interpret them and interpreting this plot simply means computing are calculating number of encirclements around an origin in the F of S plane plot, so if there is some software code which gives you this contour in the F of S plane all you need to do is really look at that and then be able to count the number of encirclements and use this idea of N equal to N minus P to do you whatever you want, so that is important thing to keep in mind.

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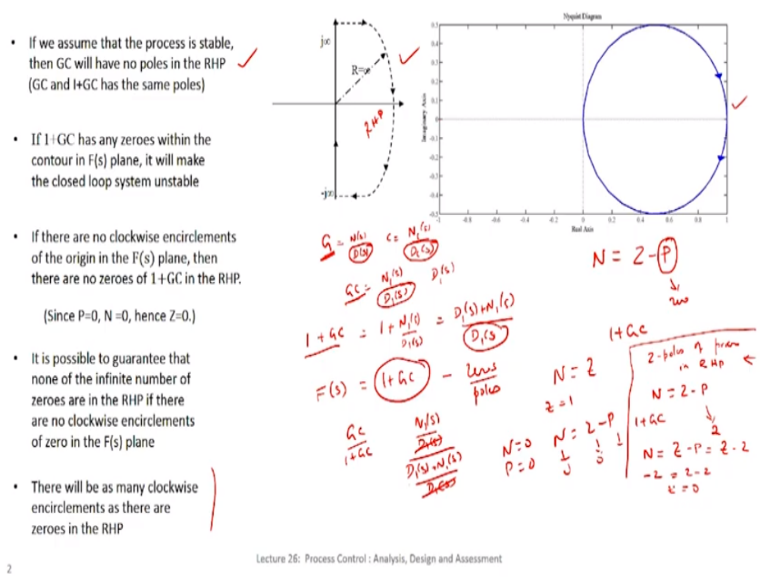

Now we will come back to the way in which we are going to use this in control we have already seen this picture here this is how we convert this notion of closed contour to cover the over whole of our HP right offline is completely cover when I take a contour where I start moving from minus J infinity all the way up J infinity and then go on the circular route, now let us go back then understand each of these bullet points so that the translation the general result to control is very clear I am going to reiterate the fact that this can be done for E power minus tau DS also because I have already said this many times all I need is an F of S function for any S I can compute F of S.

So we have already subside idea of doing this for time delay systems in all the discussions we have had till, now let us look at each one of these bullet points and then see whether we understand this carefully so first if you assume that the process itself is stable then GC will have no poles in RHP, so let us see whether we understand this so let us look at G which itself is some numerator by denominator C is also some numerator by denominator clearly we are not going to design controllers which are unstable, so we are assuming that there are no poles of the controller in the right half plane and if the process is stable then D of S is also not going to have any poles in the right half plane.

So GC which is some numerator let us say by denominator when I put this together will have no poles in the right half plane.so if the process is stable GC is not going to have any poles in the right half plane, so this is something that we should understand, now the second thing is if GC is N1S over D1S basically D1S is the polynomial denominator polynomial from which you compute all the poles, now look at what will happen if I have one plus GC, so one plus GC will be one plus N1S by D1S, so this is going to be D1S plus N1S by D1S, so if you look at GC and one plus GC the poles of GC and the poles of one plus GC are the same, so that is the first bullet point.

So basically what it says is if you assume the process is stable then GC will have no poles in the RHP, that is from this notion here and when you actually write one plus GC you write one plus N1S by D1S which is D1S plus N1S by D1S so the poles of GC plus one plus GC are the same, so this is first bullet point which we have understand, now the next bullet point basically takes this idea to this which plot and the F of S that we are going to use to plot all of this is going to be one plus GC and now this one plus GC has some zeros and poles.

So now let us look at one plus GC and then as we go along this very long infinite path, starting from minus J infinity all the way up to J infinity and go on the circular route covering all of RHP we want to ask whether this F of S which is one plus GC because we are going to draw this NYQUIST plot for one plus GC initially and then we will refine it further.

