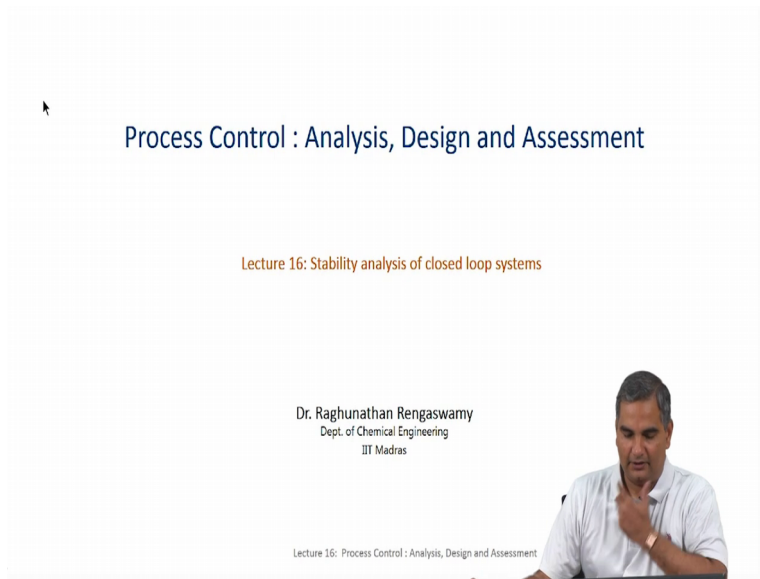


Process Control - Design, Analysis and Assessment
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Lecture No 16
Stability Analysis of Closed Loop Systems

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We will continue with the lecture 16 of this course. In the last lecture we looked at how to analyse closed loop control systems and then I explained that just like how we did with open loop systems first we talked about how to analyse open loop system in terms of inverting Laplace transform and getting back the variables in time domain and then we followed that notions of stability and how do you look at stability of the systems and so on. After that I showed you that when you look at closed loop systems essentially once you get the transfer function then the analysis ideas and techniques are just the same as open loop system there is nothing that you need to know more to do the analysis.

So the stability ideas also largely translate into closed loop systems, so remember in open loop systems we said if all the poles of the transfer function in the left of plane when we said the system is stable. Similarly as long it is a transfer function and I have already mentioned this does not matter whether it is open loop or closed loop the ideas of stability will be the same.

However we are going to introduce another notion of stability through what we call as Routh's stability table and this becomes important when we talk about closed loop systems

this was not as important before in open loop but in closed loop system it will become important and I will speak to this in more detail later but just to give you an idea when we have this closed loop systems let us say we have decided that we are going to implement let us say P controller or PI controller or a PID controller and so on.

So that just chooses a structure of the controller, so if it is P controller then the structure says you have something called gain which K_C if it is a PI controller it says the structure has something called K_C and τ_I gain an integral time constants and time constant and if it is a PID controller it is K_C τ_I τ_D . Now we have not yet said how you are going to pick numbers for these constants or parameters of the controller, so that is actually what is called as tuning and before we understand tuning I am going to explain some notions of stability so that I can then tie it in with how tuning is done.

So the difference that is going to come between open loop transfer function and closed loop transfer functions is that in open loop transfer function you have already identify a model so you know all the parameters of the model and all of these are numbers in a transfer function, however when you notice a closed loop transfer function that we saw in the last lecture you see that in the transfer function there are these parameters of the controller such as K_C τ_I τ_D which are yet to take values we have not given them values.

So in that sense what happens is the denominator polynomial and of course the numerator polynomial also the coefficients that multiply the powers S^0 , S^1 , S^2 , S^3 , S^4 and so on they become function of these controller parameters. So they are not just numbers but they now become functions of these controller parameters, so in that sense then the roots of this polynomial are not known or the root takes certain values depending on what values to give for this controller coefficients.

