Process Control - Design, Analysis and Assessment
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Lecture No 17
Controller design and tuning - Part 1

We move on to the seventeenth lecture in this course on controller design and tuning. In the last lecture I talked about how we analyse the stability of closed loop systems. I showed you that it is very similar to how we do open loop system analysis if we can write the denominator polynomial in root resolve form but I also taught you another technique which is based on a table called Routh stability table where I showed you how you could analyse the stability of any polynomial in the denominator of the transfer function by constructing a table and looking at the first column of the table and I said we can actually find out if there are any poles in the right of plane without actually computing the poles and I said you will see the use of this when we talk about controller tuning and so on.

In Lecture prior to that I talked about the different types of controller that we are going to study in this course the P PI PID controller where P is just proportion action PI is proportional integral and PID is proportional integral derivative. I also said if it is a proportional controller there is only one tuning constant KC if it is proportional integral KC tau I and proportional integral derivative has constant KC tau I and tau D. Now controller design and tuning is the task of actually picking one of these 3 types of controller for implementation and once you pick a particular type of controller, how do you choose the tuning parameters for the controller is the other aspect of punks role of controller design and tuning and that is what we are going to see in this lecture.

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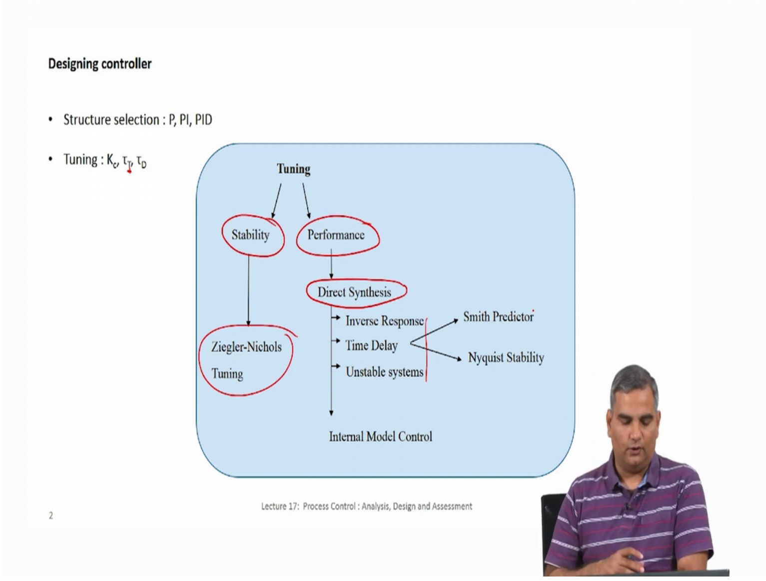

So the structure selection for as we are concerned in this course is choosing P, PI or PID and tuning would mean that we are going to find the values for KC tau I and tau D depending on what controller we have chosen whether it is P, PI or PID. Now tuning really can be done in 2 different ways and which is what I have in this picture here, so once we have chosen that we are going to use P controller, PI or PID controller then we can really look at tuning as either stability based or performance-based, so I will explain what these 2 are in this lecture and once you choose one or the 2 of these type of tuning strategies then there is a procedure that you follow a very popular way of tuning based on stability is what is called as Ziegler-Nichols tuning which I will explain in this lecture.

The performance-based controllers tuning is also called a for example direct synthesis and this is a very interesting idea you will see as we go along in this course and once you understand direct synthesis based tuning then it is very easy to understand and tuning for systems in which shows inverse response, we will describe what inverse response means later.

A time delay system, unstable system and so on, so these are types of systems that we have really not yet talked about but we will talk about them after the basic controller and design and tuning has been thought and when you come to time delay systems you can look at tuning using what is called Smith predictor based controller and we will also introduce the notion of Nyquist stability at this point which is the next idea that you need beyond Routh stability in terms of looking at stability of time delay systems and so on. We have also not talked about how we will tune unstable system, so that is also something that we will see as part of this direct synthesis approach.

So you will see these in the second part of the course that I am going to talk about, so as I mentioned in the introduction this course is going to be kind of broken down into 3 parts. The first part is the traditional feedback control as it is usually taught in an undergraduate curriculum which is going to be roughly 4 to 6 weeks of this course and we are coming to the end of that portion of the curriculum and after that I am going to spend about 1 fourth of the course which is about 3 weeks of the course on looking at little more advance ideas in even the single input single output in terms of what are inverse response systems.

How did tune and so on? All the way up to what is called as internal model control? And then we will also talk about multi variable control and model predictive control that will be the next 3 weeks and the last 3 weeks is really on how you actually look at the performance of the controllers once they are tuned, so with this notion that now we have lot of data and many of the decisions are actually database, how do we look at just data to identify if a controller is performing well or not and so on, so that will be the last 3 weeks which will be very advanced in terms of undergrads curriculum but as I mentioned in the introduction I think it is right time to introduced those ideas into a traditional control course.

