Process Control – Design, Analysis and Assessment
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Nyquist Stability Criterion – Part 1

We will continue with our twenty-fourth lecture in this course on process control analysis design and assessment, so in the last few lecture we have been talking about control of time delay systems talked about what time delay means, why is it relevant in practical situation the difficulties that come because I we have a time delay term in the transfer function which takes the transfer function away from the ratio of polynomials form and we talked about how we can control such system and we introduce the notion that you actively use the model in the controller implementation also and that allows as to control these time delay systems effectively and the end of the last lecture I also left with a question as to how we are going to look at the stability of the systems.

Now that we know how to think about controller and design implementation for a given performance matric the next question is once you have designed a controller based on performance since we are giving up a little bit for performance and so on we have to also look at whether the design of stable when will it become unstable and so on this particularly important because while we will have a G desired which by design would be stable so for the nominal system that is if the model and the process have the same transfer function the closed loop system is going to be stable however the real challenged is to really think about what happens when my process is different from my model.

So will the stability idea still hold and we will see how to analyze stability of systems with now the reason why we need a new idea and which is what I am going to introduce in this lecture called NYQUIST stability criterion it is because whatever we have been looking at till now is not useful anymore for time delay systems mainly because we have this E power minus tau DS term in the denominator and once I expand this as polynomial I can expand it like an infinite polynomial so conceptually I infinite poles which will be difficult to handle either in a partial fraction approach or route stability table, so we need to come up with some other idea and that idea that is used is what is called NYQUIST stability criterion this is pretty sophisticated mathematical idea at an undergrad level for engineers, so what I am going to do is I am going to explain the basic fundamental behind this idea without showing you proofs or getting into very complicated mathematics, so I am going to basically make you understand the result of NYQUIST stability more than actually trying to show you why the result is true.

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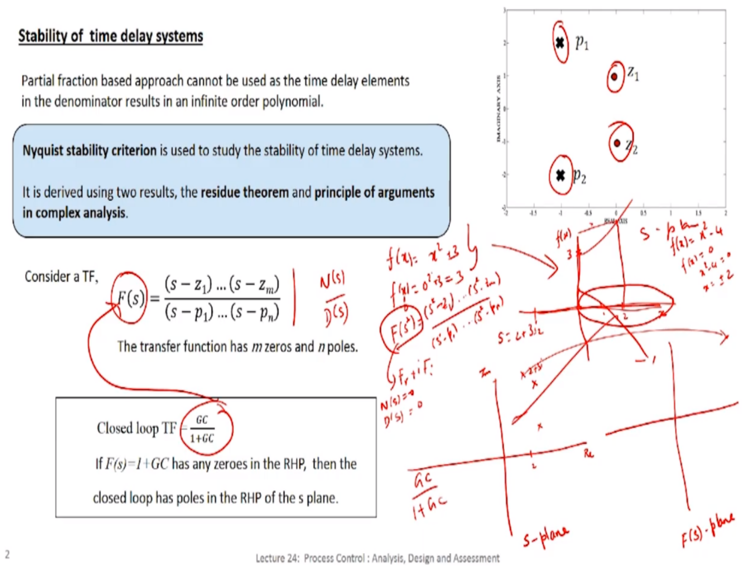

So to do that we have to think a little bit about complex variable and how they behave and we have to think about a little bit of mapping from one plane to another and so on, so let me start by explaining the key idea behind NYQUIST stability criterion and what we are going to do is I am going to generally explain this idea using simple systems which are still polynomial by polynomial form to get you to the basic idea and then we can look at how this is useful for studying time delay systems, now we have always been talking about a transfer function it could be G of S or as we have written here F of S and a transfer function has poles and zero and if you take a transfer function of this form then we say that this transfer function has M poles and M zeros and N poles.

Now at the beginning at this series of lecture we asked what is a an then I said S is a complex variable, which can take some values now we are going to expand on that idea and then explain NYQUIST stability result with that the result that I am going to show is based on two other results one is called the residue theorem and the other one is principle of arguments in complex analysis we are not going to go into either of these idea in any details at all what is more important is for us to understand how to use this ideas in stability analysis for control systems, so the very first idea that I want to explained is the following.

So think about this just like a function, so if you feel let us say I forgive you F of X let us say X squared plus 3 let us say it is a function that I give you then asked you what is the value of F of X at X equal to zero then you will say F of X is zero square plus three it is three, so F od X is three equal to zero, so you can think of this F of S or any transfer function also in a similar way and basically you can evaluate the transfer function at any value that I give so for example think of F of S just like F of X but now it is a function of a complex variable and if I want to evaluate F of S at let us say a star then basically you are going to say F of S star is instead of S you are going to substitute this value a star minus they Z all the way up to S star minus ZM divided by a star minus P1 all the way up to S star minus PN the only difference is if I want to explain this.

Let us say in a plot all I would need is basically one 2D plot by this what I mean is I could plot this X on the X axis and F of X on the Y axis and I am done, so for example X equal to zero is a point where F of X is three, so this is point on the curve at X equal to zero F of X is three now if I set X equal to one then F of one is one squared plus three which is one plus three four, so at one this will be let say four and at two this will be two squared plus three, so seven and so on so you can draw this picture what whichever way it turns out something like this, so all I need is one 2D picture F of X versus X and I am done with this.

