Process Control: Analysis, Design and Assessment
Professor Parameswaran S
Doctoral Research Scholar advised by Professor Ragunathan
Indian Institute of Technology Madras
Frequency Response Analysis – 2
Mod05_Lec24

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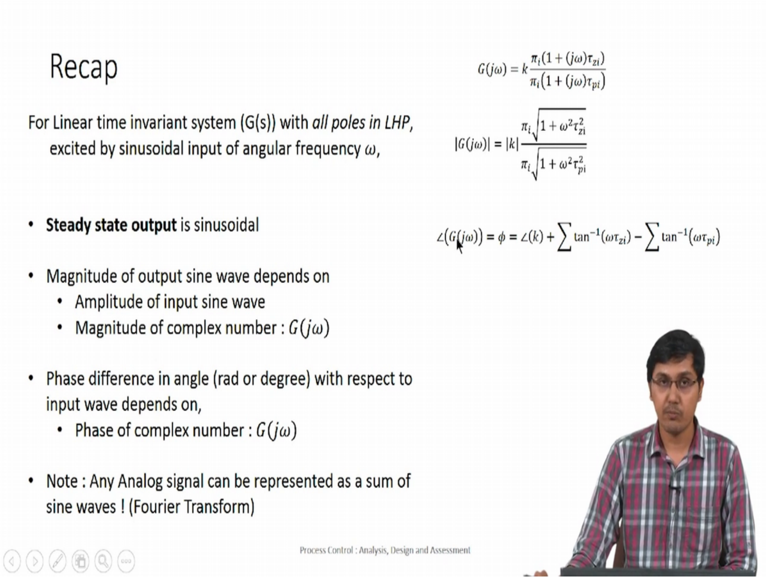

Welcome to the next lecture on frequency response analysis, so we will see what we have done in the previous lecture, so for any linear time invariant system G of S which has all the poles on the left hand side of the S plain which is given the input of a sine wave, we have at the following observations, the first one is the steady-state output is sinusoidal, that is what we saw, so here the steady-state is very important because the initial transition what we are ignoring which is due to the poles of the G of S.

And then the magnitude of output sine wave depends upon the amplitude of the input sine wave and also the magnitude of complex number G of J omega and the phase difference in the angle between the input sine wave and the output sine wave is depending on the phase of the complex number G of J omega.

And one more thing, why we are more interested in sine wave is that any analog signal can be presented as a sum of sine wave is which is given by the Fourier transform but let us just have the concept of for now, we do not need to like really going to this things for now and these are the formulas we have got for the magnitude of the complex number G of J omega and angular of the complex number G of J omega again.

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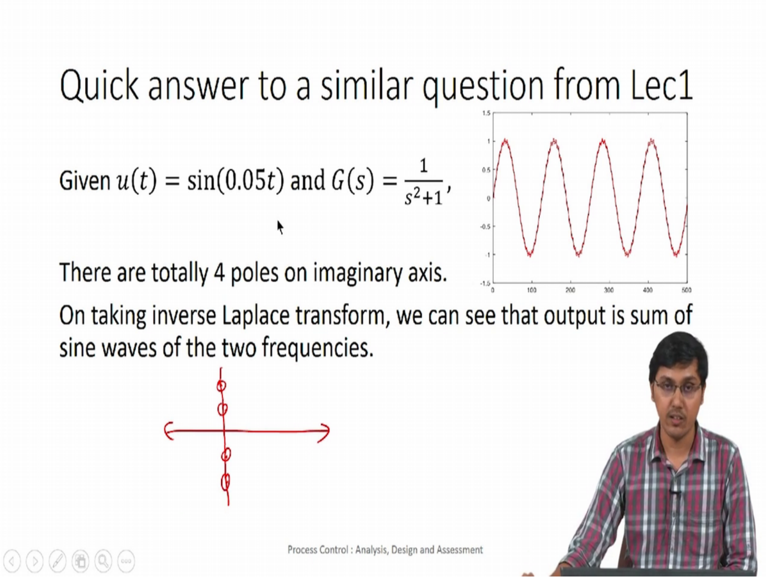

So we have ask the question in the lecture1 and ask you to try it by taking a Laplace transform also by stimulating, so a similar example is taken here where angular frequency is given by 0.05 and the transfer function is nothing but 1 by S square +1, so if you can see here and when you obtain the inverse Laplace transform what you get is like? We will get 2 sine wave is in the Y of T, so one with the angular frequency like 0.05 and another angular frequency one, so if you could see here there are 2 ways actually, if you can see there is a dominant wave and then there is another wave about it.

So these are all like 2 sine waves, one sine wave is superimposed on the another sine waves. That is what is will give, so basically if you can in the Laplace plain you will have like 4 complex numbers and if you invert this into like again, Y of T you will get like 2 sine waves that what.

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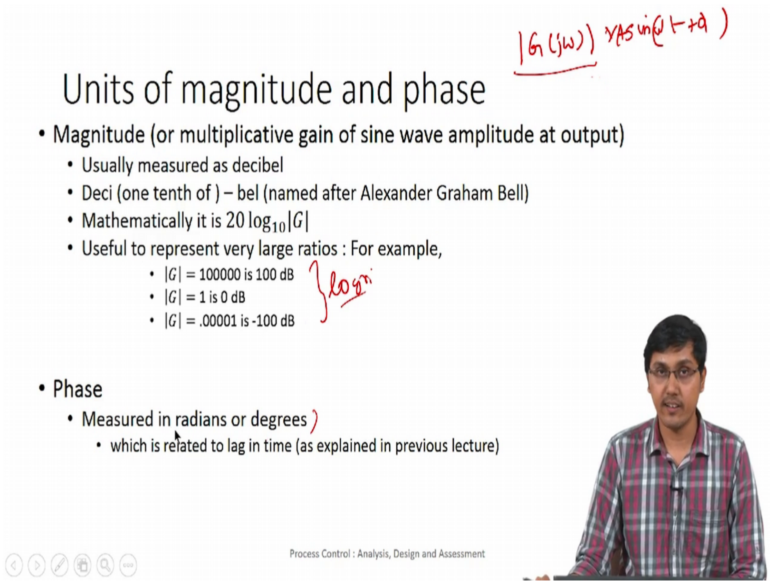

Now we will talk about the units in which you will measure the magnitude and phase, so the magnitude, when we see the magnitude, it is the magnitude of G of J omega which is nothing but the multiplicative again that we added to the sine wave, so if the input sine wave is like A sine omega T, the output steady-state sine wave is going to be some a sine omega T+ pie into magnitude of G of J omega and, so this is what we say magnitude and we are going to use as special unit called decibel by which we are going to represent this magnitude.

