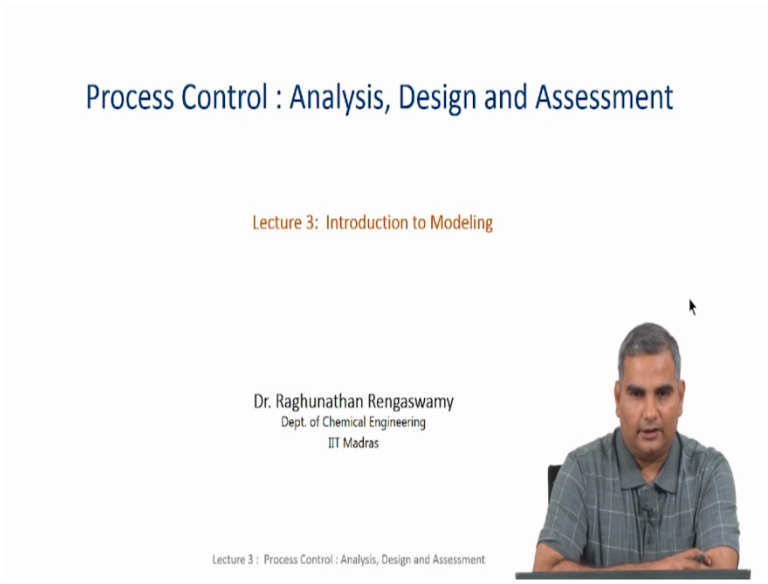
Process Control -Design, Analyses and Assessment
Prof. Raghunathan Rengaswamy
Department of chemical engineering
Indian Institute of Technology, Madras
Introduction to Modeling

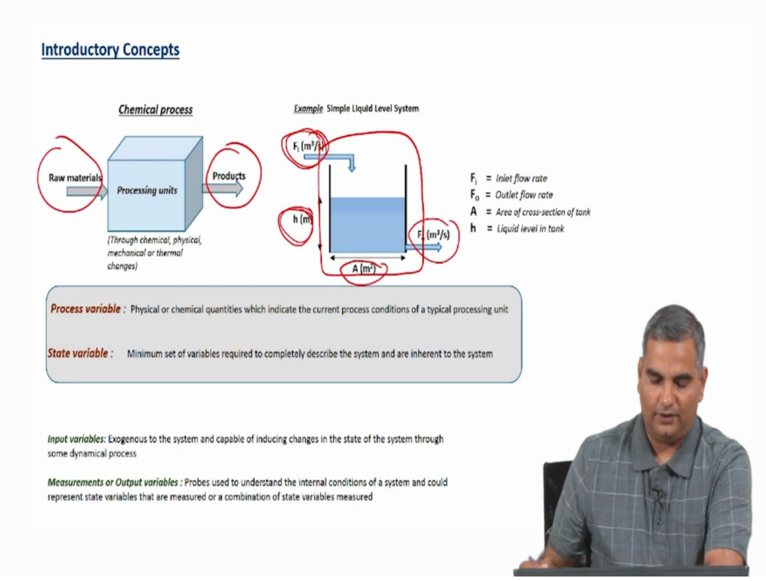
Welcome to the third lecture of the course on process control, analyses, design and assessment. In the first two lectures, I introduced the general notion of feedback control. I gave you a real-life example of control, how we use control while doing that we thought about several concepts one of them being modeling model for the process and so on.

(Refer Slide Time: 0:41)



So in this third introductory lecture I'm going to talk about modeling more generally to give you an idea of what we mean by model, what is a dynamic model and so on? And later once this introductory set of lectures are over we will look into the mathematics behind how modeling is done in lot more details.

(Refer Slide Time: 0:59)



So if you take a processing unit, it could be a chemical process for example. What you have are raw materials coming in and then you have products and then you can think of this box itself as the processing unit a plant which basically works converting this raw materials to products. So whenever we talk about models for a process basically what we're saying is, we're going to mathematically capture how this transformation occurs in the process.

So when we talk about mathematically capturing something occurring, basically what it means is that we're going to write equations that relates the outputs to the inputs to the process. And these equations can be of several types. The most common type of equations that you would have seen is the algebraic equations, these equations could be differential equations ordinary differential equations or in more complicated cases it could be partial differential equation and so on.

