Process Control- Design, Analysis and Assessment
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Lecture 19
Traditional Advanced Control - Part 1

Now that we are done with the standard undergraduate control material, we move on to the 2nd part of this course which talks to more advance ideas in controls such as control of inverse response system, control of unstable systems, time delay systems and so on. And we will also talk about other controls structure design such as feed forward control, cascade control which basically implements some form of multivariable controller within the single input signal output controller framework, then we will look at truly multivariate control, we will 1st look at how we could do pairing for multivariable control system to break them down to seesaw control systems.

And once we learn how to do the pairing then basically the seesaw control systems ideas or whatever we have seen till now so there is no change that happens there and then we will finally look at implementing these controllers in a truly multiple input multiple output framework so that will end the 2nd part of this course. So the ideas of inverse response and stable control and time delay systems are easily understood when we think about controller tuning using a direct synthesis approach because that will allow you to understand what are the issues that come about if we have these types of process behaviour and what are the fixes which will become readily apparent when you look at this direct synthesis kind of tuning.

The cascade and feed forward controller and so on are what are called traditional advanced control slightly more advance and simple seesaw control, but however you do not need anything extra other than the standard seesaw ideas to understand them. And in fact industrially when these types of controllers are implemented, they are implemented as blogs in your standard DCS systems that one has so that is the reason why these are called traditional advanced control. While the ideas are slightly advanced, they can be implemented in traditional single input single output control systems.

And then we will have the truly multivariable controller which could be looking at all the variables all at the same time. And then how do you think about either the coupling them or treating them as truly multivariable systems ok, so that is going to be the overview of the 2nd part of this course on analysis design and assessment so this is lecture number 19 of this series of lectures.

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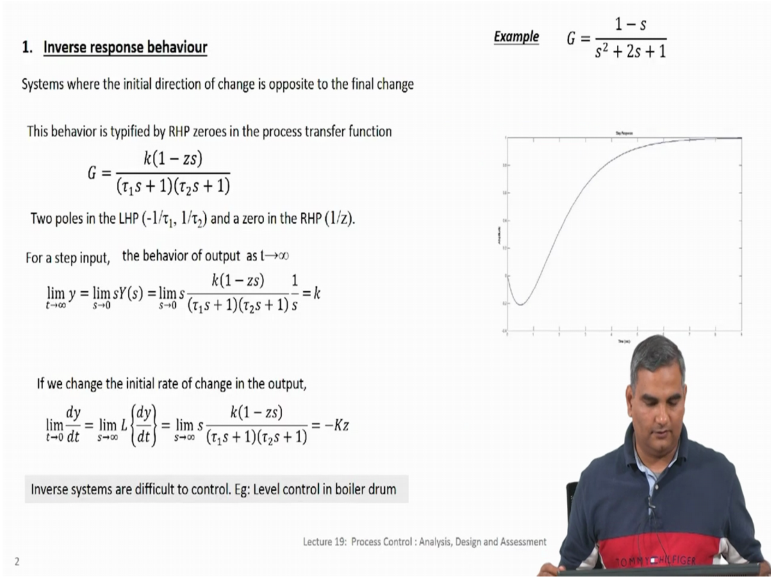

So since we are talking about dynamics which are more difficult to control we will start with the 1st dynamic behaviour which is more difficult to control than standard systems and this is called inverse response behaviour. What I am going to do in this case is actually I am going to describe inverse response behaviour purely from how the transfer function looks ok so till now I have been really emphasising that these transfer functions are embodiments or representations of physical systems. So I have been saying ultimately we keep looking at transfer functions but do not forget that these come out of physical systems and the parameters in the transfer function are actually computed based on physical dimensions of the system or operating conditions and so on.

In this case I am actually going to start with the transfer function and explain what inverse response means. So the reason why I wanted to do it like this is when you look at a transfer functions itself, you should be able to understand the complicated dynamics that might end soon from this form of a transfer function. And once we do that I will kind of explain to you intuitively why, this inverse response behaviour is difficult to control and that I will go back to my standard bike example which seems to provide us enough richness to explain many of these ideas. And then I will also talk about physical systems where you will see inverse response and I will tell you something this inverse response is not very uncommon, there are several systems which show inverse response behaviour and if you look at literature you look at different systems which will do this.