So this is a number which is gain and we are going to use it decibel to measure it, there is reason to do it like this and like the Deci is 1/10 and bel is a unit which is named after Alexander Graham Bell and its roots in like when they were trying to do the communication etc, they were trying to come up with this unit and this represents like the power of sine wave in an electrical systems, so. But let us not go in deepen.

So what now we are going to understand is like basically the magnitude is represented by the decibel unit and decibel unit is nothing but taking the log of this magnitude and multiplying that with 20, so 20 log to the base 10 of the magnitude of G of J omega is the decibel of it, so why we are going for such kind of units? So basically let us say the magnitude is like 1 followed by 5 zeros and the magnitude is 1 and the magnitude is like 0.00001.

So while we are telling all this values, this does not make like immediate sense to us right, but if we can convert it into 20 log, see if you do it logarithmic unit than basically we can shrink the values and we can easily feel like under DB, 0 DB and minus under DB, so these are like easy to also have a into to sense when we work with logarithm scale for numbers which are having a wide range and as we saw the phase was measuring degrees or radians, so which is related to the lag in time, so row we called the degrees or radians.

It is actually we saw that sys was later to the time period of the sine wave and of the lag in a sine wave with respect to what fraction of time period it is, so we saw it in details in the previous lecture, so this is the unit of phase basis in either radians or degrees.

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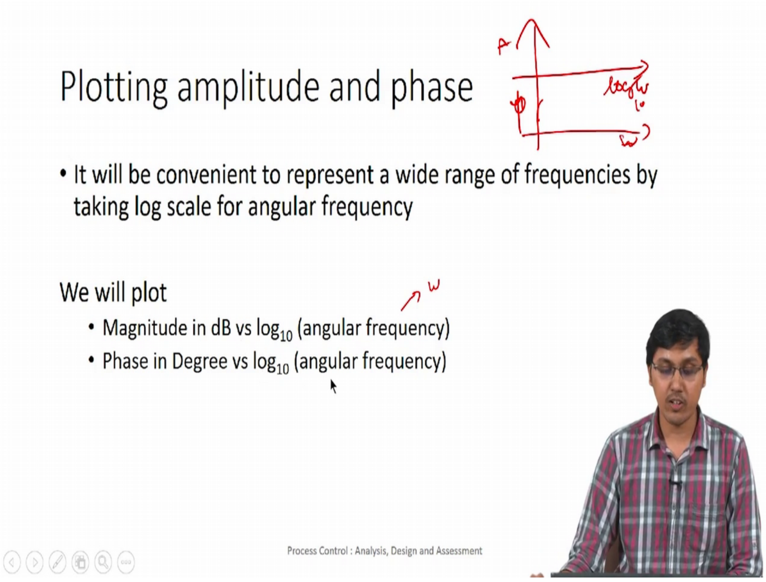

So now we also obtain, we also came to know that there are amplitude and phase, there is a magnitude and phase depends upon the frequency at which we excite, a frequency of our sine wave we are giving of input, so one usual thing is, one thing we ask is like what if I can plot frequency in X axis and then plot the amplitude and then the phase in the Y axis and then how am I going to get this graphs and that is what normal question that you will have.

So we will try that exercise for now and then since again the frequencies is going to be a wide range of frequencies, we will again adopt logarithmic scale for the frequency, again logged in, so now will attempt to plot magnitude in DB but it is just log 10 of angular frequency, this omega and phase in degrees and also we assess the logarithm of the angular frequency to the base 10.

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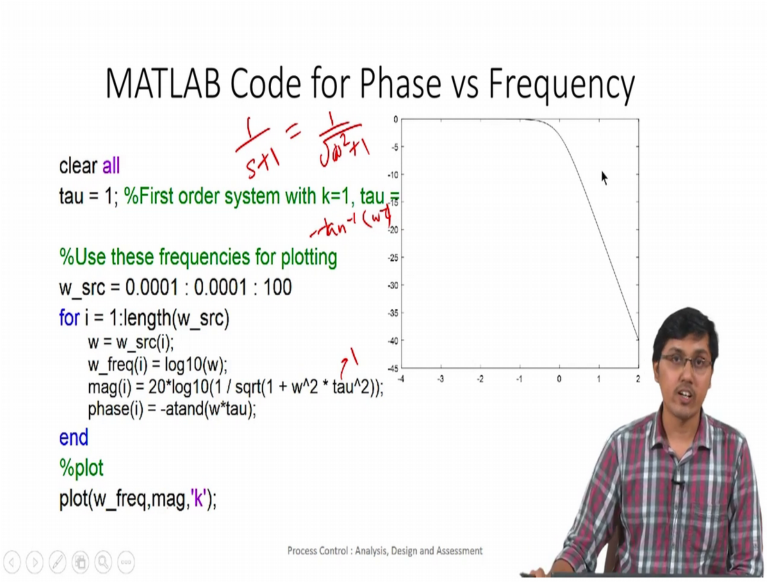