So in general, for any complicated process if we mathematically model the process we are ultimately going to end up with equations. That's the first idea that you want to remember. Now how do I come up with these equations, there is a very structured way of thinking about how will I come up with these equations which is what we are going to see as we go forward in this course.

Now this is for a complicated process in this lecture will start with a very very simple example to give you some nomenclatures and to give you some idea of what we mean by a dynamic model and a simple model, here I'm not going to talk about the whole process of modelling which I will do later. But simply appealing to intuition we can write a model for this, so that we can understand how we define this variables and how we use this variables in a model.

So if we take a tank for an example, a very simple example, I have flow let us say into the tank and I have flow out of the tank and there is a height of the tank. So if I want to model this process then basically what we are saying is we going to write equations that relate this variables to one another?

Now also there is an area for the tank which is given in meter square, let us assume that this tank is rectangular tank or a cylindrical tank with cross-sectional area A. Now there are several variables here so we have Fi, F not H, A and so on, so I'm going to use or define from nomenclature. Process variable that indicates the current process condition, in this case I would say F i is some process variable it's a very specific process variable which will come to later, F not is a process related variable, H height is a process related variable however we would call this A as a design parameter, the A value the area of this tank is fixed once we have the tank in place that is once it's been designed.

So it's not changing while the processes operating. So we might think of variables as being process variables or designed variables such as A. Now, these process variables could be further categorised as input variables, output variables or measurement variables or state variables, so I'm going to explain each one of these in this lecture. State variable is an important concept and it's also slightly difficult concept in the sense that, you know sometimes it feels that we called some variables as state variables arbitrarily, sometimes it feels that we could have equivalent sets of variables being called as state variables and so on, so you need some experience to understand this fully.

But in very simple terms, a State variable is one that is very critical to the process or which is inherent to the process or knowing the value of which I can characterise the process very well. So that is very simple game and definition of State variable. So if you look at a process like this, then the set of variables that I'm going to call as state variables are the ones which are the minimum set of variables, which completely describe the state of the system.

So let us see what this means in this simple example, so if I draw this tank as my system, then you notice that this flow F not is coming outside the system so in some sense it characterises the system. Now this H is an inherent part of the system because its height of the tank. However if you look at this Fi, Fi is a flow into the system, so if I draw my battery limits for this process as this block, then Fi feeds in to the block or its input variable or it is exogenous to the system which is outside the system. Something else is changing this and this comes into this tank, as this picture is now.

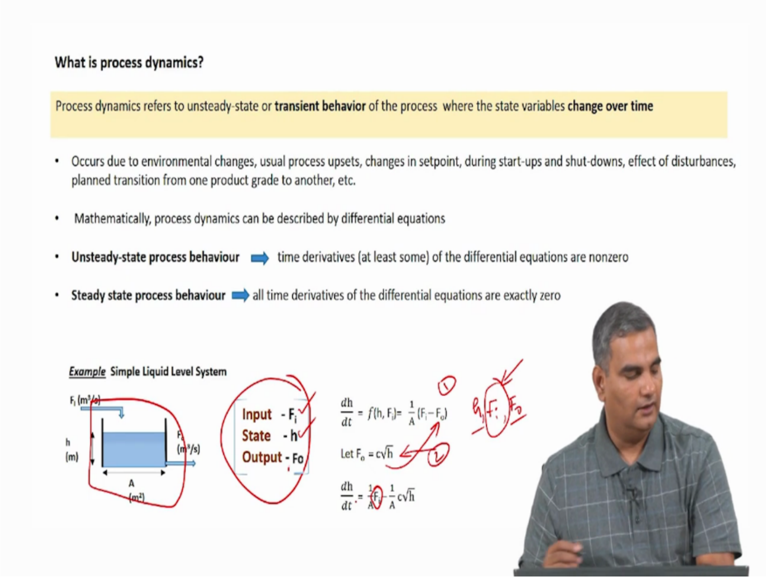
So now if you look at this, then I would call this H as a State variable because once I know the value of H, then I can completely characterise the system. What I mean by this is, if I know H at any time, then as long as I know this exogenous variable value at all times, then I can completely characterise the system in terms of how much water is there in the tank and what will be the output flow rate because this output flow rate will depend on the height.