So when we look at this F of S as one plus GC, so we are doing this plot here and then we are going to get a corresponding plot here now within this will this F of S have any poles in the RHP if you ask that question, now the answer is it will not have any poles in the RHP this process is stable why because if the process is stable we already said GC has no poles in the RHP and the poles of one plus GC and GC are the same and since GC has no poles in RHP one plus GC cannot also have any poles in the RHP, so within this big circle by looking at the encirclements we already know we can see the difference between the Z minus P, but as I said before we cannot individually compute Z than P but in this case if the process is stable we can guarantee that this P is zero for one plus GC right.

So why is P is zero plus one plus GC because poles of one plus GC are the poles of GC and G is stable controller will choose it to be stable, so we will have no poles, so for stable systems then I look at this contour like this when I go from minus infinity to plus J infinity and all the way through a large circular path then when I look at this encirclements if the process is stable and the F of S that I am considering is one plus GC poles are zero so the result is N is Z.

So the number of clockwise encirclements will be equal to the zero of one plus GC this the most important and interesting result from a control viewpoint, so basically what this says then if that if I have let us say one clockwise encirclement of plus GC right, so that basically means Z equal to one so within this large contour this one plus GC has at least one zero is what it would mean if I see a clockwise encirclements are on origin in the NYQUIST plot so that would mean then the closed loop is unstable, why is the closed loop is unstable because we already said GC by one plus GC and if I take this GC as some numerator by denominator one plus GC will be B1S plus N1S by D1S, so this and this will get cancelled.

Now the denominator of GC by one plus GC will be the numerator of one of one plus GC, so any zero of one plus GC will become a pole of GC by one plus GC, so if I have one encirclement that means there is one pole for one plus GC one zero for one plus GC in RHP which becomes a poles for GC by one plus GC in RHP, which makes the system unstable, so basically what we have done is we have reduced the identification of stability to simply counting the number of clockwise encirclements of one plus GC around origin that is a beautiful and simple result here.

So this then says if there are no clockwise encirclements of the origin in the F of S plane which is one plus GC then there are no zeros of one plus GC in RHP because no clockwise encirclements will mean N equal to zero we already know P is zero N is equal to Z minus P, so this is zero, this is zero, this has to be zero of one plus GC in RHP that means no poles of GC by one plus GC in RHP.

So remarkably by just looking at this F of S plot or the NYQUIST plot we are now able to guarantee that you might have infinite expansion polynomial in the denominator because of your E power minus tau DS and so on but we are going to guarantee that is going to have no zero in RHP that means you might have infinite number of zeros but I am still guaranteeing that nothing will be in RHP by just doing this contour plot without actually ever computing the poles our zeros of the transfer function.

So that is the most interesting and beautiful idea here right, so all I am saying is you simply get this plot look at the clockwise encirclements around origin if there are no clockwise encirclements around the origin that means that means your closed loop is stable and there will be as many clockwise encirclements around origin as there are zeros in the right half plane for one plus GC right, so if one plus GC have three zeros then I will have three clockwise encirclements because N equal to three minus P, P is zero already because we know the system is stable.

So that is how the whole idea of NYQUIST stability is translated to time delay system and it is translated to time delay systems because F of S can be any functional form I need to simply compute the F of S value as I go through this contour and this is done using software and you will basically have these plot which we have to interpret, so if you understand each one of these points clearly and explained this here then basically understand how to use this NYQUIST plot for stability.

Now just a point a point if you know that there are no encirclements clockwise encirclements then you know that the system is stable and this result is true for processes that are stable but I do not want you to think that this result is restricted to analyzing stable processes a simple extension is supposing you know that your process is unstable and you know that your open loop process have let us say two poles in RHP.

So let us take that case just wanted to kind of explain to you how this result extend to unstable processes also supposing you know that there are two poles of your process of processes in RHP, Now when will I say once I design a controller the closed loop system is stable, now you do not have to change anything this result you are going to use to understand this, so we know N is Z minus P and when I look at one plus GC we have already said the poles of GC N1 plus GC are the same, so if there are two poles of G seen RHP from this fact then there has to be two poles of one plus GC also in RHP, so when I do this infinite contour for stable systems I put P equal to zero for unstable systems I have to put P equal to as many poles in the RHP as there in your process transfer function open loop process transfer function.