Okay so let us take transfer function, till now whenever we have talked about transfer functions we have talked about numerator by denominator and we have talked about the roots of the denominator, we talked about how we can do partial fractions in terms of the roots, we have talked about writing the denominator in root result form, we have talked about identifying the roots of the denominator polynomials without ever computing them in terms of where they are located and so on.

So our focus has largely been denominator if you look at all of the traditional control that we have talked about and I also mention it is not that the numerator does not play a part in our partial fraction expansion, each of the coefficients for each of these terms actually more captures collectively the numerator polynomials. Nonetheless our basic assumption was numerator polynomial was not creating any special difficulties for us so that we could really focus on the denominator polynomials and look at this. And purely from an analysis viewpoint if I wanted to do an inverse Laplace is, I really needed to know only the denominator polynomials root result form and then each term can be individually inverted right.

There again numerator polynomial plays a role but not explicitly, it is just the coefficients are computed based on the numerator polynomials. And when we come to stability again, if you are really looking at the denominator polynomial then again you are not paying much attention to the numerator polynomial. And when it comes to tuning and when we talk about stability based tuning, again you are looking at the limits of the stability so how far I can push the system with control parameters and then you are backing of based on that. Again the controller design did not seem to bother too much about the numerator polynomial because the ultimate period and ultimate gain all of these were computed using the denominator polynomial ok.

Then when we go to performance-based tuning which is what we call as direct synthesis approach where I am given a certain performance metric in terms of a closed loop transfer function and then you ask what is the controller that will get me this closed loop transfer function that is real first time that this numerator is going to start playing a bigger role and understanding the numerator polynomial is going to start playing a bigger role because if you notice the expression for the controller, you have a 1 over G model ok. So since you have the model transfer function coming in the denominator of a controller calculation which is what we do when we do direct synthesis.

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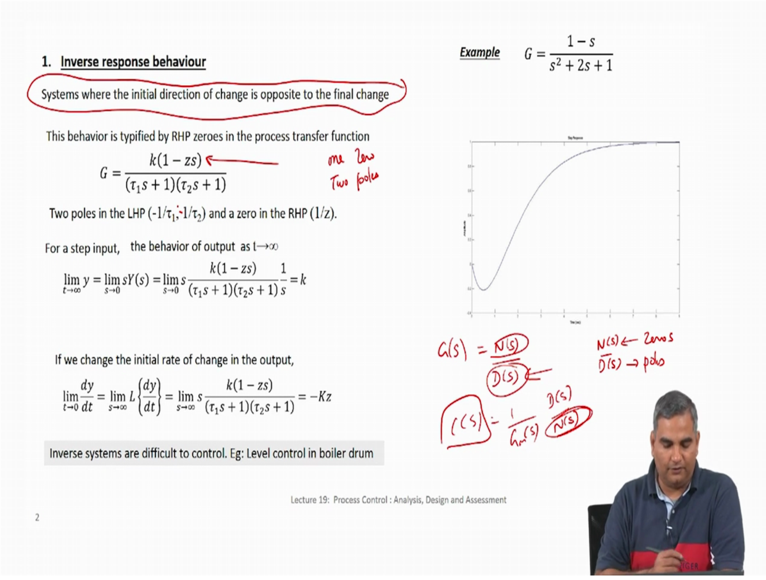

Now interestingly what happens is, if I have a transfer function like this, this is a numerator this is the denominator. Somewhere in the control calculation I start having this G model in the denominator so that basically conceptually what happens is then it flips it to DS by NS ok, so the numerator polynomials somehow seems to come to the denominator and if you want to now think about the stability of the controller for example right, then you have to start paying attention to the numerator polynomial because it has come in the denominator.