So terms of tuning for now we are going to look at stability based tuning and performance based tuning and in performance based tuning we are going to look at direct synthesis and we are going to do tuning using direct synthesis for simple systems which do not have inverse response things that are not unstable and so on and these will pick up after the basic details are sketched out. So before I go on to show you the maths behind stability based tuning, so I will try to explain this intuitively what is done here, so that you understand why we do some of the things that we do in stability based tuning.

The very first thing that we should understand in stability based tuning is we have to first find the limit of stability, the idea is the following supposing let us say you were walking up a hill and then you are on the top of the hill you want to get a best view of the ground below. The best location in terms of performance which would be the best view I can get would be to really go to the edge of the cliff right and then look at it, so when you go to the edge of the cliff and look at the ground you will cover everything that is there to be seen.

So you get the best view you get the best performance, however there are many reasons why you will not do it, so if I just asked you what is the best location for the best performance then the answer is very clear it is the edge of the cliff so that I can see everything that is done. Now if I pose the question slightly differently and say what is a safe location for the desired performance that I have to see as much as I can see while getting whatever is the best I can get while being safe okay so let us say if we repose the same question in this manner then clearly the edge of the cliff is not the safest position all of us know common sense tells us that that is not the best position.

Now if you kind of bring that into controller or control systems viewpoint the reason why the edge of the cliff is not the best location could be one is that if there is sudden large winds that blow then if you are at the edge of the cliff you could be toppled over and then fall off, so that is the point at which things become unstable and unsavoury, right. So this you can see from a control systems viewpoint is like saying okay if there is no wind that is ever going to blow and I can predict that there is no such disturbance that is going to occur then I can stand at the edge of the place and feel reasonably save because I am saying nothing else is going to happen okay.

So the reason why standing on the edge of the cliff is not good is because there are disturbances which you cannot ever account for a completely, so you have to pack of a little so that even when there are disturbances that occur your system does not become unstable, so that is one reason why you want to back off or move away from the edge of the cliff, so that is one part the disturbance.

The second aspect is when I am walking towards a cliff you know the land might look very stable but you never know towards the edge of the cliff you might have loose sand right, so when I am walking towards the cliff and I am thinking about a model for the ground below my feet I might say this ground is very good it is very stable, so it is slightly to be stable towards edge but reality is not like that, so is always difference between what is the reality and what actually happens and any model that you build, however complicated or sophisticated the model might be.

So if you expect solid ground and then you go to the edge of the cliff and suddenly that is very weak ground then you simply fall off, right so that is the… from a control systems viewpoint that is like saying whatever be the model that you have if you identify the stability of a system based on the model in this case I am assuming that the ground is very hard and I can go to till the edge of the cliff. In reality the model might be different the ground might not be hard, so I have to account for it, so it is better to back off.

So these are the notions that I used in stability waste controller, so what you do is you first find what are the limits of stability, how far you can push the system without becoming unstable? And once you push the system to the limits of the stability then you do not operate the control system at that limit because you know disturbances can be there, there can be planned model mismatch all of which we have to account for somehow and in that stability of this tuning the way to account for these is to simply say okay let me back off a little and while for every step I back off I lose little bit of performance but my stability in the face of disturbances and model uncertainty keeps getting better and better.

So the worst extreme is never go anywhere near the cliff right, see completely inside then your performance will be very poor you will see very very small part of what is that to be seen but if you go to the other end there is a possibility of unstable system, so you have to find some middle ground somewhere you want to stand and get the best of the view and while being very very sure that nothing is going to topple over there is not going to be a physical harm in the case of a control system the equipment is not going to break down and so on. So this is the basic idea so we will see how this is operationalize and using mathematics they we will see that we look at the limits of stability and then we back off.

So mathematically what is back off mean is something that we will see in this lecture, so I hope this notion of tuning based on stability is cleared. We will now see how we do the mathematics of that after that I will come back as performance-based tuning where… Now what we are going to do is I am going to say look if you find the limits of stability and back off you are kind of arbitrarily losing some performance without having a clear idea of how much performance we are losing.

Instead I am going to ask this question I want to see let us say 80 percent of whatever is there to see where should I stand? Right so here look at the way we are posing the problem it is conceptually slightly different we are saying I am going to specify the exact performance that I want and tell me where should I stand right, so I might say okay 75 percent of the ground I want to see whatever it is then I calculate where I should stand in a performance-based approach whereas in a stability based approach I find the limit of stability and I back off by a certain amount which is based on intuition and so on, so we are going to see how the maths works for both of these techniques.

