Process Control: Analysis, Design and Assessment
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Lecture 15: P, PI and PID Controllers

We will continue with our lectures in this course on process control in the last lecture we talked about how to analyze closed loop systems we basically showed that once you understand the structure of the transfer function and how do derive these closed loop transfer functions then analysis of closed loop systems is not very different from analysis of open loop systems, we showed that in a typical open loop system the output will be a function of the input and disturbance variables and there will be transfer function corresponding to the process disturbance once we close the loop we showed that the input itself becomes a function of the error and as a result the input or the manipulated variable is not an input the closed loop block diagram and that would be replaced by Y set point, and the disturbance variable will still be an input so the output will be a function both Y set point and the disturbance variable.

So this is what we had seen in the last class and then we also describe what are ideal transition function from viewpoint of servo control and viewpoint of disturbance rejection control we showed that if the transfer function for this set point effect on Y is very close to one then the output will follow the set point very closely and if the transfer function which models effect of the disturbance of the output is close to zero then the output will be largely immune to the effect of disturbances and then we also saw that these are not objectives which are counter to each other and we showed that if we take very large control actions then you could have both the transfer function with respect to the set point going to one and the transfer function with respect disturbances is going to close to zero

So now that we have looked all of this what we are going to do in this lecture is actually see how this works we are going to take very simple process transfer function and then we are going to take these controllers as PI and then look at how this transfer function look and what are general principles that we can learn from this exercise, so you might want to think of this lecturer as going to more details where we put in more structure to the controller transfer function and then see what effect it has in terms of performance matrix that one would be interested.

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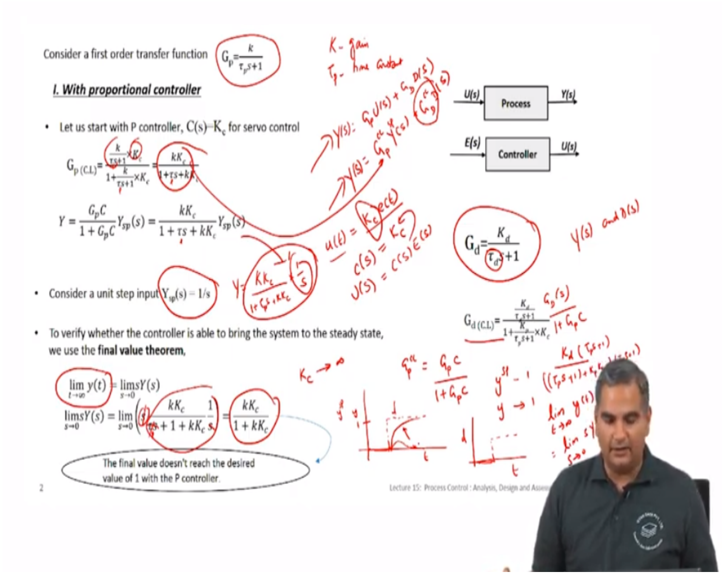

So let us start with very-very simple first order transfer function is something that we had seen before though is of the form K or tau P, S plus one K is the gain of the transfer function tau P is the time constant of this transfer function now we are going to we are going to assume that this a process and then we are going to see what happens if I control this process with the proportional controller and then can we see some performance related computations for proportional controllers and see how well they behave when we do this we will also cut disturbance transfer function that is also first order which has different gain KD and which is a different time constant tau D X plus one but nonetheless the form is the same so remember that basically what we are saying is that this way of S is GPU of S plus GD D of S.

So this is something that we have seen several times now when we close the loop be said this way of S will have a process related close loop Y set point of S set point as input and we are going to have a closed loop disturbance transfer function which is going to multiply D of S so this is open loop and this is close loop and we already derived how this transfer functions are going to look we w derive what the transfer function will be in terms of GP closed loop and the transfer function for GD close loop.