So when you explicitly do this direct synthesis and because in the direct synthesis formula you have 1 over GM, the numerator in the model becomes now the denominator and then you got to start worrying about what is happening in the numerator so that is an interesting thing that happens and this comes out very beautifully from that simple calculation when we did the direct synthesis controller. Now we will find and explain what happens when there are issues with the numerator polynomials and clearly the issues will happen if the roots of the numerator polynomials which we call as the zeros of the transfer function have certain behaviour.

So just to recap if I have NS over DS, the roots of this polynomial we call as poles of the transfer function and the roots of this polynomial we are going to call as zeros of the transfer function. So now let us look at a particular transfer function form and then try and see whether we can understand what happens and then we will come back to what inverse response behaviour is which is basically in words defined by this. We will say system where the initial direction of change is opposite to the final change okay, so initially it goes in one direction but ultimately it settles down into other direction.

So the way to think about it because in control we have always been talking about deviation variables so and the initial value is always we think about the initial value as being zero and if I have to interpret this and then say finally the variable becomes positive then if it is an inverse response system, the initial direction of change will be negative so it will become negative for a short period and then maybe it will become a positive value. Or in the flipside if the final value is negative it might start being positive and then finally settled down to negative value. So the initial direction of change, how long that change happens is a different question but if the final value you see if positive or negative at least for a short period of time, the initial values would have been positive or negative respectively so that is the key idea of inverse response.

Now you might ask it seems like a very strange thing to worry about, why would someone worry about this is one question and the direct synthesis approach is going to give you nice answer as to why you should worry about this ok, so that is something that will follow. But before we actually we worry about this we also have to think about how so if this is something that I need to worry about how do I just look at the transfer function at and then see whether the system is going to have this inverse response behaviour, so it is another question that we need to answer ok. So the 1st answer if you just look at the transfer function, if you want to understand whether this process which is represented by this transfer function has an inverse response so what you do is for the 1st time I am going to tell you focus on the numerator ok.

So when you focus on the numerator polynomial, if you look at this numerator polynomial you will see that this has 1 zero because there is only one root because the numerator polynomial is of order 1 and the denominator polynomial is of order 2 so I have 2 poles, so I have one zero and 2 poles in this transfer function. Now let us just quickly think about the poles because we have done this many times so let us quickly kind of dispense with this poles in this example. So if Tao 1 and Tao 2 are positive then the poles are in -1 over Tao 1 and -1 over Tao 2 and these poles are in the LHP if Tao 1 and Tao 2 are positive that means the system is stable.

Now that we have talked about the poles let us go to the zeros of this transfer function. And if you look at this transfer much and it has one zero where the numerator goes to 0 which is at 1 over Z, this is one minus Z S 0 so S is 1 over Z. Now if Z is a positive number then you notice that this 0 is in the RHP okay remember so this is how we look at this, so in this case you might have -1 over Tao 1 here -1 over Tao 2 here, but this is 1 over Z right. If a poll had been in the right half plane we would have said anyway the system is unstable but actually the system is not unstable because all the poles are in LHP however, we have 0 in the RHP ok so inverse response systems are typified by zeros in the RHP.

So the first question that I asked we can answer which is to say that if I look at the numerator polynomial and I find that there are zeros in the right half plane then I have to worry about inverse response system, and if I have only one zero in the right half plane that system will show an inverse response behaviour. And the generalisation of this result is if I have an odd number of zeros in right half plane the system will show inverse response behaviour that is a generalisation of this but as far as this course is concerned we are going to largely look at maximum of 1 zero in the right half plane most of the time, there might be an odd case where we will have more but and when we have one 0 on the right half plane then you have to assume you are guaranteed that that the system is going to show an inverse response.

Okay let us for understand that how can we say that this is going to show an inverse response. Mathematically how are we going to understand this inverse response behaviour something that we can see. So here we said look at it carefully, we said the initial direction of change is opposite to the final change so this is talking about the rate whereas this is talking about the final value ok. So the minute we say the final value then we know that from our bag of tricks we have something called the final value theorem so we will take this system and then try and see what is the final value for a step change ok. So if I have a step change from before we know, this Y of S will be G of S times 1 over S, so Y of S is here, I have G of S over 1 over S so this is Y of S.