So we will get to a stage where we have to analyse the denominator polynomial to understand stability, however the denominator polynomial have coefficient, polynomial coefficients which are actually functions of controller parameters. So that will bring in complexity in the analysis that would require us to introduce the next notion of stability. If that were not there if you are actually going to say look I am going to put this numbers for this controller parameters and then I want to check stability then for each set of controller parameters we can use the same ideas you do not need anything new but when we actually let them be functions of the controller parameter then you need some new ideas and that is what I am going to cover in this lecture today.

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Stability analysis of the system

If the open loop transfer function is $G(s) = \frac{N(s)}{D(s)}$ and controller is, $C(s) = \frac{C1(s)}{C2(s)}$

The closed loop transfer function

$$G_{cl}(s) = \frac{N(s)C1(s)}{D(s)C2(s) + N(s)C1(s)}$$

If the poles of the denominator of this transfer function are in the LHP, then the closed loop system will be stable.

- Write the denominator in root resolved form
- Coefficients become functions of parameters

Routh stability test

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So remember the open loop transfer function G of s we will have numerator by denominator form. The controller can also be thought of as a numerator by denominator form for example if the controller is simply K_C then the numerator is K_C and denominator is 1, if it is a PID controller and then you have $K_C 1 + \tau_I s$ then this can be simplified as $K_C \tau_I s + 1$ by $\tau_I s$, so this is the numerator polynomial this is the denominator polynomial.

So in essence whatever we are doing we can always write this in the numerator by denominator polynomial form and then once we have this we know the closed loop transfer function which we have seen several times the closed loop transfer function is basically GPC by $1 + GPC$, so if I write G_P as N_S over D_S and this $C_S C_1$ over C_2 and I do the simplification then I will get something of this form which is again some numerator by denominator, right so the closed loop transfer function also is some numerator by denominator form.

So when we talk about stability we are going to say why is some close look transfer function times y set point okay so what we want to know is whenever this is stable that is we do not ask for set point changes which are unreasonable crazy then when will this still be stable is a question that we are interested in answering and we know the answer to this question. This will be stable if the poles of the transfer function are strictly in the left of lane right remember so I have this just a reminder so I have an imaginary axis this is a right of plane which includes all of this area, this is a left of plane.

Now as long as every pole is strictly in the left of plane then I will have a system which is stable even if one pole goes either to the right of plane or on the imaginary axis then the system can become unstable even for a reasonable y set points. It is very clear why if there is a pole here then the system will become unstable, we also discuss this and I just want to recall this her is if I have let us say a pole on the imaginary axis then if I give a sign oscillation at exactly this frequency then I talked about residences and how will things become unstable, so if there are poles on the imaginary axis still you have to worry about it they are not strictly going to be stable for all reasonable set point wise, right?

So for that to happen the poles of the transfer function should be strictly in the left of plane, that basically means it does not matter whether you are looking at open loop transfer function or closed loop transfer function because we are always going to write this transfer function as a numerator by denominator form then basically what you are going to do is you are going to look at the roots of the denominator and as long as the roots of the denominator are strictly in the left of plane we call the poles as being in the left of plane then we say the system is stable.

Now if you actually knew the value of this and this then once you substitute this here all you are going to get is you are going to get a transfer function form where there is a numerator and denominator. What you can simply do is take the denominator polynomial and maybe send it through let us say a MATLAB code to find the roots of that polynomial and that is what I called as a writing the polynomial in a root resolve form, so for example if I have a polynomial which is 3rd order if I simply write this as whatever the values a, a3s cube plus a2a square plus a1S plus a0 so this is third order polynomial.

If you specifically write this in some form where you have something like a3 times s minus root 1 times s minus root 2 times S minus root 3 right if you write like this then this is what I call as a root resolve form and we know the advantage of this because when we do the partial fraction in this root resolve form we know that the terms that are going to come or of the form e power R1T, e power R2T, e power R3T and so on and if these roots repeat then you know the corresponding form from what we have seen before okay.