So let us start with GP closed loop for controller which is a proportional controller remember from the before we said U of T is some KC times EP is the equation for a proportional controller were we said the effort that we take in controlling the process or the manipulated variable value that we keep is directly proportional to the error that is seen UT is KCET and this KC is the tuning parameter of a proportional controller and the structure of the proportional controller then simply becomes C is equal to KC because if we do plus of this I get U of S is C of S, E of S and C of S is KC so remember the closed loop transfer function form for GP we said is GPC divided by one plus GPC, so we substitute this here so that process transfer function is K over tau P and tau P here so K over tau PS plus one times KC GPC so this is the GP this a controller divided by one plus GPC, now if you simply multiplies this denominator by tau PS plus one on both side you will get one plus tau PS plus KKC and in the numerator you have KKC,.

Now this is the transfer function with respect to this here which is what we saw from the previous class, so if there are no disturbances and you are only looking at servo control then why is GPC buy one plus GPC Y at point S, so that is going to be KKC divided by one plus tau PS plus KKC, now we have derive this now what you want to do is we want to see how the controller behaves and how the output behaves whenever there is a change in the set point, let us for example assume that I give a step in this set point, so basically the pictures that you want to keep in your mind is the following so I have Y as a function of time and let us also say I have Y set point as a function of time so initially all our deviation variable so Y is like this Y set point that is also like this let us say at some time I give a steps in my set point so I increase the unit to one.

So I say basically the set point suddenly changes so at this point there is an error between Y and Y set point and because of this the control action will start happening so the Y starts moving so this Y movement is as a result of this Y set point input so if I am going to give a step input in Y set point then from before we know that the Laplace transform of that one over S so basically what I will get is this equation Y is KKC divided by one plus tau PS plus KKC times one by S this one by S comes because I have given step input to the set point now ideally what you would like to do is have the controller set in such a way that after a while clearly it cannot happen right away but after a while this output reaches the set point so in other words ultimately I want the output to reach a value of one.

So Y set point is always one because I have given step and then from then on it is remaining to be one initially Y will not be one but as the controller starts working after a while you want this way to go to one, so the final value of Y should be one, so what we would like to do we would like to check if the final value of Y really goes to one or not and from what we have learnt from before we can use what is call the final value theorem which say if you want to find the final value of Y of T tends to infinity Y of T then that is given by limit S tends to zero SY of S and we already have the way of us here so what we are going to do is we are going to basically used this limit S tending to zero SY of SS limit S tending to zero here is this and this whole thing is way of S from here we are just stop be here so what happened now is that this S and S get cancelled then when you set S to zero this term goes to zero then you have this limit as KKC divided by one plus KKC.

Now know you notice something very interesting here you see that as T tends to infinity ideally you want Y to go to one because I have given a step of one and if it goes to one would say the controller is working very well however you see that this can never be one it can approach one but it can never be one and as we say before for it to be one essentially what you need to do is you to keep pushing KC toward infinity a larger and larger value that KC takes the closer and closer this will become to one so if you want very close to one you do not want any error then basically that means KC he has to be very-very large.

So this illustrates what people call as offset in proportional controller which says that whatever you do the proportional controller there will always be a minor offset between the true value that you want the output to take and the actual value it takes and the offset can be small or large depending on the values of K and KC but for sure you know whatever be the value of K which is the natural gain of the process as long as you make KC larger and larger this offset will keep coming down but the problem with making KC larger and larger as I mentioned in the last class is that you will take unnecessarily large control moves because for a small error remember this UT is KC times ET for a small error if KC is very large your UT is very large, so you do not want to do it so it basically says if KT big KC become very-very large even if you have a minute error you UT will simply shoot up which first is not a good idea for control and number two it might not be fizzy possible.

So imagine that in our regular life we use taps to get water and you can open the tab fully and there is a maximum limit of flow that you can get you cannot get larger than that, so it is simply not physically feasible to get very-very large values with standard control equipment so that is a problem so basically what we need to do we need to basically live with this offset and there are several process where this it is really not a problem because if you are let us say maintaining the level of thank you know water in the tank you do not really want the level to be exactly mean it really does not matter as long as it is in the ballpark it might be good enough so there are several systems there the proportional controller is very useful because it is very simple tinning is simple the structure is simple just one constant and it does not arbitrarily introduce oscillations and so on so there are several interesting useful properties of proportional control ease of implementation simplicity does not introduce unnecessary oscillations and so on.