So once I know height and I know Fi which is exogenous to the system, then I could for example write a model an integrated and I will know the value of height at all times. And I will also know the value of F not at all times, so that is the reason why we call this a State variable.

Now in this example there is only one State variable, but in other examples there could be more than one State variable and in some examples that could be hundreds of state variables and so on. State variables definitions are not unique, if I have three state variables I can come up with three equal and state variables and so on, so that's something that we should remember.

And one simple way of defining a State variable for the type of models that we're going to look at or in general these are the variables for which we write a derivative and I will explain this in more detail later. So as far as this example is concerned, I have the State variable H, I have the input variable Fi and I might say this F not is output variable is a design parameter. So which is what, we have written here.

(Refer Slide Time: 8:03)



Now the process dynamics refers to the unsteady state of transient behaviour of the process where the state variables keep changing over time. So, if you change the input or if you suddenly changed the State variable from where they are, these two changes are going to have an effect on the state variables and how they change over time and capturing this change in the State variables overtime is what is called a dynamic model. It basically captures the unsteady-state of transient behaviour of the process. And you might ask why do I need to worry about dynamic models?

In real processes there are always changes that are occurring. the changes could be due to environment, they could be due to process upsets, they could induced by us we want to changes at point, remember the Servo control that I talked about before or that might be a plant which you are starting up so there will be dynamic changes when you start ups. And there are times when you want to shut down the plant for maintenance and so on, so you will have transience there, you could have disturbances, you could actually move the plant from one state to another because you're producing let's say polymer of type A, and then you want to change the process to produce polymer of type B and so on.

So these are all the reasons why dynamics is introduced into the process. So mathematically, we want to represent these changes and mathematically we want to relate the variables in a process. So the typical type of equation that one writes are differential equation and once you have a set of differential equations, then if you have unsteady behaviour at least one of the time derivatives will be nonzero.

So once the time derivative is nonzero, then there is some dynamics. Now if the all the time derivatives are zero, then you have what is called a steady state process behaviour where there are no dynamics involved and those types of equations will be algebraic equations. So, if you take the simple liquid level system as an example, you can write simple set of equations here the fundamental basis of this equation is basic mass balance and we will derive this in more details carefully in a little lecture.

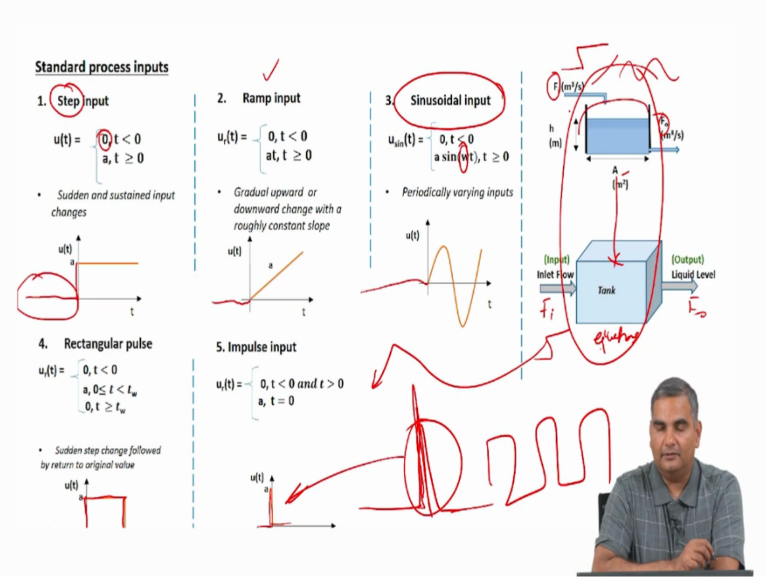
For now, in this introductory lecture, I just want to show you form of these equations and how we're going to relate these variables through equations. So if you write a simple mass balance on the tank liquid, then you will get this equation where DH by DT is one over A, Fi minus F not, Fi is input variable, H is State variable, output is F not. Now if you look at the equation, this one equation, then I have three variables in this equations, I have H, Fi and F not.

Now, we generally know that if I have to solve for three variables I need three equations. Whereas here it looks like I have only one equation and if you think about this little more carefully you will notice, that since we're writing the equation for the tank and remember I drew a battery limit something like this, that means the system equations themselves cannot define Fi. So Fi has to come from outside.

