Process Control: Design, Analysis and Assessment
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Traditional Advanced Control Part - 4

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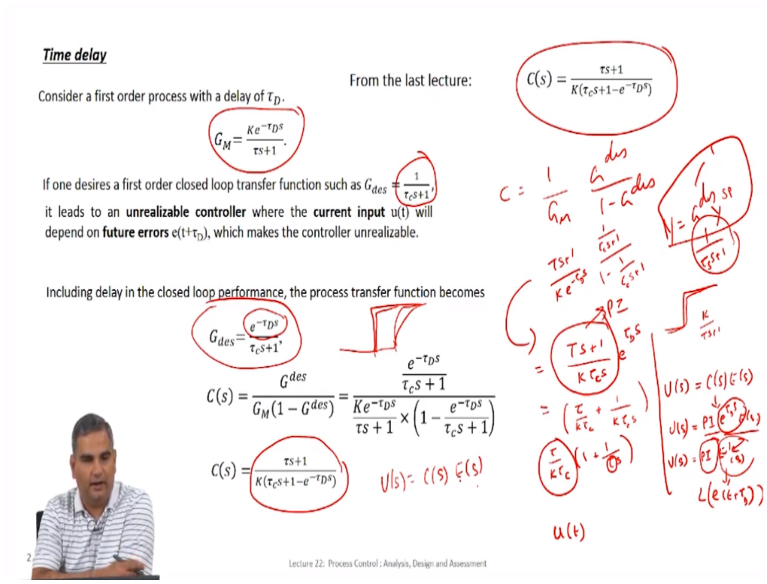

We will continue with the lecture control of time delay systems in the last lecture I showed you what a time delay system transfer function is I explained physically why time delay occurs in terms of transport delay and so on and drive the controller which basically take this form right here and that is where I stop and then it said for the first time we have a situation where the denominator is really out of polynomial in the transfer function model, so we have to think about how we are going to handle this and as I said in the last class since there is the first time we are saying this we have identify what are the challenges that we might face and what we need to do to solve those challenges and basically the idea is in most of what we try to do once we find the solution for a type of problem anytime this is some variations of the problem we try to look at whether whatever solution strategy that we already have is applicable and it might not be directly applicable.

So we might do some approximation of our current situation so that the solution that we have already seen could be readily apply so we will try and see if we can do something like this in this to see if we could use whatever we learn to solve the challenges that we face so quickly as a recap the one thing I just wanted to point out again is that if I have a G model like this then when I design a controller you should be very familiar with this equation now one by GM he decided by one minus G decided and then if we were to take G desired as a first order that transfer function which is what I want ideally where tau C is else as small as possible so that.

Whenever I give a set point remember just as a recap Y is G desired Y set point if the model and the process are exactly the same that so we derived this equation for a controller now of this is first order tau C a plus one with a very small time constant then basically what it means whenever I have a step up the output is going to follow this step very quickly throat because again this is one so that is reason why we are always wanting the G decide to take this form in fact you could think of this as the simplest from in the most ideal form we already discussed is one Y is Y set point but that basically means there is no dynamics associated with the response which we already seen it is not possible.

So we introduced the dynamics but we want introduce dynamics so that the dynamics is not very whelming so it is very short dynamics so this tau C has to as small as possible so that is the reason why we take decide like this and another thing I would like to point out here is the gain of the G decide transfer function itself should be one so there is no point in taking decide to be something like K tau S plus one where K is not one then what you are basically saying is Y is not going to be Y set point but some gain times Y set point which something that we do not want so in all the examples that we keep choosing you will notice that the gain of this G decide transfer itself will be one keep this equation in mind every time you think about G decide this will kind of keep you on track in terms of understanding this ok.

So from time delay systems we have already seen this I just wanted to recap this so that you understand again why we make this change here so we said if we simply substitute everything here G model is for example tau S plus one by K E power minus tau DS as G desired is one over tau CS plus one divided by one minus G they say tau CS plus one, CS plus one and tau C L plus one will get cancelled tau CS plus one minus one will be tau CS so this will lead to tau S plus one divided by K tau CS and then this is what I am deriving a power minus tau DS will become a power tau DS now it is very easy, so whenever you get this form you notice that this is a PA form and we will see this again and again, so if I simply expand this out then this is basically the first tau S by K tau is so that will be tau by K tau C K tau C plus one by K tau CS so this is typically form that you keep getting.