So if I know that there are two poles in the RHP then P is two, so I have N equal to Z minus P which is Z minus two, now if the system if stable then what will happen is Z has to be zero that means I have to see two anti-clockwise or counter clockwise encirclements around the origin so if N is minus two, so for your closed-loop system as you plot this one plus GC if you look at the plot and then you find that there are two counter clockwise or anti-clockwise encirclements of origin then N is minus two so this will be Z minus two Z, so there are no zeros of one plus GC which means a closed loop is stable right.

So the open-loop is unstable with two poles and if you design a controller in such a way that your NYQUIST plot shows two anti-clockwise or counter clockwise encirclements of origin then that basically means that there no zeros for one plus GC in RHP which is what is critical for closed-loop stability, so this is how you use the same result analyze closed-loop stability of unstable systems.

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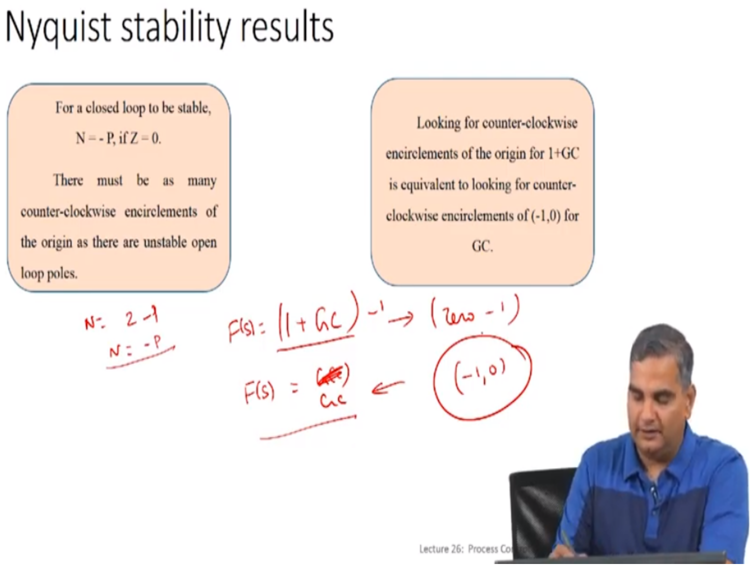

So now the last bit of result that we are going to use is the following this is summary or restatements of whatever we have seen for a closed loop to be stable, basically Z has to be zero that is the number of zeros of one plus GC in RHP and the number zeros of one plus GC in RHP will give you the number of the closed-loop transfer function in RHP as long as that is zero the closed-loop transfer function does not have any pole in RHP, so from the rule N equal to Z minus P if Z is zero N equal to minus P, so whenever Z is zero N will be minus P, so if I have as many counter clockwise encirclements of the origin as there are unstable open-loop poles then the systems is stable.

So that is the main result because that comes from N equal to minus P if they have three open-loop unstable poles I should have three anti-clockwise or counter clockwise encirclements and similarly if I have one I should have one and son on, so this is a result which basically generalizes this was stable and unstable processes, now if the open-loop is stable then P is zero and should also be zero there should be no encirclements now one last thing whenever these plots are given since we are interested in one plus GC we were looking at encirclements around origin on zero but typically the plots what they do is they do not plot one plus GC F of S is not as one plus GC typically the plots are for GC.

So if you want to get GC from one plus GC you have to subtract one here so whatever is an encirclement around zero when I plot for GC I have to look for zero minus one, so I have to plot look for encirclements around minus one and zero in GC plot, so till now we have just been talking about looking at encirclements around origin because F of S we were considering S one plus GC but for sake of convenience what people do is they do this NYQUIST plots not for one plus GC.