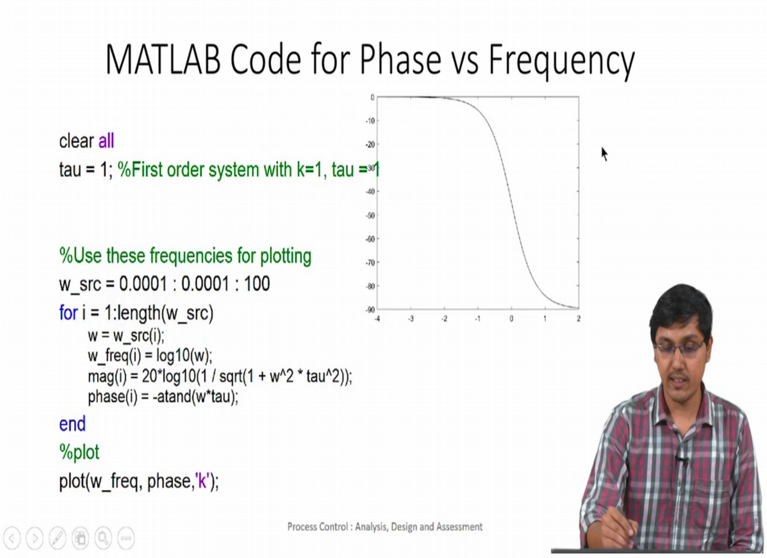

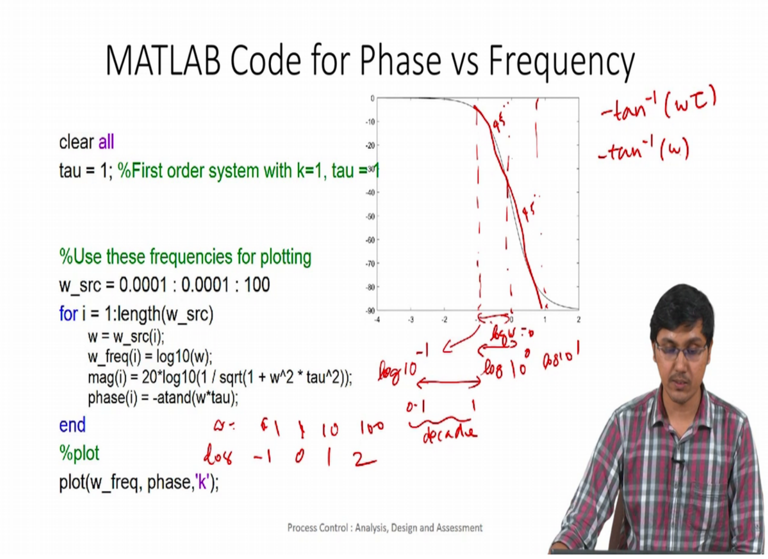

So now we are writing the Matlab code for the same bit for very simple system like 1 by S +1 with Tao equal to 1 and we plotted over the frequencies 0.0001 to 100 and then we take the magnitude as 20 log 10 of 1 by square root of, so magnitude of root is nothing but 1 by omega square +1, so now Tao is equal to 1, so now Toa has appeared here, so basically we get magnitude as this one and phase is a thing, but we saw minus tan inverse omega Tao, so it is minus tan omega.

So if we can see, so if we can plot between the angular frequency and the magnitude you get this graph and similarly if we can plot the phase and the frequency we will get plot like this, so we let us understand this in some little detail, so again, this is a logarithm of angular frequency, we have plot, this is a logarithm of the angular frequency so we have plot against logarithm of angular frequency versus the magnitude in decibels, so when you see Tao is equal to 1 basically we can see the pole is that S equal to -1 right.

So this is the pole and if you can see here the magnitude is nothing but 1 by root of omega square plus 1, so if you could see, if you substitute the values of omega and see basically when omega is very, very less than 1 what happens is this particular term is nothing but approximately 1 by root 1 because omega square, if it is less than 1 we can ignore that and when omega equal to 1 this term becomes 1 by root 2 and when omega is greater than 1, then it becomes like approximately 1 by root omega square which is nothing but 1 by omega.

So with all this in mind we can see that, when actually before omega equal to 1 we get almost the constant amplitude of 1 and when omega is very high than 1, then it actually decreases at the rate of 1 by omega, so if you take the 20 log of the magnitude then you would probably of is of that this value is nothing but 20 log of 1 by omega or -20 log of omega and if you see here for every log omega okay, this is a straight line having a slope of 20 so the magnitude in decibels is having a slope of -20 4 every you need of log omega, so basically the X axis here is log omega and after omega equal to 1 which is nothing but the log of omega equal to 1 is nothing but 0 when log base 10 to the 1 is 0.

So after this point what you can see is this becomes almost straight line with the slope of -20, this is log omega and this is the decibel thing, so this is something that we can observe from this particular plot and we will be using it little while, little later in this condition and now coming to the phase, if you can see here the phase again varies from like 0 to -90°, so again if you can use the same logic and then we see tan inverse of omega Tao, here, specifically tan inverse omega, if you can get this tan inverse omega and for very, very low values of omega than inverse 0 is 0 and for very high values of omega the tan inverse infinity is pie by 2.

So this will vary from 0 to pie by 2 or 0 to -90°, pie by 2 is variance and -90° in the degrees, again, if you can also here like in a it is by observing I can say that from -1 to 1 okay this curve is changing and, so this is changing for upon 0 to 90°, so basically for every unit of this has to fall like approximately 45° okay, so for every unit is falls by 45° and also another thing to notice like omega equal to 0 is where, sorry omega, log omega equal to 0 is valid we have the pole and this has started falling 1 unit before that itself.

So when we see 1 unit here, this is nothing but from log of 10 power -1 and this is nothing but log of 10 power 0 and this is log of 10 power 1, so between 10 power -1 omega to 10 power 0 omega which is 0.1 omega to 1 omega, it has fallen by 45°, so this we call as a decade like 10 times, to get 1 from 0.1 we multiplied by 10 right, so again from 1 to 10 power 1 we multiplied by 10, so this is like a decade, so 0.1, the units on the X axis is nothing but omega is 0.1, 1, 10 that etc, in logarithmic units it will become like -1, 0, 1, 2, that is what this is showing, this is log omega scale here. so this observation also you will be using a little while, little after in this presentations.