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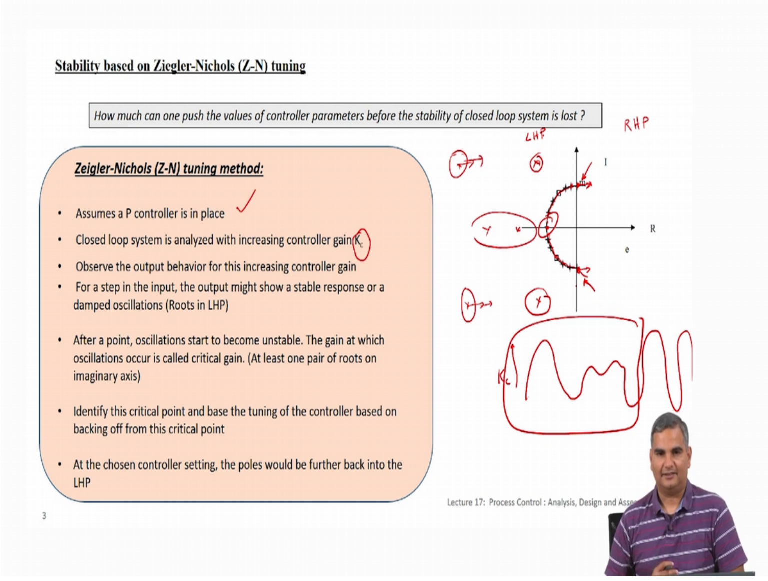

So as I mentioned before the question that we are asking is how much can one push the value of the control parameters before the stability of closed loop systems lost? Okay is the question that we are asking first for Ziegler-Nichols tuning, so I am going to say the approach in terms of let us say method that you do in terms of the series of steps and I will show you what is the mathematical equivalent of the series of steps that we talk about.

So first what we do is we think about the closed loops systems so we have open loop systems then we close that system with let us say a P controller, so we assume P controller is in place and then once we have the P controller in place then I am going to get the denominator polynomial for the closed loop transfer function and as I said before right now I do not know where the poles of the closed loop transfer function are because I have just said I am putting a P controller, I have not said what the value KC is.

So what you do is you substitute different values of KC this is the concept the math…the way this is done is slightly different you will see that but basically what you do is supposing I give you a value of KC then what happens is the denominator polynomial is something that you can actually compute every coefficient is known then you can find the poles of the denominator polynomial. Now for the KC that you have chosen let us assume the system is let us say stable means for example let us say I have you know maybe something like for a particular KC I have this root let us say repeated twice for example just for the sake of illustration then what you do is if this is root let us assume the other roots are somewhere here and they are okay.

Now so you have chosen a particular value for KC and you start increasing KC okay, so increase KC so actually Numerically put different value and then for each of this you basically find let us say what are the poles of the closed loop transfer function, so as increase KC may be this poles will start moving okay, so if they moved to the opposite direction it is great but if they move to this direction then we have to think about this and worry about this, so here I have my left half plane, here I have my right plane as long as all of this roots are in the left of plane I have no issues okay, so let us conceptually this is just a thought experiment that we are doing.

So let us say we start increasing KC and as KC starts increasing these poles start moving, so maybe this pole which is repeated twice would become a complex conjugate pole and so on and then let us say they start moving like this and this starts moving like this, so you keep increasing KC. Now you have to remember that the polynomial coefficient has now become a function of KC and at different values of KC, the coefficient KC I am saying that is the reason why the roots are changing and if I continuously change KC value I would expect the roots also to continuously change if it is a reasonable function right I can use the continuity argument, so basically as this keeps changing this is going to move.

So as this keeps moving at some point if increasing KC leads to stability problems at some point either this pole or this pole or this pole or this pole one of these is going to come to the imaginary axis right, so what we are basically saying is I am going to increase the gain and I am going to see these poles move in the complex plane and if the first again I chose if all the poles are in left of plane I have no problems and as it keep increasing KC if at all I am going to get a problem at some point then just before that problem occurs all the roots were in the LHP.

So they have basically slip on to the right of plane and they cannot slip on to the write of plane without going through the imaginary axis because I am continuously increasing KC so all the roots are also going to continuously change, so when they continuously change and then when it is going to slip off from here to here a continuous change will have to place of poles or the roots that are going to create problems by moving to RHP to be on the imaginary axis just before they moved to RHP, so this is the most important idea here okay.

So as you keep increasing KC this is what is called the ultimate gain that KC value at which the first set of roots are single root touch the imaginary axis that KC value is called the ultimatum gain and it has got the ultimate gain because this is the maximum value you can keep KC at because beyond that when you increase this is going to slip to the RHP and make the system unstable, so remember the mountain example that I gave you this is the point the edge of the cliff right, this imaginary axis is edge of the cliff, anytime I am away from here I do not have to worry about what happens and anytime I come to the right of plane I have to worry about it.