So as long as you get this in a numerator by denominator form the closed loop transfer function and then if you have the value KC and tau I you can really figure out whether the closed loop is stable or not it is very simple computation to do, however now if I ask you a slightly different question, I ask you this question I am not going to give you the values of KC tau I and tau D because those are things that I need to find, I do not have values for this

and instead of doing this here what we are seeing is give me KC τ_I τ_D then I will tell you whether the system is stable or not by simply doing this good finding but if I flip the question and then I ask saying okay I have KC τ_I and τ_D as parameters themselves you tell me for what values of KC τ_I and τ_D the system will be stable, so there is a subtle difference here, right.

So what I am asking you to do now is I am telling you I am not going to give you values for KC τ_I and τ_D but I want you to tell me what are reasonable values for KC τ_I and τ_D or what are values for KC τ_I and τ_D that will still keep the system stable okay so that is a very different question than what is the stability of closed loop okay. So when we want to do this then it becomes problem, the problem is the following you might simply ask things... think about this and say well there is a polynomial in the denominator and basically I can write the roots of the polynomial in some form and then look at the roots and answer that question for this KC τ_I and τ_D .

So for example all of us know this formula that we have been told several times and it is drilled in our memory is root of this quadratic equation which is $-\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$ so you might say let me write this polynomial get the KC τ_I and τ_D as DAC as functions of those and then I look at this and then from this I will be able to find out whether the route is going to be in the left of plane or not and what values of case KC τ_I and τ_D will retain the roots to be in the left of plane.

So this is possible if the denominator polynomial is let us say quadratic and when it becomes cubic this is much more complicated formula and yarn that there are other mathematical problems in terms of writing analytical expression for the roots, so I am not going to get into the theory of that and then the complications that surround that but as far as this course is concern you can assume that beyond a certain order it is not possible to write this analytical form for us to do this exercise.

So that brings away interesting question where I have a polynomial and I have the coefficient being functions of KC τ_I and τ_D and if the order is large enough I cannot get analytical expression for the roots but I want to answer this question where I asked what values of this constants will keep the poles in the left of plane, so conceptually it is an interesting thing to think about right, so what we are saying is can I guarantee that the poles will be strictly in the left of plane without actually calculating the poles.

So that is a very interesting question it is very different from the partial fraction idea that we have to now, so in the course introduction I mentioned that I will introduce different ideas whenever they are very required and not as another idea that people talk about and here is a point where partial fractions are not useful anymore I cannot really do this with partial fraction because for partial fraction I assume that I know the roots but here I do not know the roots and if the order is large enough I cannot compute the roots analytically, however I still want to find out when roots will be in the left of plane without actually computing the roots.

So it looks like it is very interesting mathematical problem and the nice thing is that there is a solution to that which is called the Routh's stability test, so what I am going to do is I am going to explain this Routh's stability test in terms of the procedure and the roles that you should follow to do this Routh's stability. Why these roots are true and why Routh's stability test works and so on are things that I am not going to explain, so you might just take this Routh's stability as very interesting procedure which is going to tell us if all the roots are strictly in the left of plane without ever computing the roots okay. There is complete mathematical proof for this which we do not have to worry about in this course. Just be assured that it is actually mathematically proved.

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Routh test

Consider a polynomial of the form

$$a_0s^n + a_1s^{n-1} + a_2s^{n-2} + a_3s^{n-3} + a_4s^{n-4} + \dots + a_n = 0$$

Routh table is constructed to test if the above polynomial has roots in the RHP.

Procedure for analyzing the stability of closed loop system

Step 1: Check if a_0 is positive. If not, then all the coefficients can be multiplied by -1 to make a_0 positive

Step 2: If any of the coefficients of the polynomial is negative then there is at least one root in the RHP and the system is unstable

Step 3: For an n^{th} order polynomial, construct a table as shown below.

s^n	a_0	a_2	a_4
s^{n-1}	a_1	a_3	a_5
s^{n-2}	b_1	b_3	b_5
s^{n-3}	c_1	c_3	c_5
s^{n-4}	d_1	d_3	d_5
s^{n-5}	e_1	e_3	e_5
s^{n-6}	f_1	f_3	f_5

where $b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$ $b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$

$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$ $c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$

For an n^{th} order polynomial there are $(n+1)$ rows

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So this general problem I am going to pose is the following, so let us say I have a polynomial of this form $a_0 s^n + a_1 s^{n-1} + \dots + a_n = 0$ all the way up to an equals 0 and then clearly the roots of this polynomial are going to be let us say or we can plot the roots of this polynomial in a complex plane, so this is imaginary axis and this is a real axis, so the roots are real they will be on the real line then when there is imaginary they will be here and if they

are complex they will be existing in this region of this complex plane, so what our interest is in basically guaranteeing for a polynomial.