So what you need to realise is that this Fi comes from outside so we do not have to count this as a very able for which we need to write a model equation. So that leaves us with H and F not, 2 variables but I have only one equation, you can write the other equation for relating F not and H in this form, we will again describe this in greater detail later. So now I have one equation here, 2 equation second equation here, in three variables H, Fi and F not.

Fi is external to the system, so I can solve this system of equations only if I know the value for Fi and once I have the value of Fi, since I have two equations I can solve for H and F not. In fact, what we can do is we can substitute this F not here and then you'll have a equation which has these following form DH by DT is one over A Fi minus one over AC root H and you notice that these equation is a differential equation in H and Fi is what is called the input variable or forcing function for this equation.

(Refer Slide Time: 12:49)



Now, that we have written the equation then the next question is how are we going to use this equation in modelling this process? So this equation or these equations are supposed to tell us how the output will change when I make changes to my input. So in this case if we notice that this is the input and this the output, so physically what we're asking is the following, if I change this input so let's say this is a constant flow and if suddenly step up this constant flow, how do you think the outlet flow will change?

Or if I suddenly ramp up this inlet flow, how will the outlet flow change? Or if I modulate this in sinusoidal fashion how will it change and so on. So, if you look at the left side of the slide we have shown various standard input forms and the idea here is, if you take the step input for example so what we're saying is, prior to time 0 so we arbitrarily say time zero, whenever we start that experiment that is time zero.

So it does not matter when we start, so if you started at the third minute that it is time zero, if it started at fourth minute that's time zero and so on. So, if you think about this so I have an input which is constant till here it is at zero and T is less than zero. And right at time T is equal to 0 I increase this input to value A. So I step up the value of the input, so physically that basically means that this F let's say it was at some flow rate five units I suddenly step it to 10 units or if it's at seven units are suddenly step it up to 10 units, okay.

So you might wonder I say five, seven and so on here, but here I have zero, so basically what we do is whatever value I am at I make that zero, so for example if I am at five and then I step it up to 7 that is equivalent to saying I'm at zero and I am stepping up by two, right?

So, whatever value of U I subtract the steady state from that, then the first value is always zero. So we will discuss this in more detail in terms of how we remove the steady state from these inputs and so on later. But the basic idea is what we are interested in is in the change and we can always make the original value zero by subtracting these study state values. So, just as the name suggest step is stepping up the input.

Now, ramp is this form is zero for T less than zero and U is AT greater than equal to 0. So basically I have something like this straight and then its ramps up from here. And the slope is given by A. So this is the ramp input, this is another standard input. The third type of input that we show here is is what is called sinusoidal input, here we say okay this use zero at all-time and suddenly I start moving it in sinusoidal fashion.

There is a frequency related terms here omega or W and this is a very important input so, we will later see something called frequency response analysis and you will see this is a very important input in terms of this frequency response analysis. If we had let us say 2 steps, so if we have initially liked this at T is equal to 0 I bump this up and then hold it for some time and after a while I bump it down, then this is called a rectangular input which is shown as 0 T less than zero, it is A between T and TW and after TW it becomes zero again, so this is called a rectangular pulse.

You could also have an input which is very short acting, okay. So for example, in this case I have zero up to here and then suddenly I give A and then immediately come back to 0. So, it is not really realistic to have value for only time T is equal to 0 because it cannot have, so this looks like at time T is equal to 0 I have zero value A back again to 0. So this is an idealisation of something that is sharply changing, so if I have here and something like this, so, this is an impulse input.

Mathematically you could say something like this it is at A when T is 0 and every other time it is zero. So this you might think of as a mathematical idealisation of this, so you can get closer and closer to mathematics by doing the sharper and sharper and so on. So, the reason why we describe all this different types of input is that these are types of input you’re going to see when we discuss the control and design and the frequency response analyses and so on.

Now, I just don’t want you to lose the connection between the physics or the engineering system and the mathematics. So if you want to kind of map this all we are saying is if I suddenly step of the input flow that is step input, if I slowly rapping up the input flow that is this.

Supposing this flow I am increasing, decreasing, increasing, decreasing in a periodic sinusoidal fashion that is this input, rectangular input I suddenly increased hold it for that value for while then suddenly decrease it back to the original value and impulses I suddenly open and have a lot of flow grain and quickly close so that is input. So, what has happened now is the following.