So the edge of the cliff is really the imaginary axis and the point that we are saying is if you are continuously working from ground and suddenly falling off you have to touch the cliff, edge of the cliff first before falling off right, so this is the limits of stability. So also notice that at this ultimatum gain when I have folds on the imaginary axis and let us assume this (()) (18:04) mode here okay so let us say I have this 4 poles now.

Now from our previous lecture we know that these 2 poles will create oscillations but because the real part of these complex numbers are negative that a power minus 8t will keep coming down so the oscillations will be damped out after a while, so after little bit of transient dying out, the effect of these pole in terms of oscillations will die down because while I will have cos and sin term because of the imaginary part of this roots, the real part is less than 0, so we know that it will basically make the amplitude 0.

So if you wait long enough at this gain then what will happen is these 2 roots will lead to oscillations that will die down and once those oscillations died down there will be only oscillations due to these 2 roots and because the real part of this is 0, so there is no way to dampen the oscillations. What we will see is what we call as sustain oscillations, so basically what will happen is may be some transcends will happen and then you will have the sustained oscillations okay.

Now you also know the root here, the imaginary part of the root actually gives you the frequency of the oscillation, so that is also something to remember, so I have an ultimate gain at which point the roots are on the imaginary axis and the location where they are on the imaginary axis actually determines the sustain oscillation right and also you should remember that there will be some transcend behaviour before the sustain oscillation because you might have other roots which might be here, might be here.

If all the other roots are on the real axis but on the left half plane there will not be any oscillations because of these roots and they will die down and you will have a sustain oscillations but if you have roots in which are on the left half plane but they also have a complex components the imaginary part then they will lead to oscillations initially but because they are on the left half plane the oscillations will die down before this sustained oscillation takes place.

So this if you have understood very well then Ziegler-Nichols tuning is a very very easy approach to remember and notice how we are basically taking a polynomial, but we are talking about each set of roots individually in terms of the behaviour. This is simply because again I just want to reiterate this that this is because of the way the Math works there if I have a polynomial with multiple routes you remember that I can write each one has a separate term, so the overall effect is an additive, effect of all the root, so these 2 roots effect will add on to these 2 root Fx, so to add on to the overall effect.

So that is the reason why we can talk about the substrate and then say here these oscillations will die down, these oscillations will be sustained but when you add them up initially there will be something which is crazy looking with some part of sustained oscillations some dying down but once the transcends are over the oscillations due to these have died down then that term does not matter anymore and you will have only sustained oscillations.

So you see how beautifully you can start understanding just looking at the roots of the denominator polynomial plotted on a complex plane that you can start basically constructing how a time profile for the output is going to look, so if you can actually get to this place where you can imagine the time profile easily based on this understanding then you have really got a good understanding of how these things work, okay.

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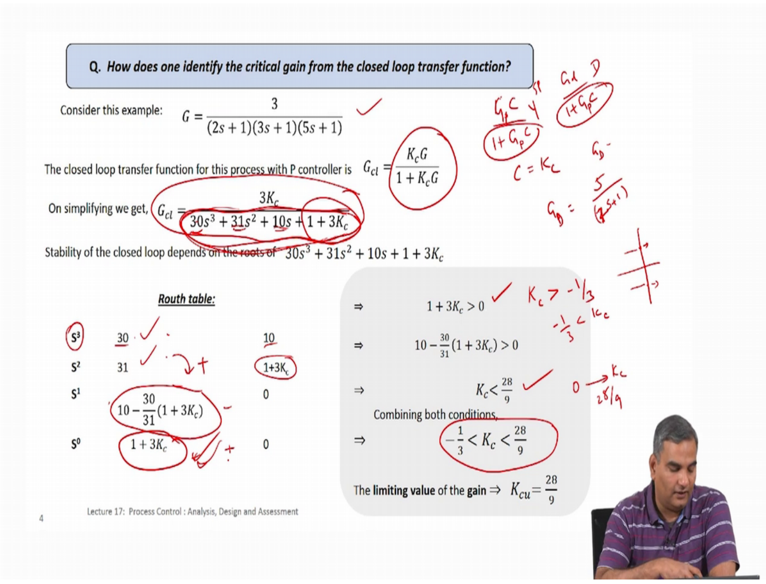

So now there is all good terms of intuitive understanding and the theory behind how we are going to do this but how does one actually find the critical gain right so you cannot really go to a physical system and then implement a P controller and then keep changing gain and keep watching the output and then keep watching the output till steady oscillation is setup this might have been done many many years ago but right now this is very poor way of doing this and no one does it that way, so there must be some mathematical way of doing it.

So the mathematical way of doing this is basically first you get the open loop transfer function and I have already told you how we get the open loop transfer function, either you model the process and get first principle's model linear I said and then dual plus transform or there are more sophisticated ways of actually doing what is called testing of the process either increase your input in step or some other fashion and then basically identify forces model. And once you identify a process model all of this computation we are going to do based on the process model, so that is a critical idea.