So it also let me set context so this polynomial if you are looking at GP of S is NS by DS or it could be some closed loop transfer function which is NS by DS, so we are always looking at the denominator polynomial okay, so the denominator polynomial is what this is and what we are trying to see is if we can guarantee the roots of this never fall back into the right of lane, if I can guarantee that then I know this polynomial will lead to a stable open loop, stable closed loop and so on so that is what we are trying to do but through some mathematical trick we are trying to do this without actually computing the root and the reason why we do not actually compute the root is because some of the coefficients are functions of the control parameters.

So that is a logic that I want you to keep in mind so that you see why we are doing all of this and not as some Routh's test that we learned, so now let us go through procedure and then what I am going to do is I am going to take several examples and run through this procedure so that you understand how this computations are done and once that is done then we will go on to tuning of controllers and looking at the values for KC tau I and tau D based on the ideas that we have here, so the very first step is you should check whether a naught is positive okay.

Now if a naught is not positive and then basically what you do is you multiply the whole equation by minus 1 and make a naught positive, so the first thing is you can always make a naught positive is something you should know and once you have a positive a naught then if even one coefficient in the polynomial is negative then you are guaranteed that there is at least one root in the RHP and the system is unstable, so you automatically immediately know that the system is unstable, so this is the first reason okay so first make a naught positive and then look at all of these coefficients do not worry about a not which we already made positive then a_1 , a_2 , a_3 , a_4 all the way up to a_n .

Even if one of these coefficients become negative then you are guaranteed that there is a pole in the right of plane and your system is unstable, so that is a very interesting (15:59) okay so then you know already there is some problem here. Now if you really want to figure out how many folds in the right of planes then what you do is you do this procedure, so you take the highest power and then put the coefficient corresponding to the highest power in the first row, so you also write this so that you kind of keep track of what is happening and then you leave the one which is corresponding to S minus 1 and then go to S n minus 2.

So basically you skip one term and then keep adding constants here till you are run out of terms okay very simple procedure and will show this through many examples in this lecture, so that you are comfortable with this procedure, so take the first one and then put the corresponding power here and then leave one term go to the next term, leave the next term go to the next term, leave the next term go to the next term and so on and you keep filling it till you run out of terms.

The second row you put s power n minus 1 here okay, so if this is 5th order polynomial this will be a S power 5, S power 4 this is just to show you how many rows will come in, so you are not going to really use this anywhere, you are going to only use these numbers here. Now what you do is you start with the second term in the polynomial and then you skip what was already used in the previous one use alternate that way all the left out coefficients starting from here to here gets filled up in the second row, so the first 2 rows are really directly filled out based on the polynomial under consideration okay.

So once you have that every other row is computed based on this coefficient and we are going to show you a rule to figure out how you can use this construction to identify if there roots in the RHP and how many roots are in the RHP, right so I have to summarise again. The first 2 rows are filled out of just the polynomial that we are looking at currently and for the first row you are going to just show that the highest power S power n we are going to take the first term and then put the coefficient and then skip on term put the coefficient, skip on term put the coefficient and so on and in the second row you are going to write s power n minus 1 and all the left out terms we are going to just put the coefficient here okay.

Now once you do this then you can start filling one row after another anytime you fill another row you basically say S power n minus 2, S power n minus 3 in the reducing order here, so if I want to get b_1 , so the rule is very simple, to get b_1 what I do is I am multiply this and this and subtract the multiple of this and this and divide by a_1 , so which is what is her a_1 , a_2 minus a_3 buy a_1 okay. Now if I want b_2 then what I do is I am multiply $a_1 a_4$ minus $a_1 a_5$ by a_1 okay so that is b_2 so and so on, so you repeat this procedure.