This is the physical process, this is a block diagram conceptualisation this is the math okay. So this tank now the physical process has become a block, now this input flow is Fi which is physically the input flow coming in and you could say the liquid level or F not both are equivalent here is the output of this process. Liquid level and F not are related to each other. So, how this input could vary we have shown through this multiple possibility.

And how do I convert this input physically when I change something clearly the flow is going to change, so for example, if I step this flow up because initially if this liquid is at steady state that means Fi and FO are the same, otherwise the liquid won't be steady. So at steady state Fi and FO are the same. When I step up this Fi that means the improved flow rate has increased whereas I have not done anything to F not so the output flow rate is still at F not, then what will happen is.

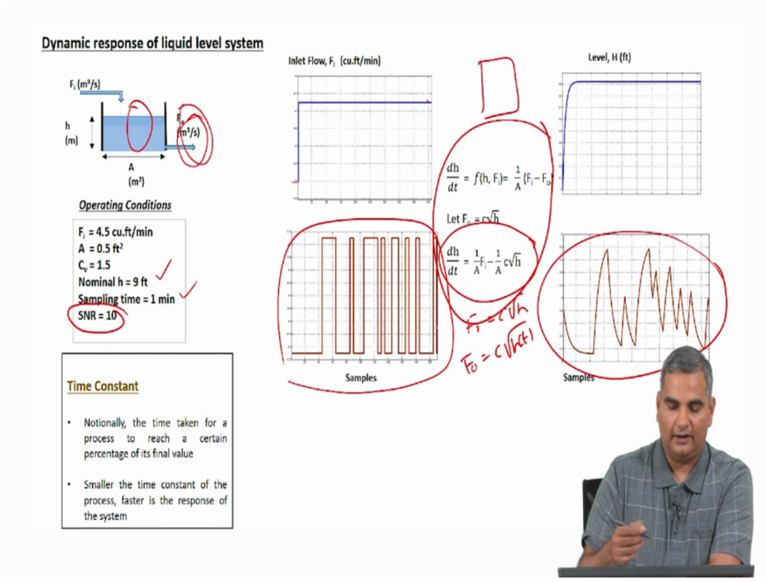
Because input flow rate has increased without output flow rate changing the height of the tank level will keep increasing, the minute this height increases the output flow rate will also start increasing, so after a while you will get to a steady-state. So what is that which is physically going to happen, so what we're trying to do we're trying to mathematically capture that physical behaviour by saying if I give you this what will happen to this F not.

And the way will capture this tank, this tank has now become your set of equations. So this block is the is representing the set of equations that basically tell you how physically if I make this changes here what I will observe and measure I want to mimic using this equations. So this is an important concept that I would like you to understand. The block is always some mathematical representation of what is physically happening so, after a while when you're doing this course, we will start talking about only the math, we will have this input and this output comes out and so on.

But I don't want you guys to lose this physical connection. So whenever we're talking about input which is a physical input and whenever we are talking about output it is a physical output and in fact, what we're saying is once we model this we do not have to actually go to the tank and to these experiments, we can run this model and then tell you what will happen, so this might be the input, so supposing I asked you something like this if I keep changing Fi like this what will F not be?

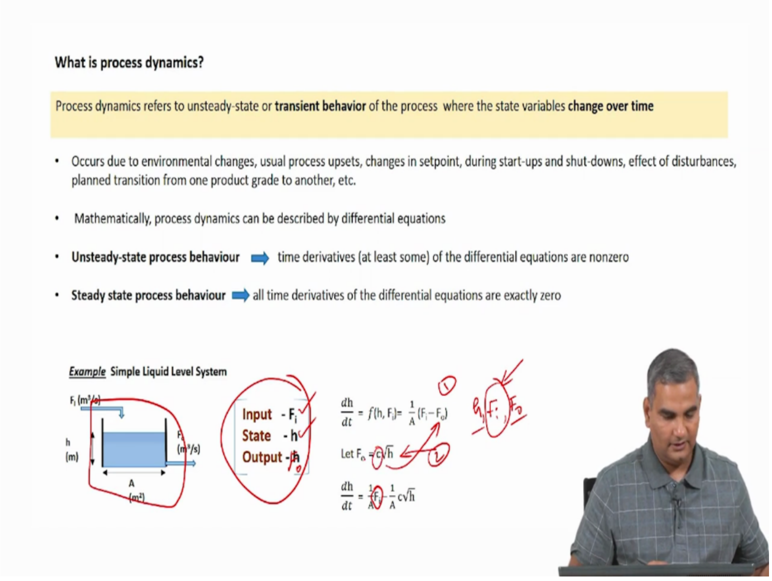
I do not have to go and do this experiment here in the tank to be able to get that; I can basically get that out of my simulation.

