Process Control: Design, Analysis and Assessment
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Traditional Advanced Control – Part 3

In the last two lectures we looked at some interesting dynamic kind of response systems and then the difficulties that arrives out of the inverse response and the difficulties in tuning controller for inverse response system and so on so before we go on to fully multivariable control there are three different component of a transfer function which can lead to difficult dynamics as far as control is concerned that we are going look at the first one was inverse response and the second one that we are going to touch upon and today and the next lecture would be time delay so in fact that time delay is very-very important idea and this is in some sense all pervasive there is no some system that does not have some sort of delay in response and all of us have feel this time delay in real life and I will explain that you as we go through this lecture and it also turns out that controlling systems time delay is it is really difficult problems.

So in fact and this is one of the most difficult problems in control and that is the reason why I can move this all the way to this time in the course I do not want to confirm time delay ideas with other ideas which are reasonably clean and straight forward and you could teach most of it using very simple ideas such as partial fraction in motion and so on which we have already seen so now that we have started splitting the complexity of the transfer functions and the kind of tools and techniques that we need to understand this the types of systems I think now is it time to look at time delay and you will see that till now we were looking at a transfer function even if there were difficult dynamic such as inverse response we were looking a transfer function of the form numerator or denominator and they been looking at transfer functions where both the numerator and denominator are polynomials.

So we did not have to go for anything very drastically different from basic stuff that we had learnt at the initial portion of this course now you will see that we will have a term which is going to be an exponential and that will kind of to do all kinds of complications in terms of both analyzing transfer function looking at the stability of open loop system and stability of closed loop systems and so on

So basically first I will describe what a time delay system is and from an open loop I will show that it does not really matter much because it is very simple in version that happens when you have this exponential part but when it comes to close loop you will see that this going to be all kinds of difficulties because we are going to get this time delay term in the denominator and that is going to completely changed the way we do the inversion of this transport functions that we have been looking at till now and then of course stability is going to be total way of looking at stability so this is what we are going to see as we go through the next few lectures.

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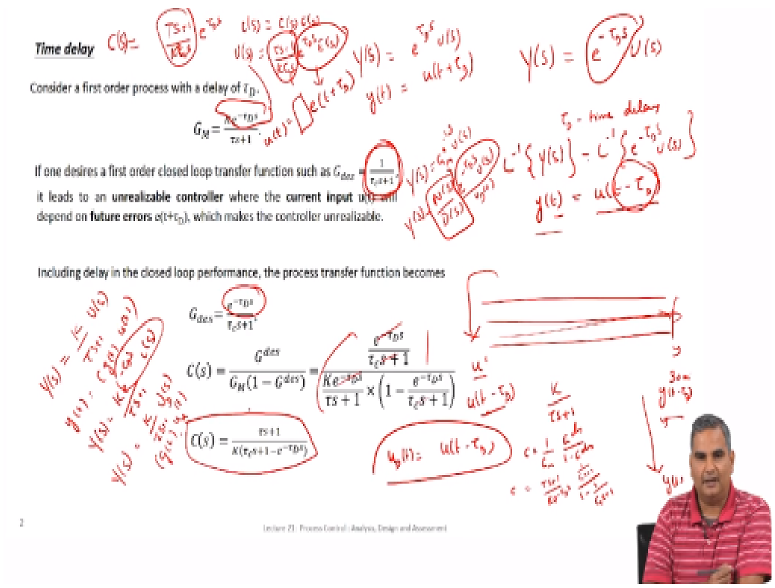

So before I describe the time delay in a transfer function let us understand how the function form looks and how do you go from time to Laplace domain and so on when I have this time delay term, so if I have something like this Y of S if I simply write this as let say E power minus tau DS, U of S so if I write something like this then this is the time delay term in the Laplace domain and we call this tau D as the time delay and if I have something like this and I want to do an inverse Laplace transform of this Laplace inverse of Y of S is Laplace inverse of E power minus tau DS, U of S so I have something like this then and the if you go to the Laplace table and see this you will see that this way of way of T will be U of T minus tau T so this is the Laplace inverse for this equation.