So let us consider a simple example let us say I have open loop transfer function which is of this form 3 divided by 2 s plus 1 times 3 s plus 1 times 5 s plus 1. Now the thought experiment we did conceptually what we are going to do is we are going to put in a P controller in place and then we are going to say the gain is KC which is not yet determining, so once we do that and look at the closed loop transfer function, so the closed loop transfer function we are looking at is from a set point viewpoint which is remember kd process transfer function G PC divided by 1 plus GPC is what we had before and so that…we remember the disturbance transfer function is GD by 1 plus GPC okay.

So hours long as we are looking at the stability aspect of the system, so how far can I push this controller parameter till I get to (())(23:53). It does not matter whether you are looking at the transfer function with respect to voice a points or the transfer function with respect to D, and we make this point before the denominator is the same. So we really are looking at the denominator polynomial for stability, so once we look at one plus GPC it works for both the set point transfer function and disturbance transfer function.

So there is nothing extra that you need to do and look at what happens from a stability viewpoint if there is disturbance and so on, so in this case G is given here KC is the controller because we are talking about a P controller, so we have simplified this to C equals to KC and then we compute this KCG by 1 plus KCG and you can do the simplification quite easily, so you will get this as the closed loop transfer function okay, so the transfer function with respect to disturbance will also be something similar like this except the numerator will be different and denominator will be the same okay.

Now you notice what does happen here I want to find the stability of this transfer function let us say and I want to look at partial fraction opposed to doing this then basically what it means is I have to find the poles of the denominator polynomial and when I try to compute the poles of the denominator polynomial I have a problem right, so this is a number, this is a number, this is a number but this is something that I do not know. Now it is become a function of a constant, so I cannot directly write in many cases, in this case since it is a cubic it is possible but it will be a (())(25:34) if higher order polynomials it is going to be very difficult to write analytical expression for the roots of the polynomial.

So immediately the thought that strike us is okay so I cannot write the roots of the polynomial but I am really not interested right now in finding out the exact root of this polynomial because remember from a Ziegler-Nichols stability viewpoint what we are really interested in is finding out when will this become unstable, we are not interested in the roots till then or after that you want to only find till when will it become…at what point of KC will this polynomial lead to unstable behaviour.

So if we pose this question then basically what we are asking is look a do not care about the actual values are the roots but I want to ensure that no roots are in the RHP, if that is the case then this is stable, so immediately we know that we can use Routh table which I thought in the last class and that is the beauty of this whole approach of this Routh table, so we are going to find out actually without a (())(26:41) computing the roots, what values of KC will take this closed loop system to the edge of instability is the idea that we are going to do.

So before we do this let me clarify one more thing just so that we can understand how all of this works. Supposing….so I mentioned that you do not have to worry about the stability of the transition function for GCL and GD separately because the denominator polynomial is this that comes… It is just the theoretical point that I just want to make here, so we are also assuming whatever is the transfer function for the disturbance GD that is stable, so when I do GD by 1 plus GPC I will obviously get this polynomial in the denominator, sometimes I can also get poles from the GD transfer function in the denominator and as long as they disturbance transfer function is stable we do not have to worry about that pole.

So we are not going to really consider that that is the reason why as long as you simply consider this polynomial it is good enough whether you are looking at the transfer function corresponding to set point of the transfer function corresponding to disturbance, so there is another quick thing that you want to check for example you can take something like GD some 5 by let us say something like 2s plus 1, so I have something like this you can actually put this transfer function and see what happens you will see that this 2s plus 1.

So sorry here there is a 2s plus 1, so we will make this 7s plus 1 then you will have pole corresponding to this also showing up in the transfer function corresponding to disturbance however you do not have to worry about this as long as this is stable you still have to worry about the only in this polynomial, so that is something that you can look at okay. Now that we have made all of these points let us look at this transfer function and the denominator think about the Routh table.

So in the Routh stability the very first condition that we have is that all the terms of the polynomial should be positive. Even if one term becomes negative then we know that we will have a pole or a root in RHP, so this is positive, this is positive, this is positive, so we should also have 1 plus 3 KC is greater than 0, so this is the first condition that we look at, so this basically means that KC is greater than minus 1 by 3 or you can write minus 1 by 3 is less than KC, so both of this are okay.

So every term in this polynomial should be positive that gives you this condition, so if everything is positive then at least right now while we cannot say that there are no poles on the right of plane yet, we can for sure say that once we construct this table or there is a reason to construct this table because the test did not fail here at all are right if it had failed we would have for sure known that there is a root on the RHP and nothing to do anymore but since it did not fail then we have to do the confirmatory tests to figure out whether there are poles or roots in the RHP.