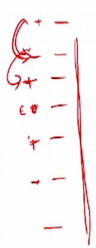
So if b_3 then you multiply this by this, this by this divided by a_1 and so on okay now when you come to the next row you basically follow the same procedure no difference except that for filling out any row you only look at the 2 rows above that row that is all okay, so once you have 2 rows to fill the third row you use row 1 and 2, to fill the 4th row you use row 2 and 3, to fill the 5th row you use row 3 and 4, so recursively you will be able to... Still there is


nothing more to fill and I will show you how that happens, so if I use the same rule over here for c_1 I showed to be $b_1 a_3$ minus $a_1 b_2$ divided by 1 that is what I have. If I want c_2 I should use $b_1 a_5$ minus $b_3 a_1$ by b_1 and so on, so you can keep doing this and since I am going up to S power 0, so if I have an n^{th} order polynomial I will have n plus 1 row here that are filled out, so I hope this procedure is clear. We will look at examples down the line to understand this procedure in terms of the mechanics of these computations.

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Routh test

- Routh stability criterion states that the number of sign changes in the first column of the table corresponds to the number of roots in the RHP
- If there are no sign changes, then the system is stable
- If one of the elements in the first column becomes zero then we replace it with a very small positive number, say ϵ . If there is a sign change about ϵ then the system becomes unstable
- If a row is completely zero, then that row is filled with the derivative of the auxiliary polynomial constructed using the row previous to the row having all zero elements





Now comes the really fascinating part of this table, so what then? So was the Routh stability criterion says is that if I start with a positive number okay and if I have all of the first row as positive number then there are no roots in the RH, so that is an interesting result, so if there are no sign changes is what we say. When you say no sign changes because I start with a naught which is positive, so if everything else remains positive there is no sign change from positive to negative okay.

So if there are no sign changes then there are no roots in the RHP. Now the number of routes in RHP can be shown to be equal to the number of sign changes in this polynomial, so for example the first one is a positive number let us say, the second one is a negative number and everything else then remains negative there is only one sign change, so there is only one root in RHP okay. Now let us say the first number is positive the second number is negative the third number becomes positive and everything else says positive then there is one sign changing here from positive to negative, another sign change from negative to positive, so there are 2 roots in RHP and so on.

So as long as you do not get any zeros in the first column the rule is very simple, the number of roots in RHP equal to the number of sign changes, so noticed something very interesting that mathematics is allowing us to do, what we are saying is we know how many roots are there in the RHP which is like an infinite region, however I can actually never compute at the roots okay, so that is a very interesting idea that you want to remember. Now if one of these goes to 0 then we are going to do some small tricks.

So we are going to use think about this as a very positive number and then we have going to see whether the next one is positive and so on, so that will tell us whether there are roots in RHP and I will show this using an example but when we get zeros then he always have to worry about what happens then, so zeros generally indicate that I have roots on the imaginary axis okay, so whenever we have 0 we might still make judgements about how many roots are in RHP but this indicates that there are already roots in the imaginary axis.

So that is something that we have to remember when let us say the first column has 0 and the reason why we worry about 0 in the first column is because remember in the calculation procedure I showed in the last slide every time a we divide by the first number in the row above, so since we are dividing by the first number in the row above, if that number becomes 0 I have a problem, so that is the reason why 0 is a particular issues here.

Okay now there are cases where whole row could be 0 in which case we need to fill that with something to complete the procedure and I will explain that with examples, so these you can consider as more cases that are usually not the rule I would not really say exception because there are many times where you have poles on imaginary axis but these are considered separately but if these were not there the procedure is very simple you simply say I start with a naught fill the table and I find out how many sign changes are there and the number of roots in RHP is equal to the number of sign changes there I have had.