So whenever I multiply something by E power minus tau DS and then basically what we are saying is this output is going to be time shifted back by this tau D amount so this is the basic expression, so what this basically said if you look at this equation it says let us say I make a change in you and I want observe the change in Y so I observe the change in Y at time T for the changes made at time T minus tau D, so whatever I made terms of changes at time T minus tau D (())())(5:18) it is self only at time T so there is a delay of tau D time.

So I have to wait for tau D amount of time before actually see there affect so this is the idea of time delay and this is something that you see in real life so for example if you have a long pipe let say at steady state the flow is constant and then it is say it is a long pipe and suddenly increased the floor here let us say now the effect of this increase inflow is not going to be felt here right away so all this intermediate liquid has to kind of go out before this high flow (()) (05:57) here so if I think of this as you and this is Y write if I see a sudden change here let u say 30 minutes let us say in the flow that is because something change here a while back quite a while back and then that is the reason why I am saying this changes so this is what is called the transport delay and the amount of time it takes before something that I do here shows up is what is time delay.

So if you want to think about it this way if you say I make a change it U minus tau D and then I look at Y T minus tau D then I would not have seen I made a change here but that is not reflected here so let us say the next Y it will not reflect so it will reflect at Y of T so this amount of time has to be lapse or delay before I see the effect of this here so this is a very important point and this happens is all kinds of real situations particularly flow system and so on, whenever you taken action it takes some amount of time before you see the effect and typically called as transport lag or transport delay, now similarly just so that you I show this because they going to see this as we go along in this lecture if Y of S is on the other hand E for TDS U of S then basically this equation says Y of T so when I do the Laplace this will be U of T plus tau D so the sign changes so this basically says that actually the input is going to lead the output so you might ask why should I look like this I can write this equation as U of S is E for tau DS.

So that is the reason why we have look at forms like this but just so that you understand whenever I multiply by E power some alpha S then you basically say Y of T is U of T plus alpha if this alpha is negative corresponding will be minus T if it is possible will be plus tau D and so on, so when we talk about let us say transfer functions now which we are considering so we can write here GM which is used for G process so first order transfer function is something we had seen which is KO tau S plus one now this basically means there is no delay so whenever I have U that immediately acts on Y is what this say but in real life you will have time delay, so when you have this time delays the transfer function usually is modified by multiplying a E power minus tau DS.

So this is a first order plus time delay system so there are multiple interpretation for this so for example if you say YS is basically K over tau S plus one US this without time delay so this basically say Y of T is some going to be some convolution form between let us say some function G of T and E of T so some convolution form is going to be here now I wait to Layman way to understand what impact this has is the following so if I say Y of is KE power minus tau DS by tau S plus one U of S then this together I can add and then say this is Y of S is some KO tau S plus one sub delayed version of U is what I can write here UDFS so this delayed version U and U of S is related by so this delayed version of U is basically U of T minus tau D because I can write this delayed version as E power tau DS U of S and from here I showed you this.

So basically what it say is so in this case the convolution form between some function and U delayed version so this basically says this first order is going to act instantaneously on changes to you when I add a time delayed systems it says the first order is going to act instantaneously in not on U but it is going to act on U T minus tau d so whatever changes I made a while back will affect the output in a first order form or passing through a first order transfer function in the previous case without time delay whatever change I made currently will immediately affect Y through a first order process so that is the idea using here.

So that is an important to understand so any time this look confusing you can (())(10:30) E power minus tau DS and the next term that you have and then you will see what is the physical meaning and that us what I try to explain here in the first case just a first order the physical meaning is that if there is no time delay whatever you I have will immediately act on output U through a first order process if there is a time delay changes that I made at time T minus tau will through a first order process and affect Y, so it will take that amount of time before we see any affect, now what is the problem with this right so now if you want to quickly understand whether this creates any problems in terms of inversion and so on.

So you can see that it did really does not create any problems in term of conversion as long as this time delayed term is in the numerator this is because of the following as i have let us say Y of S is G model and time U of S let us say this G model as a first part and then U minus tau DS or I can write this as Y of S is some numerator by denominator times E power tau D S in the numerator time US then whatever functional form we had use the partial fraction expansion and so one can we directly use tire because this portion I can actually put it on the partial fraction form and then I can do the inversion accept that here this I will instead of calling this is U of S, I will call it as U D of S and in the convolution integral wherever I have U of T, I will replace it by U of T minus tau D because U D of S is this here.