So now if you take this tau by K tau C out the first time will be one and the second term I already have K tau C I have here but I get an extra tau so offset that I will get tau S, now this you will see is proportional gain and this is the integral parameter in a PA controller so whenever you look at this form you can always do this manipulation and then basically get it into the PA form that we have been using with the corresponding gain and an integral controller any case so this is C so basically we know that U of S is SC times E of S so the CS has two portion ,so this is the PA portion so I am not even going into time domain I am going to explain this in a plus domain in itself there the difficulties.

So you can think of this C as having two portions typically we will have only PA controller acting on error which is what we have been seeing all the time but in this case I have the CS as a PA portion and an E power tau DS so that is what I need to compute to get my U of S, so this without going into time domain just a Laplace domain I can save this is US a PA portion which is a controller and E power tau DS E of S is going to be E power in time domain when convert this what is going to happen is this is going to say somehow I have to get this E prime S and this E prime S which is going to be error P plus tau D Laplace of that so if you think about this in the Laplace domain I can write this E power tau DS as I promise and this E prime S is going to be Laplace of this E of T tau D.

So if you now think about this from the beginning we have been saying if you want to invert and then actually get an U of T then you have to do a convolution integral between in this Laplace function and this Laplace function whatever the time equivalent variable that they represent then what will happen is you have to necessarily get value for E of T plus tau D that basically means to make decision about U of T the input at time T you need error at T plus tau D which is not possible that is the reason why we call it array unrealizable controller so this is an important idea to remember so we can think of this mostly in Laplace domain up to here and here it is basically two functions so it is going to be convolution of one by the other so this function is a PA function this function will be basically E T plus tau D.

So that is an important thing to remember I thought I will reiterate this so that you see why this becomes an unrealizable controller so that is the reason why if you want get rid of this E power minus tau DS going to the numerator and creating problem in terms of E power tau DS making the controller unrealizable we introduces this E power minus tau DS and EG decide transfer function itself so this is basically saying because there is time delay term in the model I am going to simply say there will be some performance loss and I am willing and to accept that performance loss otherwise I cannot even get a realizable controller so I am just going to say let me accept that and the performance losses E power minus tau DS.

So if you look at this G desired function you will notice that the gain is one again even though I have introduced a power minus tau D is the gain is still one the means ultimately I will get to my Y at set point value however I will have a time delay so if there is a Y set point change that is made here Y will do something like this ultimately it will go there but there will be a time delay here and this is the performance loss I have so ideally what I would like is something like this possible but what I get something like this of tau C is small I will get it as close as possible to this here again this will just be shifted by tau D which is basically something you do not want but there is no way around.

So you basically say let me lose that performance so the consequence of that choice is that when you actually compute these CS you get G decided GM times one minus G decided when I plug in all of this and now you notice that I have CS is this so if do US is CS time ES now here I had this (())(09:52) and a PA now I do not have that so I have some other form which does not look like a PA but this is some other form but that is basically going to act on only error at time T, so at least the unrealizable problem we have gotten rid of however we have a new problem because we have a controller transfer function which is in a form and that we have not look that before and we do not know how to handle this at this time so we have to figure out what are the ways in which I can handle this transfer function so I hope till now it is clear in terms of why we have to introduce this time delay in that said and why that create this new headache for us which is that the CS in the numerator by denominator form which we are very comfortable looking at and for which we have all kinds of tools in terms of how to implement these hoe to invert these and so on.

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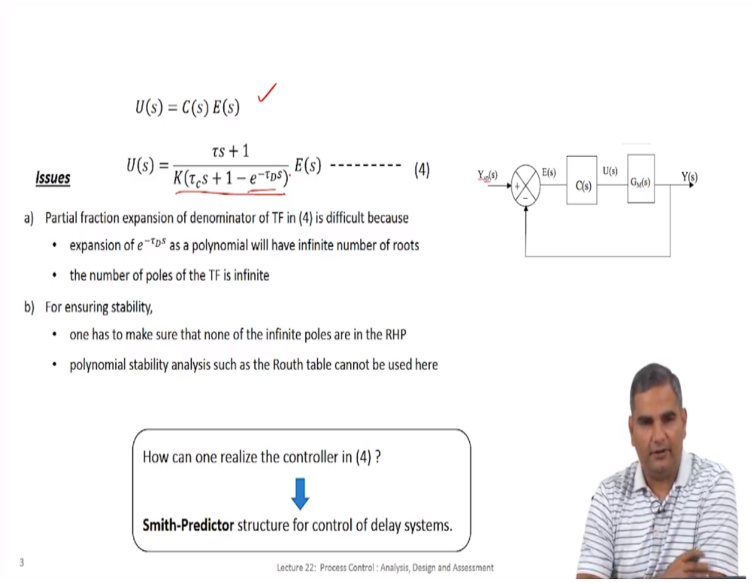