Now it is very interesting to note here again I would like to repeat this because these are kind of things that makes some of these topics very interesting that we are actually seeing how many roots are there in RHP without actually computing the roots. So I could say there are 2 roots but if you ask me what those are I do not know right, so that is an important thing and that you want to remember.

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Examples

1) Stable systems without oscillations, $s^5 + 8s^4 + 24s^3 + 34s^2 + 23s^1 + 6$

Routh Analysis			
s^5	1	24	23
s^4	8	34	6
s^3	19.75	22.25	0
s^2	24.98	6	0
s^1	17.51	0	0
s^0	6	0	0


Number of sign changes = 0
 Number of RHP Poles = 0
 Hence the system is stable
 Poles = -1, -1, -3, -1, -2.

Handwritten calculations:

$$19.75 = \frac{24 \times 8 - 34 \times 1}{8}$$

$$22.25 = \frac{8 \times 23 - 6 \times 1}{8}$$

$$24.98 = \frac{19.75 \times 34 - 8 \times 22.25}{19.75}$$

$$6 = \frac{19.75 \times 6 - 0 \times 8}{19.75}$$


So let us take some examples then look at how this process is done, so I think I have 3 or 4 examples in this lecture so that we will go through those which will kind of look at all the cases that we have discussed and then we will show you how most of the cases that we have you can handle quite easily, so let us take this first example which is 5th order polynomial so automatically you know that there must be 6 rows, so row for s power 5, s power 4 s cube, s square, s power 1 s power 0 and as I said before the first 2 rows are directly filled in with the numbers given in the polynomial, so the coefficient of S power 5 is one, so I have one then I skip one then I have 24, I skip one I have 23.

Now the next row is s power 4 row, so since I have already taken this I start here I skip one I take this, I skip one I take this I have 8, 34, 6 okay. Now let me show you just these 2 numbers and then here you can actually since this is on the screen you can simply repeat the computations by hand yourself so that you get comfortable with how the stable is? So how do I get this number? This number I get by 24 times 8 minus 34 times 1 divided by 8, so if you do this computation you will get 90.75 so that is how I got this number, so how do we get this number here 22.25, so 22.25 I get by 8 times 23 minus 6 times 1 divided by 8, so I get this number okay.

Now when you come to this number clearly, so when I wanted to do this I took this and this here right, when I wanted to do this I took this and this, so when I want to do this I have to take this and this here but these are not filled and so they are 0, so whatever multiplication you do you will get is 0, so that is how you get a 0 here. So think about this as just for the

sake of standing say this is 0 0, so this will be 8 times 0 minus 1 times 0 by 8 will be 0 okay. So that is how you get is 0 here.

Now that this is filled then you can fill the next row, so if you want to fill this right here so that is going to be 24.98 you will have 19.75 times 34 minus 8 times 22.25 by 19.75 so this is what this will be okay. Now if you want to do this this is easy to see, so for example then you have to use this and this call, so 6 the way I computed as 19.75 times 6 minus 0 times 8 by 19.75 so this and this will get cancel at 6, so you can do this computation and fill each one of these elements right now okay.

Now you are done okay then you look at the first column and then look at how many sign changes are there and you will notice that there are no sign changes, so we can categorically say that there are no roots in the RHP so this system is stable if there is a transfer function which has this polynomial in its denominator, so number of sign changes 0, number of RHP equal 0. Now out of curiosity if you were to go and do the poles for this, this will be the poles on...see all of them are in the left lane.

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2) Stable system with oscillations, $6s^5 + 4s^4 + 6s^3 + 6s^2 + 5s + 2$

Routh Analysis

s^5	6	5	
s^4	4	6	2
s^3	4.5	4.5	0
s^2	2	2	0
s^1	ϵ	0	0
s^0	2	0	0

Number of sign changes = 0

ϵ is a very small positive number

Because a row with only one element vanished to zero, there exists a pair of purely imaginary poles.