So the confirmatory tests is done very very simply using the Routh table that we talked about, so remember the first term is sq I said you have to put this number leave 1 and put this number 30 n turn and s square it is 31 and now you see 1 plus 3 KC. Then what you do is… I spent a lot of time in the last lecture showing many cases where the Routh table is constructed, so you follow the same procedure and they slide is on the screen you can after listening to this video you can actually go and do this computations yourself you will find the next element here is this which is basically 10 times 31 minus 30 times 1 plus 3 KC by 31, so the 10 times 31, 31, 31 will get cancel minus 30 by 31 this and 0.

Now this is very simple, this times this minus 31 times 0 divided by this number, so in will give you just one plus 3 KC. Look how beautifully now everything is going to work out for us to find what is the value of KC at which the system could become unstable without ever computing roots at a point, so you never computed even a single root till now remember that. So we know this is a positive number from Routh stability criterion we know once we pass the first test which is all coefficients are possible then the next test to pass is that there are no sign changes in the first column.

If there are no sign changes there are no roots in the RHP okay. So this is positive, this is positive now if this were not positive you will not even come near and we have assumed this is positive and already generated condition for that for KC right, so we are doing this analysis now assuming KC is greater than minus 1 by 3, if KC is less than minus 1 by 3 already we know that there are roots in the RHP because this term will become negative, so even if one term becomes negative there are roots in the RHP.

Now by ensuring that KC is greater than minus 1 by 3 when we start this analysis we know that the first test is passed and we can construct the Routh table, so this is also positive number with the condition that KC is greater than minus 1 by 3 or minus 1 by 3 is less than KC okay, so then it only leaves this term here, now if this term is positive then there are no sign changes, however if this term is negative then there will be a positive number positive number 1 sign change and another sign change, so there will be 2 sign changes then there will be 2 poles or roots in the RHP okay, so that is the key idea here.

So if we have KC such that this is positive and if we have KC such that this is positive then there will be no sign changes here, so we will have all the roots in LHP there will be nothing in RHP and the system will be stable, so this is one condition we have already seen, so the second condition is 10 minus 30 by 31 times 1 plus 3 KC is greater than 0, so if you do the mathematics of this and simplify this you will get this condition which is KC is less than 28 by 9 okay. So the fact that all the coefficient of the polynomial will have to be positive as giving you one condition which is KC is greater than minus 1 by 3 and the fact that every number in this first column has to be positive otherwise there will be sign changes as giving you the second condition which is given KC is less than 28 by 9.

Now if you put both of these conditions together you will get minus 1 by 3 less than KC less than 28 by 9 okay and typically we have been talking about positive KC values we will keep it like that, so if you start from 0 and then start increasing KC okay till you get to 28 by 9 we can guarantee that all the routes will be in LHP because this Routh table says that there will be no sign change but the minute you cross 28 by 9 then what will happen is this will be positive, this will be positive, this will still be positive because we are always greater than minus 1 by 3 starting from 0 here.

So this is the only thing that will become negative but the minute that becomes negative that means there are 2 sign changes and you will have 2 roots in the RHP beyond that, so in your head a way to think about this is there are these 2 common complex conjugate poles which are coming together at exactly KC equals to 28 by 9 the poles of this will sit here and here okay beyond they will move to this side that is the reason why you will get 2 poles in RHP which is what the Routh's table says because this will be negative that will be 1 sign change positive to minus and then minus to positive will be the other sign change.

So see how beautifully we have now been able to find KC where we will have problems starting to appear, so just prior to that remember the poles will have to be on the imaginary axis and we can actually see there will be 2 poles recommence it on the imaginary axis at this point.

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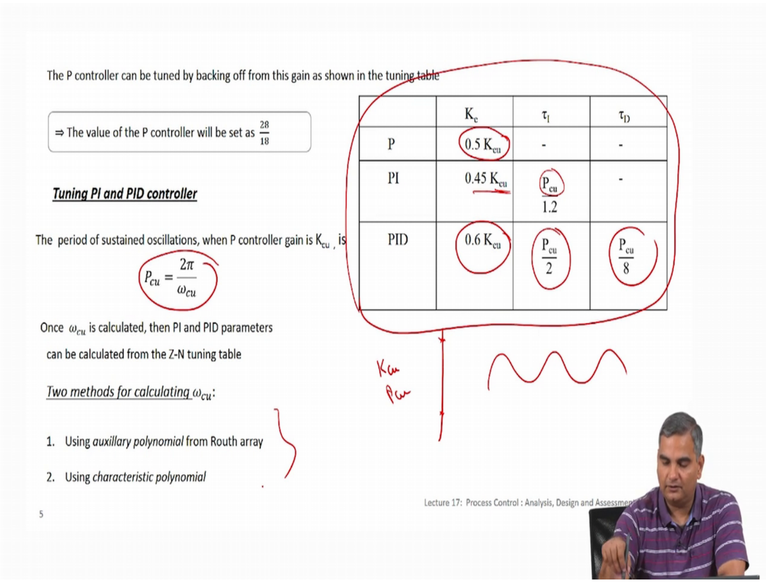