So you will have to see how we are going to do this so again I have just written the same thing here I have an input which would be a function of the controller times the error and the basically gives me this equation typically what I have been doing is I have said this is a numerator by denominator polynomial usually we can do Laplace transform and the convolution to kind of identify what you should be so those are things that we have been talking about however now I have this the denominator which is not a polynomial the problems the is the tool that I have taught you till now is partial fraction expansion but this is not possible here because I have E power minus tau DS and I am going to kind of talk about this in slightly simple perspective I am not going to go all very exact mathematical details but just to give you feel for why the problem comes and how we can think about addressing this I am good to kind of put this back into the partial fraction framework and then ask the question if I know how to do this if I have a numerator by denominator polynomial.

So is there somewhere in which I can convert this polynomial so that I can continue to use the techniques and tools that I have in my bag so one way to do this would be to kind of expand this as a polynomial and we all know that in the expand E power X you will get a polynomial of N and order you keep adding more and more terms and if you conceptually think about this without worrying about all the mathematical details you will notice that I am going to get a denominator polynomial which is going to have infinite roots so basically there are going to be infinite terms in the partial fraction so which is going to be a problem so that is problem number one when I have this time delay term of course the consequently the other problem is if I want to look at the stability of the systems it looks like an even bigger headache because if I think of a large number of poles through a polynomial expansion then I have to ensure that all of this poles are in left of plane so if even a few of these move on to the right of plane even if one of these move on to the right of plane then I have problem.

So one has to ensure that the large number of poles none of them are in HP so basically the sequence in which we have done this all of this competition we have done in using partial fraction that is not possible anymore is the first thing that I want explain and number two is that if it is simple partial fraction then we know how to do stability so if we get it in the root result form and we look at all the rules if all of them are in LHP we say the system is stable however because partial fractions is not possible we cannot do that for first stability and of course even when partial fraction ideas were not directly possible we used table to table to think about stability and if you want to see whether we can actually use root table here that is also not possible because for a root table even if the denominator is not done in the root result form we can only work with finite polynomials whereas here because can keep expanding E power minus tau DS the polynomial terms are also not finite so doing not table is also not possible.

So the question is how do I invert this so that I get my partial fraction ideas working or I have to find somebody I think about it and of course one size figure that out then I have to worry about the stability of this systems how to address the stability of the system when both the notions from partial fractions that we have talked about and roots stability both the root table both do not work anymore so one standard technique that we could adopt is to say look I am going to use a Taylor series expansion and I am going to use only a few terms and then see how good the approximation is and then proceed with doing everything that I have been doing till now notice this if I do this polynomial expansion then actually I can put this in the numerator by denominator polynomial form then all the techniques that we have learn come back into play I can use any of those techniques and then I do not need to do anything more so conceptually if I were to you live with using this polynomial expansion up to certain number of terms I do not need any other new idea to do this I can simply follow what I have been doing so typical expansion for E power minus tau DS would be one minus tau DS plus tau DS whole square two factorial minus this cube by factorial and so on.

So you can keep going now you say ok I am going to use only the linear term so if you take this then you could put this here and then put this E power minus tau DS back into your controller equation then everything is done because I can again use a numerator by denominator polynomial idea and do the inversion and then easily live with whatever we have learn however this form so we are going to do that and I am going to teach that as one technique for dealing with the time delay systems but I am going to tell you that when you deal with time delay systems in that format it is going to be somewhat approximate this is of course grossly approximates very inadequate but there are smarter and better ways of doing it and still you can live with analyzing time delay systems using the standard polynomial based ideas that we have seen till now.

So is this is grossly inadequate is there something that better I can do better approximation for this is what is called Pade's approximation which basically what it does is it takes E power minus tau DS and then it kind of split it into two and the reason why you want to do it is that you are kind of balancing the poles and zeros that you get in this expansion so if you look at this E power minus tau DS you keep freezing the poles but nothing happens whereas the zeros are concern over here there is some balance between poles and zeros this is a much better fractional approximation then this kind of approximation you have so what we do is we write so typically in books they will say first order second order and give this to as formulae and there is a logical way in which you can get this formulae derived.