So all I am saying here is if you want to go from one plus GC two GC you have to subtract one from one plus GC so if you are looking for zero encirclement around the origin in the one plus GC plot because to go from one plus GC two GC I have to subtract one similarly the zero would move to minus one zero, so basically you look for encirclement around minus one zero, so that is all the final difference because this is how most people report the NYQUIST plot in control systems, so I hope this is clear this is just where you look for encirclements nothing more.

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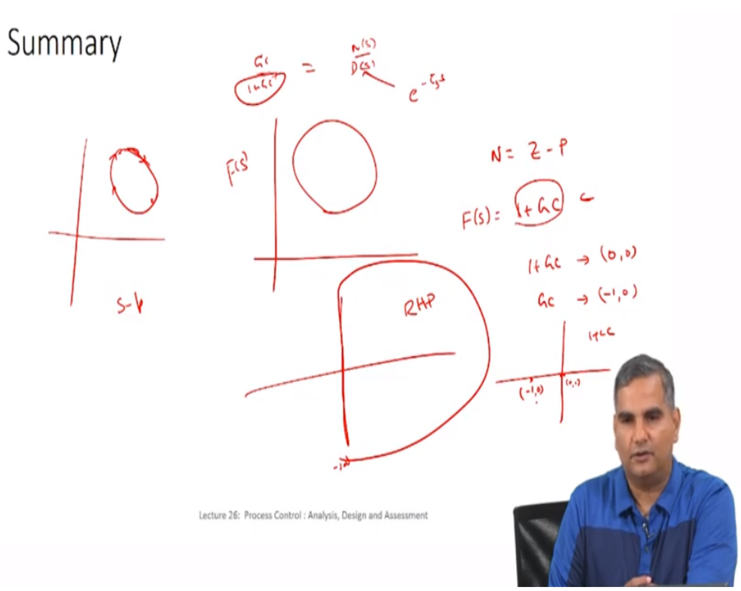

So in summary we said whenever I have GC by one plus GC typically we have been saying this is some numerator by denominator and then you can look at the stability of the systems by looking at the poles of D of S and if I can get it in root result form then I can use partial fraction then we talked about root stability table when I do not write it in root to solve for and then when we came to time delay systems we got into this issue of this term having E power minus tau DS in it is functional form then we said we have to look at some way of handling this that is where we came up with this notion of NYQUIST stability where the key idea is if I have an S plane and I am doing encirclements around something in the S plane.

I am looking at how it is going to look like in in F of S plane the same encirclement and we got this result which said that the number of encirclements of origin in F of S plane N is Z minus P where Z is the number of zeros of F of S within the contour that is used in this plane and P is the number of poles of F of S within the contour that is used in the S plane then we went to the next idea which is to say make this F of S one plus GC and we know the number of poles of GC and number of poles of one plus GC are the same so we are looking at only these zeros of one plus GC because it comes in the denominator.

So by making this F of S one plus GC we came up with the result saying the number of clockwise encirclements around origin is Z minus P but now the S plane contour is something that we take which from minus J infinity all the way up to J infinity and this long so this come you know and compass all of RHP, so if we are able to show that this one plus GC has no zeros in the RHP then we are able to show that the closed loop is stable.

So that is what the idea that we use and basically I said you do not have to worry about doing these computation because these computations are done using a software you need to only learn to interpret these computation and look at picture and be able to understand it the last idea we said was whenever we are looking at encirclement for F of S if it is one plus GC we look for encirclements around zero if it is GC we look for encirclement around minus one zero, so this is zero basically because we are talking about the complex number, so there has to be a real part and imaginary part, so basically if you are looking for encirclements around here which is zero-zero point for one plus GC because when we do GC we have to subtract one from everything.

So we have to look for in encirclement around minus one zero when we look at GC, so that basically finishes this whole idea of NYQUIST stability and how NYQUIST stability can be used for understanding the stability of time relay systems so with this we finish the portion on designing controller understanding controllers and analyzing the stability of controller for time delay systems in the next lecture I will talk about the next difficult dynamics which is actually when the process is open loop unstable, how do we design controller, how do we design robust controllers using direct synthesis approach I will see in the next class, thank you.