Okay so now the value of the control gain is what we call as K CU the ultimate gain and I also told you at this point I am going to have 2 poles sitting on the imaginary axis which is going to lead to a sustained oscillation and once I have the sustain oscillation basically to tune a controller what we need is both the KC and the period of this sustain oscillations and we will see how we compute this but here is a table that we use for computing the coefficients on tuning the controllers.

So we ordain know how KCU is achieved I have not shown you how this PCU you get but I have already shown you that it is going to be sustain oscillations because there are 2 roots that are sitting on the imaginary axis, so that hash to be a PCU, how to compute that PCU we will show here however once you compute this right using this idea then you can actually come up with a table which tells you what are the tuning constants that you should use for KC tau I and tau D for P, PI and PID controllers.

An interesting thing to notice here is we never use the PI control structures the computations we have done, you do not need to. So we always assume that we first put in a P controller find the limits of stability at the limit of stability I will get 2 things one is the critical gain at which the limit is reached and then at that limit because there will be roots on the imaginary axis. I will have a sustain oscillations and I can compute the period of the sustain oscillation and once I have those 2 I can directly write the PID tuning parameters as a function of this.

So if you are only doing a P controller then it is 0.5 KCU if it is PI remember again these are computed… You assuming that I put a P controller in place but if ultimately if you are going to use a PI controller these are the tuning parameters you should use which is 0.45 KCU PCU by 1.2. If you are going to use a PID controller it is 0.6 KCU PCU by 2 and PCU by 8 for tau D, so this is the table.

Now what happens is this is a simplest of tables and I think it is lot of understanding value to think about this table and how this was filled from a stability base condition but this is not the most current table, so there are several tables which are based on different names of people who have come up with ways to tune this controllers there is another table called Cohen coon table which you can easily find and as part of the assignments that we give in the course we will also give you Cohen coon table for you to look at and then tune the controller.

There are tables based on IMC internal model control structure that again is going to be some table you are going to build the model look at parameters and simply compute these values for the tuning conference okay. So I am not going to go through many of these tables in this theory lecture, however we will give you problems based on these tables and different types of table, so that you are comfortable with the latest tables that are used and so on okay, so that is an important point to remember when we look at tuning controllers. Now the last thing I have to show is how do you get the period of oscillations.

So the period of oscillation is computed as 2 pi by omega CU and this omega CU is something will compute using one of 2 approaches, so I am going to show you both approaches one is called using the auxiliary polynomial from Routh array the other one is using the characteristic polynomial itself to compute this WCU, so as far as this summary is concerned once we figure out how to compute WCU we can get PCU by 2 pi by WCU are omega CU and KC I have already shown how you compute and once you compute both of this if you were to use (())(39:04) tuning you basically use this table.

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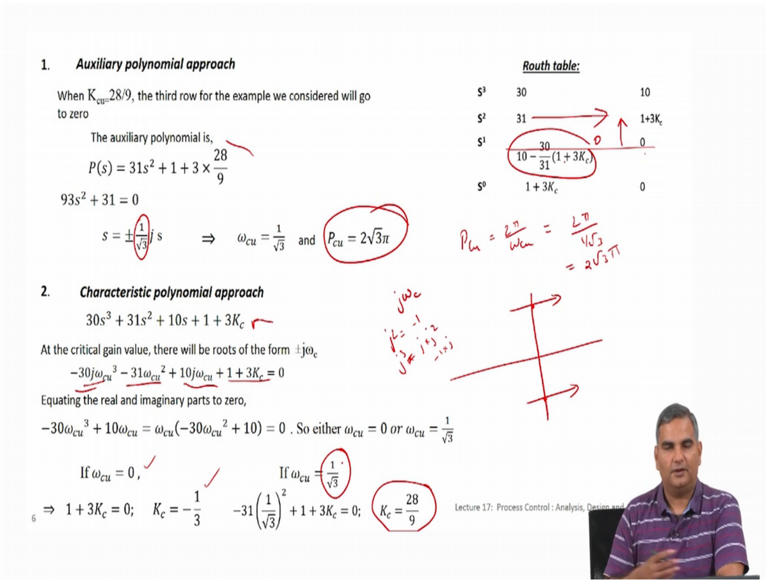

So the axillary polynomial approach is simple, what we are going to do is at KC exactly at 28 by 9 if you compute this because we said this is greater than 0 and got this 28 by 9, so if you comfort this for KC equal 28 by 9 this will go to 0 okay and in the last class when I talked about the Routh table, so we talked about if a row becomes 0 then we said there are roots that are going to be in the imaginary axis and one of the ways of computing the roots on the imaginary axis is look at the polynomial, auxiliary polynomial which is one row above which is what we talked about.