So this is how I represent this now if I were to think about plotting this in a similar way I cannot just do with one plot the reason is the following number one the argument here X has only one value of course from a complex number view point S has a value but that S has a real part and an imaginary part so this itself this X itself when it becomes S needs to access to represent variable so if I told you S equal to let us say two plus 3I then to represent this S itself I need to access, so this is the real axis and the this is the imaginary axis and this is real part is to imaginary part is three so this will be two plus 3I, to notice the difference here when I look at XF of X very simply I can look at a 2D plot and able to plot this whereas the minute I came to S basically I need a plane or a 2D plot to plot S itself so this plot of S the different value that I can let S take itself is called an S plane so this is something that is important to remember so think about the transition from here to here, now what happens to F of S right

So here if I put a value X I get one value of X, so that is the reason why I could just do it with two axis here however S itself has become an S plane and when I substitute a value for S into this equation then I am going to get F of S which is going to be a scalar however this F of S is going to be complex number again because S is complex that one could be complex P1 could be complex and all of those, so F of S itself is a complex number so it would require so for example if I put S equal star and this turned out to be some real plus I F imaginary this is the complex number value turns out to be then to plot that I need another plane which is what I am going to call as FS plane so this is a very important concept that you should remember.

So if I want to find what the F of value is for an X I go all the way up here and then simply read out and then that gives me the F of X value but if I have to pick a value here and then I say what is F of S then I have to go back to this plane and then say this is F of S, so this is a difference when we come to complex numbers so this is the first important idea that you should understand and the reason why we need these two planes is because S is a complex number and F of S is also a complex number now if I let us take another example here let us say if I have a function F of X equal to X squared minus four then if I want to find out where F of X equal to zero then I will write X squared minus four equals zero X is equal to plus or minus two.

So when I want to find a particular function where it goes to zero then I correspondingly solve this and then I will get a value for example this is a different function this X squared minus four, so I will say plus two and minus two value at which this function goes to zero and those could be plotted in this axis itself however if you ask the same question in terms of F of S plane which is of importance to us from control view point and we asked the question when will F of S is a numerator by denominator B such that or in other words when what are the points of zeroes of F of S and poles of F of S which basically will translate to asking the question just like how I put F of X is zero asking the question when will enough S go to zero and when will D of S go to zero the point at which an office goes to zero or called the zeroes of the transfer function and points at which D of S goes to zero are called the poles of the transfer function.

So just like how I put F of X equal to zero if I put N of S equal to zero then the solution would be S will in N of S will go to zero at S equal to Z1 Z2 Z3 all the way up to say, so there are M points at which N of S will go to zero notice something interesting, so in this plot if I found out where F of X goes to zero I can plot that X on this line but whenever I find the zeros of this transfer function that is this transfer function is retained in the NS over TS form and the zero of the transfer function means I am asking when will N let us go to zero those points cannot be plotted here those points will have to be plotted in this plane so basically this single line has become a plane and this single line has become a plane.

So if I ask for numerator going to zero then the corresponding points are plotted here, so if I look at this plot and then say if I am plotting this then I might plot this as a zero point this as a zero point so on so you see that plotted in this plane so the key idea is that the zeroes of F of S with which are values where N of S goes to zero will have to be plotted in the S plane not in the FS plane, so that is an important think remember just like here if you say F of X is zero then you will find the value where F of X goes to zero X on the X axis now this axis since it is become a plane and this axis it is become a plane, so I look for those in the S plane similarly if I ask there are the poles of this transfer function F of S then that basically means I have to set D S equal to zero and those again are going to be values that S takes such that D of S goes to zero and those again will have to be plotted in the S plane not the F of S plane.

So this is an important thing to remember this are all simple idea but since you might be seeing this for the first time I just wanted you to get a feel for how we go from simple real number functions like this and then go on to complex functions and the key idea is that the argument itself I need a plane to describe which is a complex number and the resultant is also a complex number and I need another to discuss that describe that one is called the S plane another one is called FS plane and I could ask questions about the FS transfer function and put and results in the S plane, so that is what I just explained if I ask for what are the zeroes of F of S that means the numerator polynomial equal to zero.

So the S value at which the numerator goes to zero will have to be plotted in the S plane and if I ask what are the poles of the transfer function those are the values at which D of S goes to zero those values again will have to be plotted in the S plane, so here is an example where I have profiles which has two poles and two zeroes in the S plane, so this will be the S plane and we have not even plotted there for F of S plane, so this is the first and important idea that you have to remember when we discuss NYQUIST stability criterion that is going to come subsequently, so why are we doing all of this just to give you a context for doing all of this.

So basically what we want to do is we want to find out the stability of GC by one plus DC which is this so it is not in the polynomial form because I have this E power minus tau DS term in the denominator I am still going to look for how to understand the stability of this and basically some sense I am going to use this for F of S and start thinking about the stability.

So ultimately once I teach in the polynomial form I am going to say the same results hold even if you have you know time delay elements in your transfer function and basically we will replace this F of S by GC by one plus GC and then we will be able to study the stability of time delay systems that is the thing that we are going to look at now that we have understood the ideas of S plane and F of S plane what we are going to do next I we are going to look at how we identify how many poles are zeros are there in some locations in the S plane by looking at combination of this S plane and F plane diagrams is.

What we are going to do so in the next lecture I will describe how this is done and then this I will do for very simple example, so that you understand the basic ideas and then I will describe how this translate to thinking about control of closed loop transfer function remember our problem is still guaranteeing that and there are not poles in the FHP for a time delay systems, so how some of these ideas translate to the real problem that we want to solve I will also describe in the next lecture thank you.