So that you do not have to you know commit them to memory so all you need to know is E power minus tau DS you are going to write it as E power minus tau DS by two divided by E power tau S by two now if you take this to the numerator that will be power minus tau DS by two times E power minus tau DS by two which will be this so you are simply writing this minus tau DS by two divided by tau DS by two E power now once you understand this first order second order approximation of part A approximation is very straight forward what you do you do a Taylor series approximation of this to the certain number of term and you use the Taylor series approximation of the denominator also do the same power understand this spare parts business very straight forward what you do if you do it Taylor series of approximation of this to a certain number of terms and you use the Taylor series approximation of the denominator also do the same number of terms.

So if you talk about D power minus tau DS by two and you are going to use only one term then you will have one minus tau DS by two and the denominator will be one plus tau DS be two now if you say I am going to do a second order of approximation then it will be minus tau DS by two plus tau DS by whole squared times two factorial which will be four times two eight here you will have one plus tau DS by two plus tau DS whole squared by eight and so on you could do the third order approximation so on, so this is a critical idea once you do that you do not have to commit these two memory it is basically this fractional formula that you are using notice that in this case E power minus tau DS you are approximating it as a system with one zero and one pole in this case you are approximating it as a system with two zero and two poles and so on.

Now we do this then basically if you notice in the previous slide I have this tau CS plus one E power minus tau DS and I have tau S plus one I have this equation here now if I just do this tau S plus one by K tau CS plus one minus some numerator polynomial by denominator polynomial depending on whatever order part A approximation I am going to use then I can multiply this by the denominator problem we will take it to the top and then simplify it I will get some numerator by denominator polynomial form so once I get my numerator by denominator polynomial form then and the basic idea are the same you can simply look at how to do the expansion of this and what with all of these other things that we have taught you so a simple example.

Let us say if I have transfer function of the form for E power minus three S divided by one plus two S then if you write your D power minus three S as a part S approximation it will be one minus tau DS by two so it will be one minus 1.5 S divided by one plus 1.5 S then you can put this into this transfer function and then you will basically get a transfer function now that you have this transfer function basically you can use all the techniques that we have talked to about which is doing the numerator by denominator polynomial you can think about getting your controller and stability all of those are things that you can start looking at so basically for time delay systems one way of designing the controller is to use for A approximation for your process and then simply go ahead and do the standard techniques that we have been looking at so if I have a G of S could with the time delay function then you can get an approximate G of S which will be I the numerator by denominator form because I am going to convert this E power minus tau DS into a Pade approximation which has a numerator by denominator and once you have this numerator by denominator form you choose the corresponding G decided that will make the overall controller one of the forms that you are comfortable with the PID form and then go through the same direction procedure you will get a controller.

Now an interesting thing to notice is this is G of S even in the model so this actually whatever we have is really a G model of S because I want to keep distinguishing between the model and the process more and more as we go through this course so even the model itself we are making it even more approximate using Pade approximation and then we do G descent and then we can get a controller so once you have that controller then if you want analyze it for a true process then you have to actually say in the control loop I have a controller which is derived based on a model for which I got an approximation based on per day approximation and then actually the GP could even be slightly different from the original model that you took so it could be four times E power minus 2.5 S so you kind of wrongly estimated the delay let us say then you can actually kind of do the analysis where if you say my process is slightly different from the model that I have used how would my controller still perform is an analysis that you can easily do now by choosing different G process that could happen.

So that we can actually study the robustness of your controller for time delay when the time delay that you have actually I have in the process is slightly different from whatever was used in the model and so what but the key idea that I want to leave you with this G model itself is an approximation of this process and we do a further approximation using part A transform it I have part A approximation and once we do that then the controller design is standard because I will get the numerator by denominator form I can actually look at how to do they design using my standard direct synthesis approach so this is one way of doing this I might also ask if there are better ways of doing this without doing this Pade approximation and if their better ways of doing this without doing the Pade approximation how do I do it how do we realize the controller and more importantly once the controllers is realize how do I understand the stability of the controller can I go away from the numerator denominator polynomial form and the idea associated with it and then bring in some other newer idea that can be used for analyzing the systems and understanding control of time delay system so this is what I am going to teach you in the next lecture thank you.