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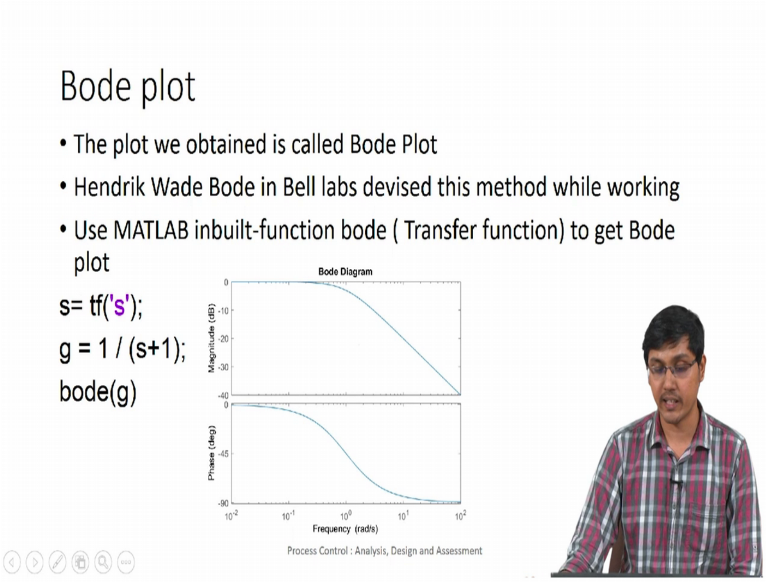