So if I want the final value for Y which is limit the tending to infinity Y ok Y of T here then I know from my final value theorem that this is limit S tending to 0 S Y of S, Y of S is this transfer function times 1 over S, I am multiplying this 1 over S because we are subjecting this system to a step input and S Y of S so I multiply this S this side and then I can cancel this S and S here and then when I said S equal to 0 okay. What I am going to get is this is going to go to 0, this is going to go to 0, this is going to go to 0 so I am basically going to get the final value as K, so if K is positive the final value is basically a positive number ok so this I think is something that we have seen several times before so it should be straightforward to understand.

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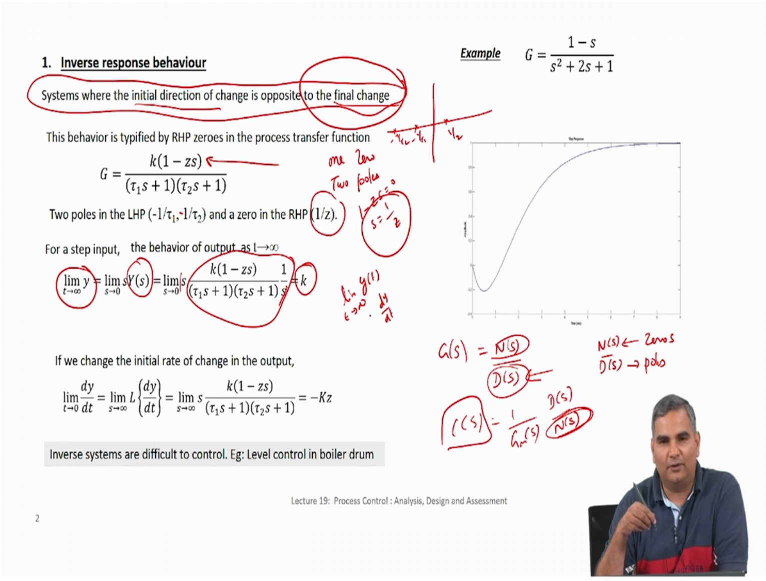

Now if this system is going to show inverse response then the initial direction of change has to be negative, so let us understand what direction of change means. So if I am looking at Y so Y of T okay limit T tends to infinity Y of T is the final value is the final change from 0 right because always our base is 0 because we are looking at aviation variables. So the initial direction would be I want to know what this DY by DT so what direction is it going to start moving. So if this is negative then I start at time T equal to 0 at value of Y equal to 0 and if DY by DT is negative initially at least right after this system starts responding to the step input it has to going the negative direction because DY by DT is negative and I start with 0 so I have to get negative numbers for Y.

And this only talks about what happens initially, we have already shown finally Y is going to take a positive value from the final value theorem. So how do I find out what limit T tending to 0 DY by DT is, and whenever we have T tending to infinity we have to use the final value theorem, and whenever we have T tending to 0 then we have to use the initial value theorem and the final value theorem you have to set S to 0 and the initial value theorem we have to set S to infinity, this is something that we have learned from our Laplace transform lectures. There is also another difference here, here we are looking at Y of T but here we are looking at DY by DT but that is not a big deal because we know the Laplace transform of DY by DT is S Y of S so this is something that we already know.

So what is going to happen is when I want to get this limit T tending to 0 DY by DT that is going to be limit S tending to infinity Laplace of DY by DT, and Laplace of DY by DT is S Y of S. And this Y of S is G of S times 1 over S because this is a step input so this term here Laplace of DY by DT so this is going to be this here because this S Y of S, this S comes for DY by DT and 1 over S comes for the step response so that and that will get cancel so I will be left with one S which is outside here.

So when you do this math here then what you will do is so you take this and then you take the S out of this and then what you will have is S square times K times 1 over S - Z divided by so if I take both S and S outside I will have S square ok, I will have since I have taken the S square out I will have Tao 1 1 + S, Tao 2 1 + S ok so this is what I will have. Now this is square and square I can cancel and then when I substitute S equal to infinity you will see that I will get a negative rate of change ok. So what it basically says is that the final change while it is positive the initial rate of change is negative okay, so let us take an example to understand this.