So in the convolution integral finally when we do the inversion we basically say it is simply a convolution of U of T I am just going to make it U of T minus tau D so in terms of doing the analysis open loop there is nothing extra that I need to do I can leave the same idea of partial fractions and then look at now I address this in the same framework and it looks like there is nothing new or extra we need to understand or learn to handle this in the open loop, however the minute we get into closed loop form it is a totally different ball game which we will see now, so now let us try to design a controller using direct synthesis for this model form which is KE power minus tau DS by tau S plus one and if let us say I were to decide that I am going to do this and then I am going to do this and I am going to ask for decide such as this tau S plus one then I want to see what happens here.

So to see that I am going to show you very simple computation, so remember the controller is basically one over GM, G decide by G des, so in this case the GM is one over GM is going to be tau S plus one divided by K E power minus tau DS let us assume the G decide is something I have here one over tau CS plus one divided by one minus one over tau CS plus one now after you do all the computations here I am going to write this C of S here the C of S that you will get is the following tau CS plus one minus one divided by tau CS plus one, tau CS plus one tau CS plus one will get cancelled I will get tau S plus one divide by K tau CS and then this E power minus tau DS will go up E power tau DS this will become.

Now for people who being are following this course and nicely will immediately recognize that this is basically the PI form, so tau S by K tau CS, S and S will get cancelled that will give you the proportional part and then one by K tau CS will give you the integral part so this they will see is as PA controller nonetheless there is a problem here because we know the controller is a transfer function between the input on her error so U of S is C of S E of s and remember I told you that when I write this U of S as tau S plus one by K tau CS, E power tau DS which is a controller E of S, then basically what it is say is now his is a PA controller and the PA controller is acting on a variable which is E power tau DS E of S that means it is acting on a time variable which is error at T plus tau D and this is still U of T because now this minus tau DS when it comes to the numerator as it becomes plus tau DS so basically what it say is the U of T the current input that I need to compute is a PA controller acting on error at time T plus tau D now you see where the problem is so to make a control move at the current time I need to know what the error will be in the future and the error is computed as the output minus the set point and I do not have the output in the future because I am at the current situation.

So how can I get an error in the future to make a control move in the current situation so that they becomes a unrealizable controller so if I just choose any G decide I want then that is going to lead me to getting a controller that is not implementable and that is not implementable because I need to have future errors for the current control move can computation I do not know future outputs any way so I can do that so this become unrealizable so an interesting thing to notice is to think about this (())(16:38) the inverse response controller in the inverse response model because of this one over GM we got an unstable controller whereas when we have a time delay system because of this one over GM you are going to get an unrealizable controller because the computation of the current input (())(16:59) errors in the future and much like how we did compromise in the case of the inverse response process is we are going to do a compromise when we do control design for time delay systems and before I get into the math of it which is very simple just like how we introduced inverse response behavior in the desired transfer function itself which is the allowance that we are making we are going to basically introduce a delay in the decide transfer function which is allowance that we are making.

So basically just like how we describe why inverse response systems are difficult to control you can also understand why time delay system are difficult to control so I will give you two example one is something that all of us have felt some time or other in life the other one is I am going to go back to bike example and then tell you what kind of complications this time delay system introduce, so all us have taken a bath we have taken a hot water bath it typically go you have now for hot water you have now for cold water and then you can open both the nobs and then say we can get timid water to take a bath but what you find is that you get extremely cold water to get the bath if the outside temperature is cold this is because when you have the systems and you have pipes where water is there because the external atmosphere is pretty cold you have cold water in the pipe and it takes a while for this water to drain out and the hot water to come it from the source into the bath tub.

So basically what it mean if there is transport delay thought you have to your hot water nob this action is not going to immediately result in hot water coming so all the water in the pipe has drain out before the hot water comes so that introduces a delay but typically what happens is when you open the nob for the hot and cold mind says ok hot water has to come in but hot water is not coming in so the first thought that comes is really not time delayed it is that may be have not open the nob enough so the flow of hot water is much lower than the cold water so that is the reason why the water is still cold so what you might do is you increase the nob than more floor is there but still hot water is not coming in but what happens ultimately is when if you open the nob too much when the hot water comes in it is going to be scalding hard so this is something that many of us felt at some time other than.