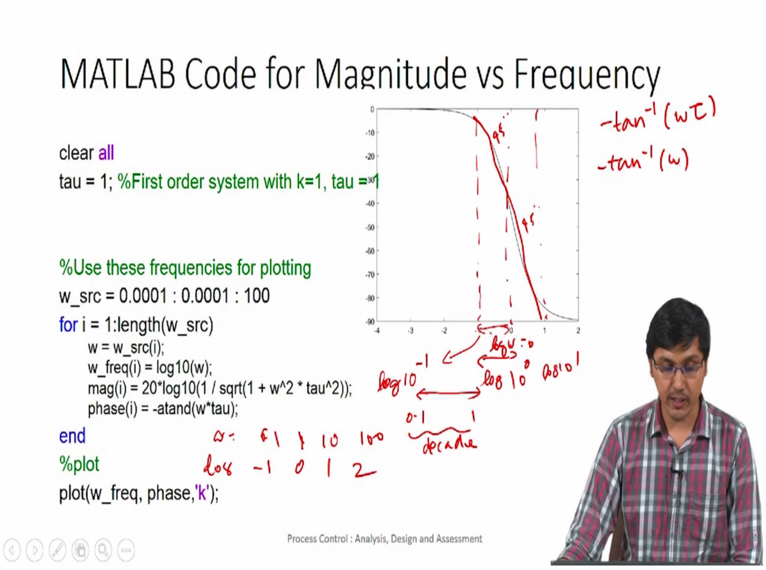

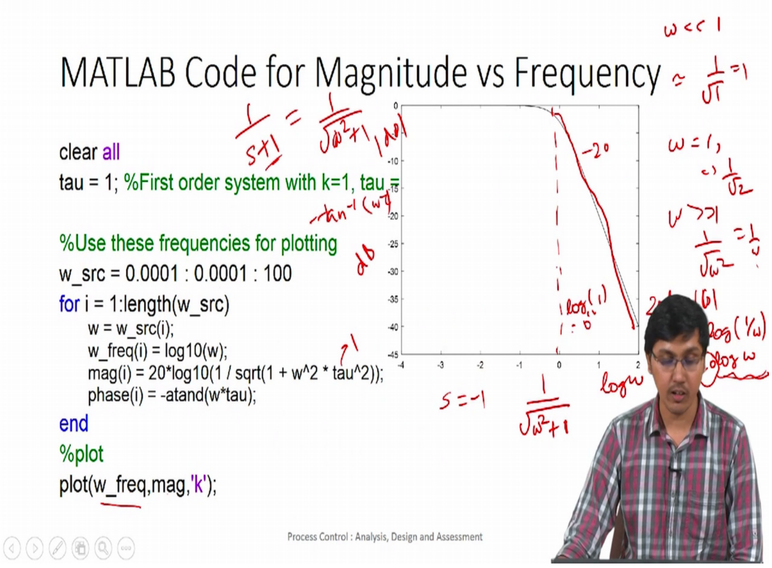

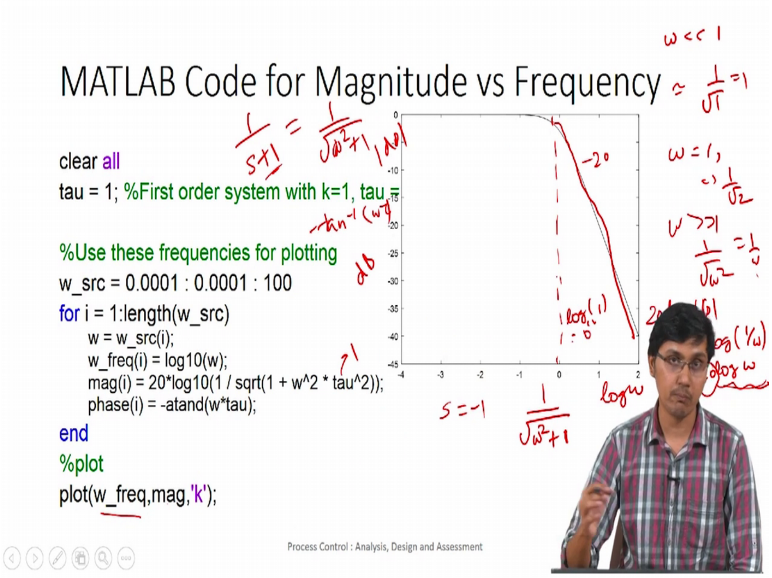

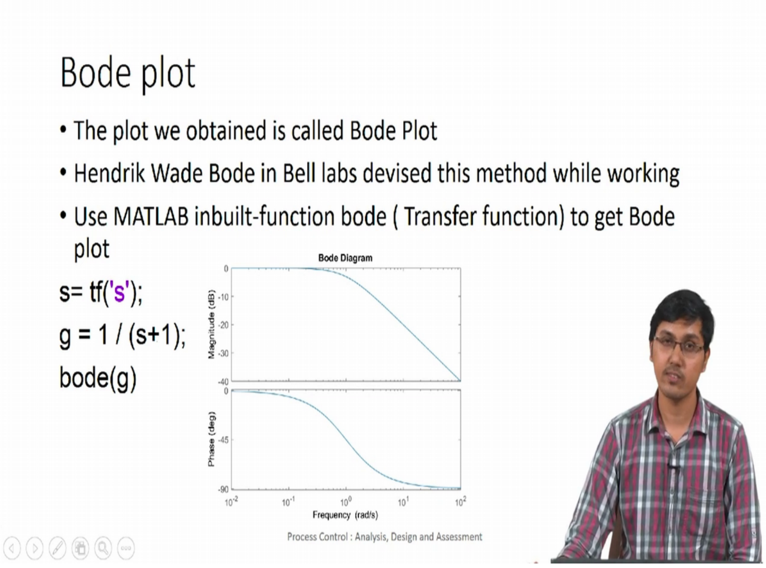

So now we have a achieved is basically we have achieved plotting the amplitude in decibels and the phase in degrees or in radiance with respect to the logarithm to the angular frequency in the X axis, so this particular plot is called Bode plot and this was developed by a person called a Hendrik Wade Bode was working with bell laboratories and there is a Matlab inbuilt function called bode and you can just fit the transfer function into the particular function, and it will display with the bode diagram, so whatever we have plotted by doing this calculations manually or like by Matlab coding the same figure we can get from the bode function of the Matlab, so this is straightforward things to do

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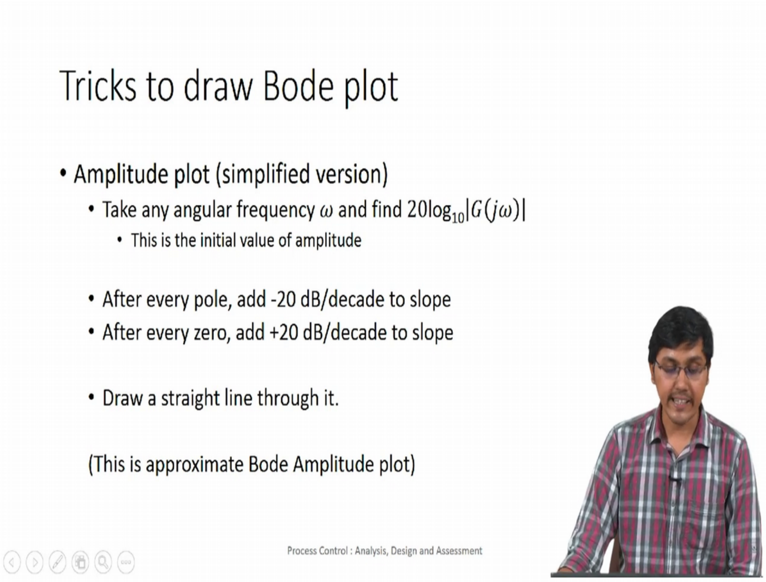