So if this becomes 0, this is s 0 room so if you look at one row above then you have this, so the polynomial, auxiliary polynomial is 31s square plus 1 plus 3 times this 28 by 9, so the 28 by 9 is ultimate gain. Now if you simplify this polynomial you will get 93s square plus 31, if you set this axillary polynomial equal to 0 and then compute s that will give you the omega, so for example you will see that you will get imaginary solutions alone and real part, so omega is whatever you have here and we said that PCU is 2 pi by omega CU, so this is 2 pi divided by 1 by root 3, so this will be 2 root 3 pi which is what we have here.

So this is one way of computing the imaginary solution that it results when I have gain such that that one row has become 0 here, so this is a very simple way of doing this. Another way of doing the whole thing is to simply look at the characteristic polynomial and then say okay so here this is the third order polynomial, so there are going to be 3 roots for this polynomial, so whenever there is a root which is going to slip from the LHP to RHP exactly at that KC will know that there has to be some imaginary solution with no real part.

Let us assume that imagination is actually J omega C so the reason why we write J omega C is simply because there is no real part okay and when I have J omega C plus J omega C I will also have minus J omega C, nonetheless so solution has to be of the form J omega C okay that is the key part that we need to remember without real number real part and the reason why we do not have the real part is we have assumed that KC we have chosen or we are going to choose is such that it is critical and the roots are going to slip from the LHP to RHP, so now since I am looking for a solution of the form J omega C I put this into this equation and then I say my 30 J omega C whole cube will give me omega C cube, J cube.

J cube is minus J because J squares is minus 1 J cube is equal to J times J square, so since J square is minus 1 minus 1 times Js minus J and then s square will be J omega C whole square, so J square omega C square, so that will be minus 1 and this plus 30 J omega C plus 1 by 3 KC. Now what you do is you collect all the imaginary terms and the real terms and since this is 0 or right-hand side both have to be 0, so both the imaginary terms and the real terms have to go to 0, so if you collect the imaginary terms you will have this term and this term which is minus 30 omega C cube plus 10 omega C.

So that will give you omega C is either 0 or omega C is plus or minus 1 by root 3 okay and once you have that then you look at the other part of the equation which is 31 omega C square plus 1 plus 3 KC will have to be 0 that is the real part which is 0, so for each of this you will be able to find the corresponding KC, now if you assume omega C is 0 and then substitute into the real part equation you will get KC is minus 1 by 3. Remember when we did this with the Routh table we said one end is minus 1 by 3 for KC but we ignored that because we were only looking at the positive values of KC.

The other KC will turn out to be through this approach KC is 28 by 9 then this omega C is 1 by root 3 which is exactly what we got using the root of regulatory criteria, right. So this is a very elegant simple way of doing the same thing in terms of finding limits of stability. So just to summarise you collect and this is an exercise I would really like you to do on your own and then make sure that you get the same results, so collect the real part and the imaginary part and then because it is 0 the real part has to be separately 0 and imaginary part has to be separately 0.

In this case from the imaginary part equation you will get values for omega C, so you will get multiple values and if you put those multiple values in the real part equation then you will get the corresponding KC values. Here when you put omega C is 0 you will get KC is minus 1 by 3 and when you put omega C is plus or minus 1 by root 3 you will get KC is 28 by 9 which is all the consistent with what we did before. Once we got this omega CU now PCU is simply 2pi by omega CU, now you got for KCU and PCU then you basically use a table to tune the controller, so this hopefully gives you a good idea about how stability based controller tuning is done.

We looked at table called (())(45:01) course table, so while this gives you an idea of what stability based tuning means ultimately all of this boils down to looking at a table and being able to read the table in terms of what controller tuning values you should keep for what systems and Ziegler-Nichols kind of table that was used before, now there are multiple improvements on this there is a table called Cohen Coon table which you could use and there are also tables based on internal model control which you can use, so those are tables that are popular in use, so somehow these tables are created based on this notion of stability based tuning.

Some of these tables such as IMC based tables are derived based on the direct synthesis or performance-based tuning, so in the next lecture I will talk about the performance based tuning, so those will be used to tune controllers in general and in some specific we you can make them useful for tuning PID controllers and I will tell you the concept in the next lecture but then ultimately again there you will look at the table where many of these have been already worked out you do not have to do all the work. Nonetheless when you look at this table and you are tuning your controllers you will have a clear idea of what concept is being used in the tuning table so that you can interpret your results much more proficiently. So with this I end this lecture and I will see you back again in the next lecture on performance based. Thank you.