So we see why these time delay create problems in control because you take an action and you really do not know how long have to wait because you cannot get a very good estimate of time delay at all time so you just have to wait and you do not know whether it is really that you not open the nob enough because at which the water has not become hot or is it because of time delay you open the nob enough and you wait for long enough time you will get hot water so it is very-very interesting question and it makes the controller design that much more complicated particularly if you do not have a very good idea of how long the delay is and so on.

Now it is interesting to kind of thing about the same thing in the context of the bike example and this will really tell you very nicely what the problem is supposing you are going on a road and you say there is right coming up I am going to take the right and then you go on the road and then you movie your steering wheel to the right and the bike if it has no time delay the minutery move steering to the right the bike will turn and everything will be nice but if let us say you have the special bike which is time delay then you go and then you want to take a right you move your staying to your right but it is going to take some time before this action will (())(21:04) so basically keep going for some more distance and then suddenly the bike will turned right but there will be no road to go because you already missed the ride so you can see how this time delay create difficulties in control.

So the real solution is actually if you know the exact amount of time delay then what do you know that after I move the steering to the right it takes about may be three second for the bike to turn right so I am going at this speed come so the intersection is going to come a head so I have to estimate how long it will take me to get to the intersection let us say the minute I figure out that it is going me 3 seconds to get to the intersection I have to move my steering and then stay with steering in it is new position and exactly when you hit the intersection at 3 seconds will move write and be on your way what look at how many difficult things you have to do here you have to understand your speed you are going to figure out how long it is going to take you to get your intersection and that has to be precisely math with time delay assuming that you know the time delay accurately if you do not know the time delay accurately it might take 3 seconds,4 seconds, 5 seconds than you can see how complicated this computation even for this small example will become and arriving might not seem as much fun as asset seem right.

Now so you see the difficulty of control of time delay systems so much like the inverse response problem for delay also we are going to moderate our expectations in terms of the performance and then we are going to say let me not ask for any performance such as this one over tau S plus one much like how I introduce an RHP zero to handle inverse response systems I am going to equivalently introduce a time delay so this basically says that I know that there is going to be a time delay and I am going to leave with it so I know that if I make a change in the steering to the right it is going to take a certain amount of time before it turn right and I am going to factor that into my and decide behavior so that I actually change the steering even before I get to the intersection so I have already constraint to include this if a do not include this then I am not going to get that realizable controller.

So once we that then I have a G decide E power minus tau DS, tau CS plus one and just like how I explain in the inverse response behavior by looking at this itself we can save this decide itself is time delay system, so we are deliberately introducing at time delayed in the decide to contract of the time delay of the process so again remember that this is basically this a model of the process the two process could have a time delay which is different from this and the effect of what that will be we will see later and it is a same process we have to put the close loop the controller once it decide like this the close loop will have a process which might not have the same time delay slightly different time delay and then you can do different analysis like what we did in the last class so that become.

Now procedural there is nothing new there as long as you understand the distinction between a model for controller design and a process to be check out in a close loop diagram anyway we know this formula we have use this many times G decide by GM time one minus G decide so I put this G decide I have this GM KE per minus tau DS tau S plus one and I have this now this and this will get cancelled and this and this will get cancelled and this will be tau CS plus one minus E power minus tau DS in the denominator and this will go to the numerator and then you will get your controller as tau S plus one divided by K times at tau CS plus one minus E power minus tau DS, so this is the controller that we define for a time delay system now what we are going to do we are going to look at this controller and then see whether we can realize this controller using technique that we have learn till now are there other newer ideas that you need to address now controller of this form you will notice that we have to bring in newer ideas to look at how to handle this types of transfer function now which are not your standard numerator over denominator transfer function form.

So what we will do is we will pick up from here in the next lecture and then talk about how you realize this controller and how did think about look at close loop stability when you have time delay systems and so on, so we will see again next lecture to describe some of this thank you.