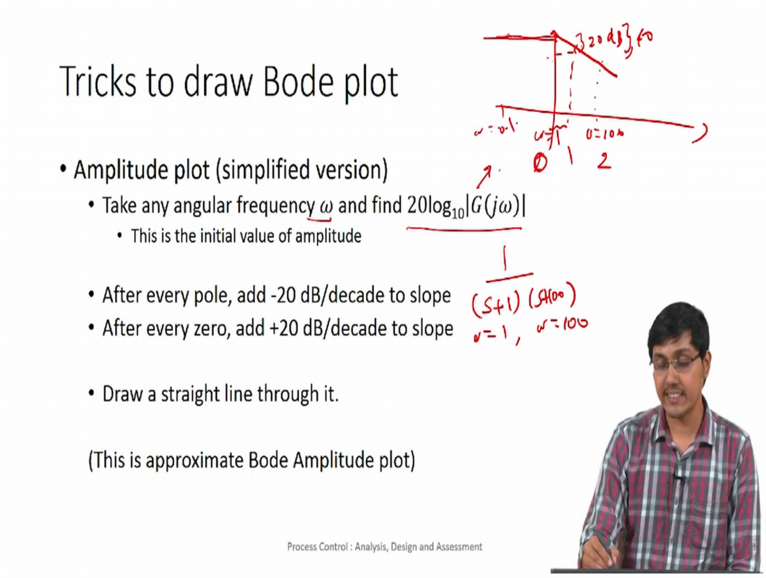

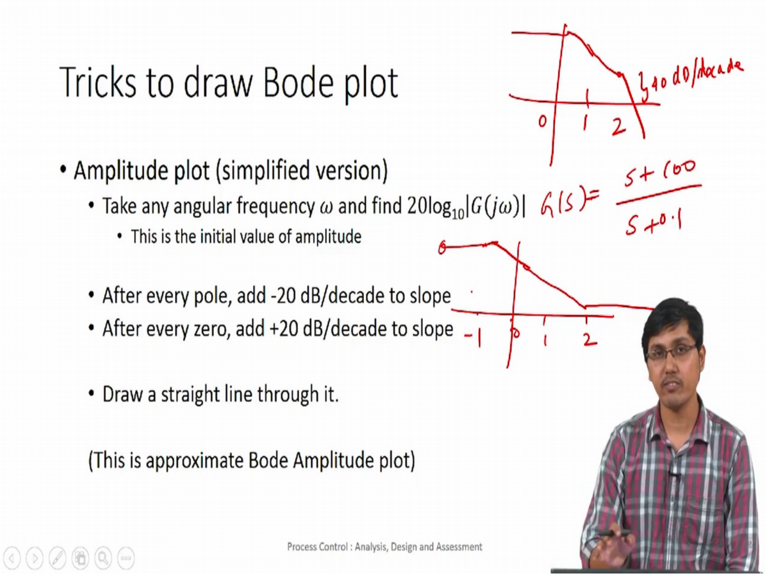

So now based on our observations let us develop some tricks that we can use without like to draw the bode plot without any TDS computations, so taking the amplitude first we will talk about the amplitude plot and right now again like we are dealing only with the transfer functions which is poles on the left-hand side of the S plain, so that is something that you have to keep in mind while we are going throughout this steps.

So for this lecture we are sticking to only that particular use case, so you can take to start with, you can take any angular frequency omega, take any value of omega and then find a value of 20 log base 10 G of J omega, just substitute the value of omega in this particular complex number and then find the magnitude of it using the formula that we have shown in the first slide and after every pole, for example if you have function like S +1 and S +2, for omega equal to 1 and omega equal to 2 after every pole, you start, you add a slope, you change the line to the slope of -20 DB per decade that what we saw right.

So after every pole the magnitude was falling by 20 DBs, so for every pole we add -20 DB to the line, slope of the line -20 DB, let us illustrate this instead of two letters have like entered here and then let us have a plot here, so omega equal to 1 is actually 0 in logarithmic scale and omega equal to and it is nothing but 2 in the logarithmic scale, so what happens is let us have a, let us assume that we are computed the gain at some other frequencies, omega equal to 0.01 or something and then this is a straight line we draw here and then after this omega equal to 1 we start making it fall by 20° by decade, which means that for every unit here, that is for every decade here it will be falling a 20 DBs here, this will be 20 DBs and again, and till this point wherein this is like totally 40 DB because 20 DB here, 20 DB here because for every decade it is 20 DB.

And then what happens at omega equal to 100 or like in logarithmic scale 2 this starts falling by an extra 20 DB now because due to another pole this is starts falling by extra 20 DB, so basically like 0, 1, 2 and initially it was falling like 20 DB till 2 and then it is starts falling the addition 20 DB, so now it falls at 40 DB per decade, so this is what we can, this is a simpler method which is proximate method to draw the bode plot and similarly for every 0, 0 is something that appears on the like in the transfer function, it appears in the numerator of the transfer function, so for every 0, we increase the slope by 20 DBs.

So let us take another example to illustrate this, let us say like G of S is nothing but S +100 by S +0.1, so if you draw the bode, the 0.1 is nothing but -1 and 0, 1, 2 is where this 100 will come because log of 100 is 2, when it logarithm to the base tan of 100 is 2, so basically if you can get this one, so at 0.1 what happens is, let us say we computed the value of G of J omega at a particular point for this and then after 0.1 what happens, it is starts falling at the rate of, after 0.1 8 starts falling at the rate of 20 DB and at 2 what happens is 8 starts racing at a rate of 20 DB, so -20 DB and +20 DB gets cancelled, so this becomes the straight-line, is it becomes a flat line okay.

So this is some like it is a tricky way of, it is not a tricky way, but it is a simpler way of trying the bode diagrams and this is mostly used when for some simple transfer functions like this and this is a very fair up approximate thing.

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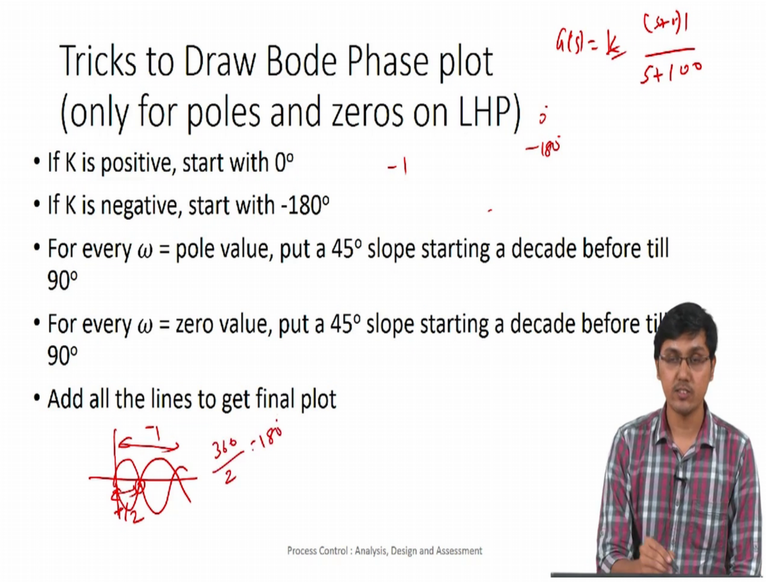