(Refer Slide Time: 21:12)



So, what we have done in this slide is for example, we have actually taken some values okay. So we'll say okay initially let us say the height is about 9 feet and sampling time of one minute and remember if you go back couple of slides.

(Refer Slide Time: 21:31)



I have this constant C here which I have is 1.5 here, A the area is 0.5 ft square and let's assume I'm originally at steady-state value of 4.5 cubic feet per minute. So since I talked about steady-state at that point F not will also be at 4.5 cubic per minute. Now, when we give an input so the first input that we had in the previous slide was a step input, let's say I step up my input flow from a particular value to a new value immediately.

Now, I want to know what will happen to the height and subsequently to F not. So in this case if you do the simulation and you can calculate the height of the tank using this and once you get to a new steady-state value of the height of the tank you can always get to your F not as C times root of that the final value or at any time you won't understand the dynamics of F not and not height it is simply F not is C times root of H at any time, right? So any time you want to know what outlet flow rate is it's going to be C times root of H and that H is going to change according to this equation.

So H and F not in that sense are equal and as far as we're concerned for the explanation of this slide. So if I step Fi to a new value, then the equations when we solve this will show that the height will change in this fashion. This you can actually observe if you actually do this experiment because once you start increasing the inlet flow rate that is a step up the inlet flow rate to the new value.

As I mentioned before because the outlet flow rate still going to be at the same value initially because of the flow mismatch there is more coming in less going out, so height will start increasing. Initially the height will start increasing rapidly but the minute height starts increasing the flow at the outlet will also start increasing because it's related as C root H. And since the flow start increasing the mismatch between the inlet flow and outlet flow is going to be reducing and because of that the rate at which the height increases will keep reducing, so the rate at which the height increases will keep reducing till the rate become zero.

So the rate at which height increases keeps decreasing till it becomes zero at this point there is no change in height, so this is a new steady-state and correspondingly you will have a new outlet flow rate as steady-state. Now, as I said before supposing you were to ask this question and say look, I want to change my inlet flow rate like this so basically physically what we're saying is.

I won't increase it wait for some time decrease it back to its original value, wait for some time increase it back to this value and then wait for a much shorter time bring it back and so on. Supposing I have input function like this and then I asked a question how will the height change, one thing is that you can do the experiment, the other thing is actually you can use a model that has already been built and then say this is how this is going to look. So this is the output of the model for the height and actually you could do the experiment and verify this is true or not and these plots have been generated using this equation.

So really physically if you think of it as a tank here where the input changes and height changes. The tank is being represented by this equation. So the model then becomes a surrogate for the physical process. So that we can really work with this model instead of the physical process in designing our controllers, analysing our controllers, testing our controllers, assessing our controllers and so on so that is the key idea of building a model.

So this is a very simple process example for us to describe how model works. Now, this will get more complicated and also how do you go from here to here by solving the model is an important question that we need to answer. These are questions that we will answer as we go through this course.

So in this lecture, I have given you an introduction to modelling and how models are used and control. In the next lecture I will talk about different control structures that we will use to look at controlling processes efficiently. And once that is done, then I would have given you good introduction of how do you think about control system, what are the important notions in control, how do you break a memo control problem into multiple SISO problems.

And each SISO problem we need a model, what does a model mean? What is a model doing which is what this lecture was all about. And then once you get a model, how do you use it with control structures so that we can actually do control. So with that I would have finished introducing you to the main ideas that will see.

And then after that you will actually talk about how you get these model equations in a systematic fashion. And how do you convert these inputs to outputs through these models in a systematic fashion, and how you actually designed controllers so that they can use these models and so on.

So with this I will end this lecture and I look forward to seeing you in the next lecture. Thank you.