Poles: $1i, -1i, -2, -1, -1$

Handwritten calculations:

$$4.5 = \frac{4 \times 6 - 1 \times 6}{4} = 4.5$$

$$\frac{2 \times 6 - 6 \times 0}{\epsilon} = 4.5$$

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So let us take the next example that we have so here again I have 5th order polynomial $s^5 + 4s^4 + 6s^3 + 6s^2 + 5s + 2$. Again the first I put here and the coefficient is one skip one 6 skip one 5 and then you do the second row s^4 first coefficient is 4 skip one 6 (())(27:22) you have this and when basically just let me show you 1 computation again just for completeness, so this 4.5 is going to be 4 times 6 minus 1 times 6

by 4 okay so 4 time 6 is 24, 24 minus 6 is 18, 18 by 4, 18 by 4 is going to be 4.5 that is how you got this okay, so you do this here.

Now when you come to this part here now you do the same formula which is 2 times 4.5 minus 2 times 4.5 by 2, so that is 9 minus 9 by 20 okay so this will turn out to be 0, the problem with the 0 is that when I come here if I leave it as 0 then I will have 2 times 0 minus 2 times 0 by 0, so 0 by 0 form will not make any sense mathematically as far as this competition is concerned, so what we are going to do is we are going to assume that this is not 0 the fact that it has become 0 tells us something but at least as far as just saying purely on the right of plane we can make this as Epsilon which is a very small positive number and then we compute this number here.

So to compute this number what happens then as you have 2 times Epsilon minus 2 times 0 divided by epsilon the same procedure that we have been using, so this will give me 2. So now when I look at this first column here I will notice that there are no sign changes because I am assuming this Epsilon is a small positive number, so there are no roots in the RHP, however the fact that this has become 0 tells me that there will be roots on the imaginary axis and it will turn out that there are roots on the imaginary axis and all the other roots are in LHP so nothing in RHP so that is another example.

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Oscillatory system : $s^6 + 14s^4 + 49s^2 + 36$

Routh Analysis

s^6	1	14	49	36	✓
s^5	6	56	98	0	(0)
s^4	4.67	32.67	36	0	
s^3	14	51.7	0	0	
s^2	15.43	36	0	0	
s^1	19	0	0	0	
s^0	36	0	0	0	

Since the elements in the row corresponding to s^1 are all zero, an auxiliary polynomial $A(s)$ is derived based on the s^2 row.

$A(s) = s^2 + 14s^2 + 49s^2 + 36$

s^4 row is filled with numbers using the derivative of $A(s)$

Number of sign changes is 0.

Number of RHP poles = 0.

Poles: $2i, -2i, -1i, 1i, 3i, -3i$

Handwritten notes:
 $s^6 + 14s^4 + 49s^2 + 36$
 $6s^5 + 56s^3 + 98s$

Now the next example I am going to show you is what is called the use of Oscillatory polynomial, so here let us take (())(29:16) polynomial which is of the form s power 6 plus 14 s power 4 plus 49 s square plus 36, so I am going to do the same thing so s power 6 I am

going to take the first term skip the second term which anyway is not there s power 5, so I will take 14 here and then skip s cube which is not there anyway and then I get the square term and s power 4 one is not there then I get 36.

So basically to think of this as s power 6 plus 0 times s power 5 plus 14 times s power 4 plus 0 times s cubes plus 49 times s square plus 0 times s plus 36 and then use the same procedure that I am going to take one skip one 14 skip one 49 skip one 36 that is what I have but unfortunately I need 2 rows to be filled up if I do not do anything basically I will have 0, 0, 0, 0 here right, so these are the only things that I have, so the way this is taken care of in this Routh table again this I am going to tell you like a rule like a heuristic but there is sound mathematical reason why all of this is done.

Somehow I am going to fill this up already I know this rows all 0 right and I have already said when you have row 0 right and I have already said when you have row 0 then...there are roots on the imaginary axis, right but nevertheless for us to finish this process I have to get something going here, so the way we are going to do this is we are going to take this polynomial s power 6 plus 14 s power 4 plus 49 s square plus 36 okay, so we know differentiate once I will get s power 5 as cube and s terms.