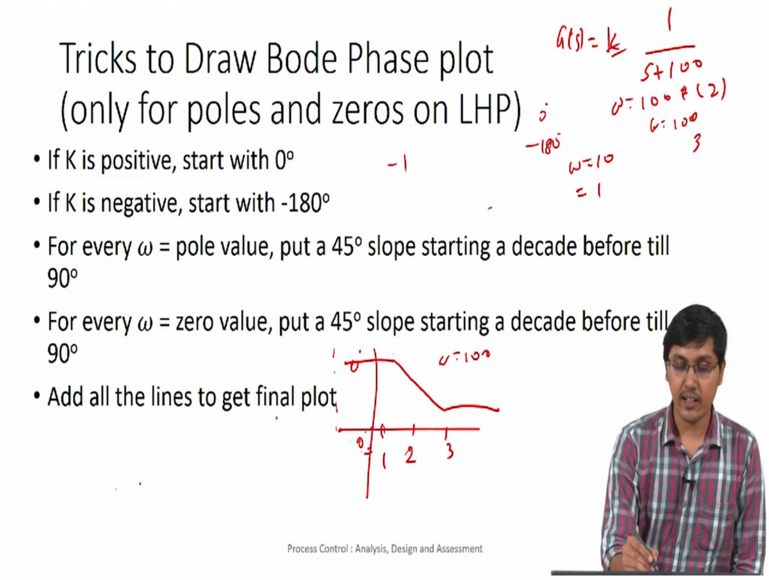

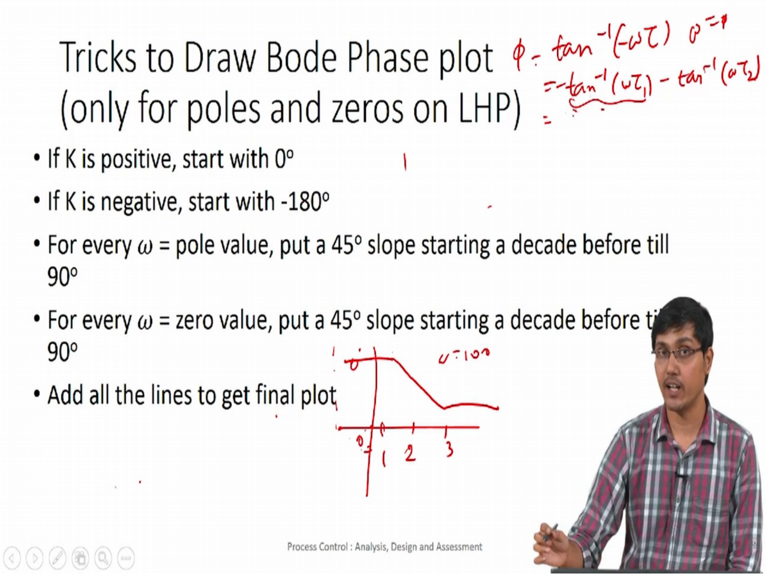

And similarly of bode face plot is again like let us have a transfer function like G of S equal to K into some zeros and some poles, so for every K that is, if K is positive than it starts with 0° and if K is negative it starts with -180°, so why it is -180° because in the previous lecture you were asked to let see what is the phase of -1 right, - contributes to your -180° phase shift, you can simply think of it line something like this, you have a sine wave and we invert the sine wave it becomes like this, so the starting of this sine wave is actually like T by 2, like if this is the time period it starts after T by 2 of the original sine wave.

So this is nothing but 3 60 x 2 which is like 180°, so it is 180° phase shift, so now if and then for every pole we observe that it was 45° slanting line, which started like a decade before and then ended with a decade after that and so for every pole value omega equal to pole value, for example when omega equal to 100 we start a decade before which is like omega equal to 10 or like logarithm of omega equal to 1 and this is logarithm of omega equal to 2.

So at omega equal to 1, we will start dropping the phase by 45° slope and then we will stop it at omega equal to 1000 or like logarithm equal to 3, so basically if you can plot something and then show how it behaves, so at omega equal to 0.1, 0.1 is 10 power -1, so here is what like, okay, first simplicity last 1 for now, so omega equal to 100 is what we are interested, so logarithm of omega equal to 100 is nothing but 2 and 1 here and 0 and 3 here, so the phase is something it will be like, it will be falling when omega at 45° from here until here and it will become flat here.

So this particular transfer function is having K as 1 or positive, so this starts with a 0°, so from 0° it will go down here and then after 90° it exaggerate, to get again mathematically feeling of why this -1, -90° comes etc, you can imagine like for example the phase angle is nothing but given by tan inverse of omega, minus omega now for poles, so it is nothing but minus tan inverse omega Tao and at omega equal to very, very small number, omega equal to 0, this becomes like 0° does not contribute anything and then when omega becomes very high again it becomes like -90°.

So for every pole will contribute to -90°, so if you have like 2 poles then it will contribute to like see Tao 1 and Tao 2, then it will contribute for first pole it may contribute initially to -90° and this pole also will contribute to phase shift of -180° when omega is very large.

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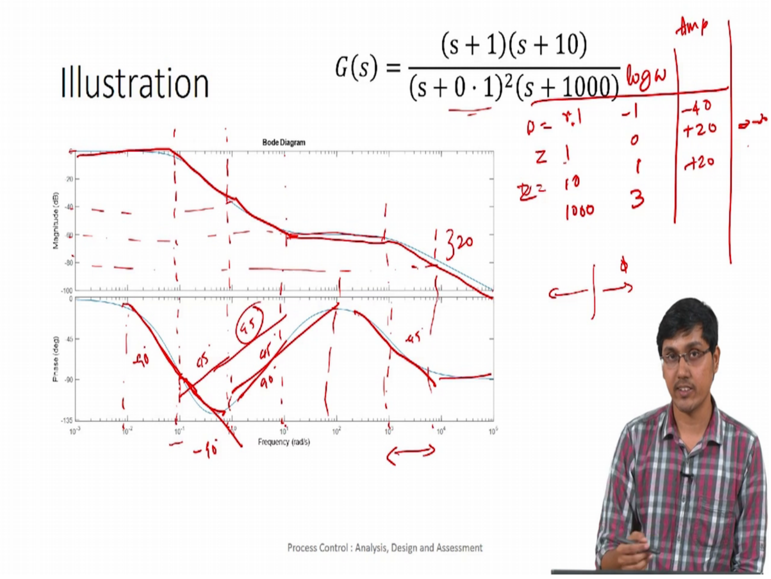

Let us take a another example and illustrate everything together, so this is as 0 at omega equal to 1, omega equal to 10 and omega equal to 0.1 is double pole and omega double pole is like 2 poles at the same point and omega equal to 1000 is a single pole here, so let us go through some formal way of doing things, first thing we will put everything in ascending order, we will say 0.1, 1, 10 and 1000, if you take the log omega it is -1, 0, 1 and 3 here, so again, so now if we take the following amplitude thing, the magnitude plot, the we know that for every pole it falls at -20 DB per decade.