Supposing I take this example here, G is 1 minus S divided by S square + 2 S + 1 and this is a MATLAB simulation of response to a step input and clearly you can quite easily see that this S square + 2 S + 1 is actually S + 1 whole square so basically this is -1, -1 are the two poles so the system is stable because the poles are on the left half plane but the 0 is S equal to 1 so this 0 is on the right half plane. So this satisfies this one right half plane 0 so basically you would expect the response to be of the inverse response time and you can actually see from the simulation here that the initial rate of change is negative.

So I am starting at 0 here and because the initial rate of change is negative so the curve has to go in this direction because DY by DT is negative here, but the final change is positive so it goes back to a positive value so it goes down and then goes to the final value. So you might actually think and say okay why do we worry about this so much, why should this create any control related issues? So before I answer this question and again like I said before I will look at this bike example to explain why this creates a problem, but before that I have to at least give you one example of a physical system which will do this inverse response. A typical system that people talk about is level control in boilers drums where will you see this inverse response behaviour and the idea here is the following.

Supposing you have a boiler and you are heating it so that you have steam coming out and supposing you say I want to increase the level of the liquid in the boiler now in our tank example and other examples we have seen that if you want to increase the level of liquid in a tank what you do is either look at down what goes out of the tank from steady-state that will increase the level of the tank or from steady-state we can increase the amount of input into the tank so that the level increases. So if you do the same thing and then say okay I want to increase the level so let me feed some liquid into it.

Typically in a boiling liquid we will have these bubbles in the liquid and it is producing steam but when you add new liquid which is at a much lower temperature than it might cool down the top of the liquid in which case these bubbles might collapse and because of that the volume occupied comes down so the level comes down a little bit at the beginning. But ultimately once you have really increased the amount of liquid and if the steam rate that you are taking out is remaining the same then ultimately the level has to go up. So typically you would see level decreasing as soon as you increase the amount of liquid into the tank and then after a while the level will of course go up.

So you see that you will have an inverse response so this is a very typical example that people talk about in terms of the level of the liquid showing inverse response in a boiler drum. So that is an engineering system and then are lots of biological systems which show inverse response, there are several other systems where you can see this inverse response behaviour. Now the last question that one might ask is why is this important right, why what is the difficulty that this introduces in control and this is where the direct synthesis approach really tells us what is the difficulty of this because if we recall from the direct synthesis formula you have 1 over GM.

And if the model has 0 in the right half plane in the numerator venue in what this for a controller calculation then basically what happens is the right half plane 0 becomes right half plane pole for the controller which will make the controller unstable which is something that you do not want to happen. So basically what it says is if you have let us say a desired response that you want till now whatever was the desired response I could design a controller based on the desired response by using the formula for direct synthesis controller. But now for the 1st time if I have a model which has right half plane 0 then when I want to design a controller and I know what I want so I give you G desire and if I am not willing to make any compromise and then say this is the G desire and you have to design a controller for this.

If we go ahead and design the controller based on this G desire then your controller will become unstable so which is something that you do not want. You do not want for many reasons; number 1 it is not realisable, you have to understand in mathematical terms when you say a controller will become unstable that basically means that supposing it is water line and you are asking for water flow to be increase, when a controller becomes unstable it is going to ask for more and more water to be flown through that line but there is a physical limitation right so we cannot flow more than that amount of water because if we try to flow more than that amount of water you will burst the pipe right so that is how the physical connection to the unstable thing comes.