So we differentiate once and then this will become 6 s power 5, so this is just a procedure I am not telling you why you should do this but I am telling you by differentiating this you will get these terms which were 0 so that they can actually fill the doubt in the table and at the reason why it works beyond the scope of this class but this is a procedure that you can follow. Now this is 14 times 4 is 56 s cube plus 98 s plus 0 okay, so if I were to put these terms here then instead of 0 which was the S power 5 term I put 6 here and instead of 0 which was s cube term now I have 56 here and 0 instead of this term I have 98 and the last term is 0 because I have run out of terms here.

Once I have this then basically follow the same pictures whatever I have shown you and then I look at the first column and I see that the first column has no sign changes, so there are no roots in the RHP okay. However because of the 0 I know that there are roots in the imaginary axis and in fact it turns out the roots are $2y$ minus $2y$ I minus $3i$ minus $3i$, so you see all the roots are in the imaginary axis none of them are on RHP which is again guaranteed using the Routh stability table.

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Unstable system: $s^5 + 2s^4 + 2s^3 + 12s^2 + 21s + 10$

Routh Analysis

s^5	1	2	21
s^4	2	12	10
s^3	-4	16	0
s^2	20	10	0
s^1	18	0	0
s^0	10	0	0

Number of sign changes-2.

So there are two RHP poles.

Poles: -1, -2, 1+2i, 1-2i, -1.

Handwritten calculations:

$$\frac{2 \times 2 - 12 \times 1}{2} = \frac{4 - 12}{2} = -\frac{8}{2} = -4$$

Lecture 16: P... ssime

So let us look at one last example to understand this and then we are basically done with this portion of you know understanding the stability of closed loop system. So now I have simple 5th order polynomial we are going to do same thing the first one s power 5 1, 2, 21 and a corresponding s power 4 I have 2, 12 and 10 then I do the same procedure, so let us just do this one term here, so this is going to be 2 times 2 minus 12 times 1 divided by 2, so this is going to be 4 minus 12 by 2 this is going to be minus 8 by 2 equal to minus 4, so that is the number here.

So if you do this computation the same way that we have done the next number will become 20, 18, 10 and so on. Now if you ask the question as to how many roots are in RHP? You would say that there are 2 roots in RHP because there is one sign change here from positive to negative and then another sign change here from negative to positive, so if you look at this right now this is a root in the LHP this is also a root in the LHP, however this is a root in the RHP, this is a root in RHP, this is a root in the LHP.

So this has 2 roots in RHP and 3 roots in LHP which we can actually find out by just looking at this because there are no zeros here there are 2 sign changes that means there are 2 roots in RHP and since it is a 5th order polynomial the remaining 3 roots are in LHP but look at how remarkable this is. We do not know what the roots are this is actually I am showing you the roots or the poles after computing but if you just stop here okay this is...you did not know this that these are the roots you would still simply be able to say look there are 2 roots in RHP and 3 roots in LHP I do not know what those are but this is the 2 situation and that is actually correct, so that is quite an interesting idea when it comes to.

So basically I introduced this notion of Routh stability tester, Routh table because I think this is an idea which goes beyond partial fraction in the sense that for us to do partial fraction we have to be able to find the polynomial in a root resolve form and follow through and once you have it in a root resolve form then it is basically a question of simply looking at the roots and then seeing whether they are in LHP or RHP figure out stability, however I try to explain to you the idea that when you have this closed loop systems and you have this controller parameter you do not know the values of these controller parameters.

So we are going to make judgements about the stability of closed loop system keeping the controller parameter as they are without assigning them values and that becomes complicated because higher order polynomial you cannot write analytical forms for the roots, so we need to find some other way and this Routh test is remarkable idea which allows us to actually figures out how many poles are there or roots are there in LHP and RHP without actually computing the poles, so that is what I showed in this lecture. In the next lecture I am going to show you how we are going to use this in tuning controllers and so on so I will start with what are the different ways in which we can think about tuning controllers and how all of this false together so I will see you in the next lecture, thank you.