So now 0.1 is a double pole, so for single pole it is -20, for double pole it will be -40, so -40 DB per decade, so if you can see here from at omega equal to 0.1, this line it starts falling at the rate of -40 DB per decade, so if you could see here, this is like -40, -40 DB per decade and then what happens that 1, 1 is a 0 now, it is a 0 here, so it is gets incremented by 20 DB per decade, so now this kind of flat ends out, so now this will become until omega equal to 10, this becomes like only 20 DB fall is that because -40+ 20 is, so in this way, it is nothing but -20 DB per decade, so this falls but at the rate of -20 DB per decade.

And then what happens at omega equal to 10, omega equal to 10 again, it is a 0 here, so again a +20 DB is added here, so now it becomes 0, so 0 is a I also underline, so there is no slope for this, so I underline here, until we go to 1000 which is again a pole and then we will see at 1000, omega equal to 1000 we will have again a fall of 20 DB per decade, if you can see here, so this is again 20 DB for a dedicated, so this is how you can plot the amplitude plot and this is, if you can see here, if you draw a straight line and then you can draw in this it would have been fairly accurate and remember all the physical systems, all the real systems are to eventually they have a decrease in the magnitude, like there is a limit to which the physical system will actually like can response.

So very, very, if you go to very high frequencies all the systems that are in nature will actually decrease the amplitude of the sine wave, that something you can keep in mind which might be useful and we can, we think of, or concepts relating to frequency response and similarly for the phase if you can see here at omega equal to 0.1, since it is a double pole, so this will start falling at the rate of -90° just from starting from before 1 decade, so from before decade it starts falling at the rate of -90° and is actually will try to fall at the rate of -90° here but what happens interestingly here is like we have a 0 but at omega equal to 1 add this line.

So here the 0 will try to increase it at the rate of 45 because it is a single 0, so this will try to increase at a rate of 45° but -90 +45 is again -45, so this will, this drop is like a -90 but this drop is like -45 and then here again, this 45 will continue till again 90, so the two things like 1 decade before and 1 decade after is what the pole will have effect on the phase angle, so now when it is equal to 10 again, it is a 0, so here it will be trying to do is at the rate of 45°.

So if you can see here this angle here will be 90°, so if you can see here the 45° contribute to this 0 and this 45° contribute to the another 0 at this point, so add this line it will be an increasing phase angular of + 90° and again at 1000, so after this decade there is no effect of this pole on the phase diagram, so this becomes like a constant thing until like 1000 comes, like omega 1000 comes and at that point it again starts decreasing at the rate of -45° for until this point and then after this, this has no effect.

So every pole or 0 will have effect for a decade before and for decade after as far as the phase angle is concerned, so that is what you can remember and you can just draw something, plot it using Matlab and draw something and then see how whether you are getting correct this equation or not, so that is why it is pretty simple.

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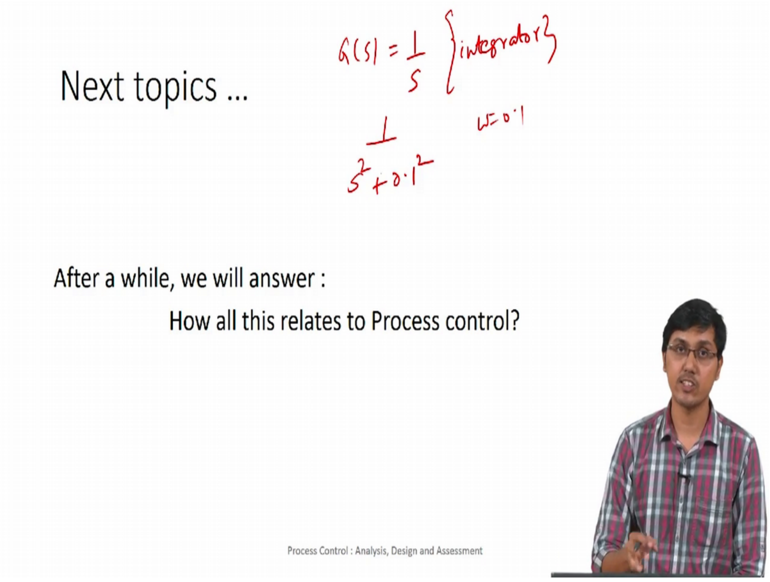

So after this will answer the question of how, so we have done all this frequency analysis, frequency response analysis, but we have not talked like how exactly it relates to the process control, so we will try to answer is after a while, after we study more concepts and you can just play with bode plots and frequency, different transfer functions, different poles and zeros and then see what happens.

So some questions you can actually think of it like draw the bode plot of G of S equal to 1 by S which is nothing but integrator right, this is the integrated thing, so you can draw the bode plot of 1 by S and also try to use Matlab to draw the bode plot of what happens when S square plus 0.1 square and specifically what happens when omega equal to 0.1, we can see what happens in omega equal to 0.1, what happens to the amplitude.

So we are previously seen that omega equal to 0.1 and for hatch transfer function, it is nothing but resonance, so resonance is nothing but an increasing amplitude, so whether **a** bode plot is able to capture this in amplitude where you can just plotted and see, here is now these are some exercises to play around with and to get a better feeling of what frequency analysis is, so that you will be comfortable when you go, when we go in details of how we can apply this frequency analysis into the process control. Thank you.