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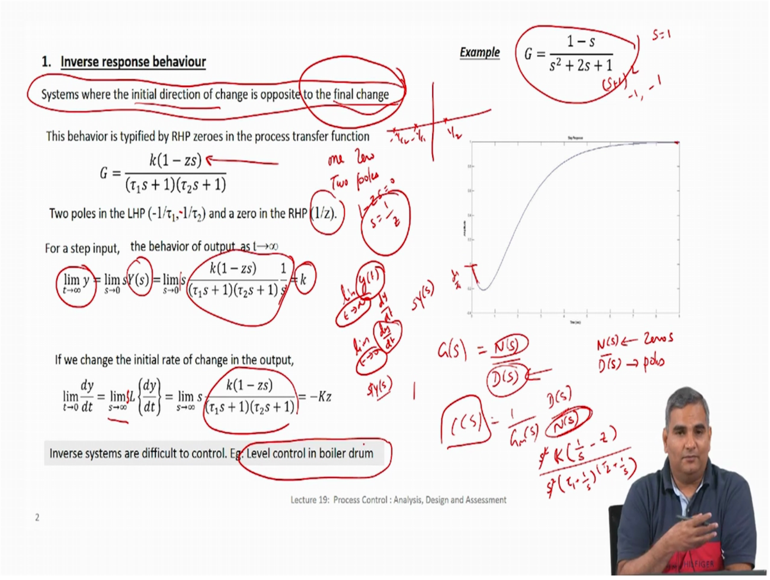

Mathematical it is okay, you keep increasing so what is the big deal but typically you cannot increase beyond a certain number because physical limitations are there. So it basically says that now for the 1st time we are faced with the process which limits the performance that I can get from a controller and it limits in the sense that I do not have unconstrained ability to say this is what I desire and please give me that right. So if I ask for something that is desired is the controller has to give me what I desire it has to become unstable ok, so there is a physical limitation on how much I can desire in some sense so that is the performance limitation that comes.

So the philosophic answer to this problem is to say okay I have to moderate my desires because the process is not going to help me get whatever I want so I am going to moderate my desire to be in line with what the process allows me to get and that is the concept of designing a controller when we have this performance limiting factors in the process model itself and this is an interesting idea which comes out beautifully from just the direct synthesis equation. Now I have explained this to you mathematically in terms of what is this inverse response behaviour, so basically you will have to look for zeros in the right half plane and if I have odd-number of zeros then the system will exit the inverse response behaviour.

I have contractually told you why this inverse response behaviour is a problem in fact, before that I actually showed you what the behaviour is in terms of simulation so the initial direction of change is different from the final value output takes. And I have also conceptually told you how this limits the performance that a controller can provide because of the 1 over GM term. Now the last thing that I also told you that there are engineering examples, I gave you one very-very commonly used example for this inverse response. Now if I want to understand this little more physically why inverse response systems are difficult, so think about driving a bike on a road and at some point you desire that you desire to turn right ok. So the desire from the control system viewpoint is that I want to turn right and then typically you are going to…

So the manipulated variable in this case is the steering wheel so you say if I move my steering slightly to the right then I will go right so that is our intuition right. So a normal bike when you are driving and you want to turn right you will turn the steering a little bit and you will go right. Supposing for some crazy reason the manufacturer has given you an inverse response behaviour bike then what will happen is the following; so when you are going and you want to turn right then of course you have to move the steering to the right because the ultimate change in the direction is going to be right, right.

However, because of this inverse response let us say I move my steering a little bit to the right so the initial direction of change is opposite to the final, the opposite to final is it will move left. So you are driving a bike and you are trying to turn right and then suddenly this turns left so you move your steering to your right but the bike is moving to the left, so what is going to happen is that you are going to get completely confused, you are saying okay I wanted to turn right but how is this going left so you are going to try to compensate then you will say maybe I should turn left and then it turn right and you will turn left and that will completely create havoc because you will keep trying to turn right and left because you are not understanding what the bike is doing.

So this is the same thing that happens with control systems if they are not designed properly because what we expect to happen will ultimately happen but because something else happens, the brain will say okay there is something wrong and I have to do something about it. And the same problem will happen in terms of inverse response behaviour when we control for inverse response behaviour. Of course, the solution to this is obvious how you should solve this bike problem if you are smart into the job, in the next lecture I will talk about how you solve this problem as a smart individual and then we will go onto what it translates to in terms of the map as far as controlling inverse response behaviour is concerned okay.

So in the next lecture I will talk about how to control this inverse response system which basically creates performance limiting behaviour Thank you.