However the negative offset so you will see in real plants several loops which are noncritical but where you need simple control you will see proportional controller so the advantages are basically rooted in it is simplicity and ease of implementation and the disadvantages is basically in the offset now let us see what happens for the disturbance transfer function, let us say if I had DFS remember we said loop D times D of S so the closed loop transfer function for D basically will be we said GD of S dividend by the denominators in the same one plus GPC so if you can you this and then you simplify this you will get the following KD divided by tau P plus, so KD now you will have tau P S plus one in the numerator because in the other case tau PS plus one tau PS plus got cancelled in this case because tau P and tau D can be different you will have this in the numerator and this will be tau P S plus one plus KP KC so this will be here inside times tau D S plus one.

So this is what you will have in terms of the transfer function between Y of S and DFS know if you do the same thing and then say I am going to give step in my disturbance and then I want to see what happens so basically here this is a plot between Y set point and Y but for a disturbance let us say if you have a different plot let say disturbance so till here I have zero disturbance and suddenly if I have disturbance in this case you wanted to do follow it but in this case what you would want Y to do is stay flat but in real situation it would not stay flat so what you want to do whenever a disturbance hits you want Y maybe it will have to increase but at least it has to come back to zero because we want to reject the disturbance so remember these two pictures are very important when there is set point step we want Y to follow the set point but when there is a step in the disturbance we want Y to stay flat.

So we want it to be zero always but there will be the minor thing so ultimately if you do the same thing and then and try and see what happens to limit T tends to infinity Y of T when there is a disturbance which will be equal to limit S tending to zero SY of S you will notice again that would not go to zero, which is ideal right so which is what we want it go to zero it will be close to zero but it would not go to zero and again you will see that as you keep increasing the KC value the controller again you will see that you will go closer and closer to in this case so this I am going to leave it exercise for you to finish it is very simple algebra that you do very similar to this and then see what happens.

So the upshot of all of this is proportional control is very good and we have started analyzing the impact of proportional control on processes based on whatever we have learnt we have only use this two transfer function that we derived in the class so the trans function between Y and Y of S is SPC by divided by one plus GPC the transfer function between Y of S and D of S is GD divided by one plus GPC, now once your GPG D and C is K you can see there will be an offset in my zero tracking that is my output would not go to one ultimately which is what I want and there will be an offset in my disturbance rejection also this is case where I want white be zero as T tends to infinity but that would not happened either nonetheless because of it is case and simplicity proportional controller is quiet heavily used in the industry.

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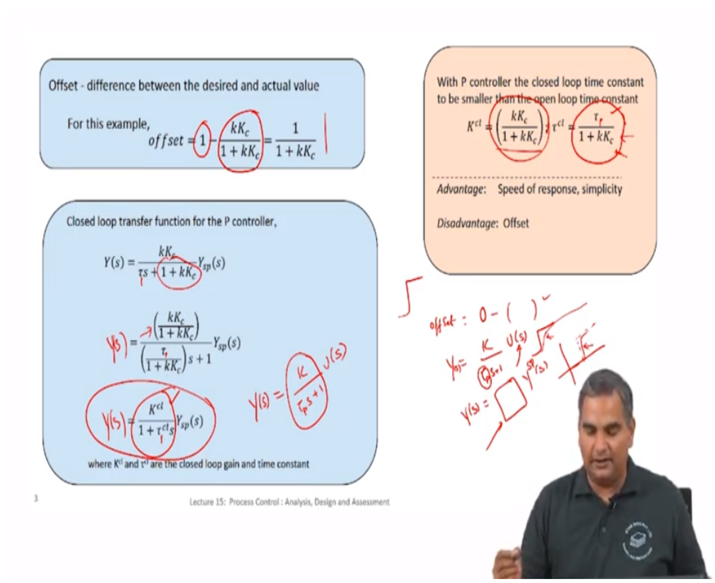

So the offset you might define as the difference between the desired and actual value so for that example that we had offset because I give step of one the ultimate desired value of one but the value that output takes T tend to infinity Y of T is KKC by one plus KKC, so this if you do some simple algebra you will see that this is one by one plus KKC, so this is offset from servo control viewpoint of course you could also do the same offset for the disturbance let us say if I gave a disturbance step and then I tried to find what the offset is so in that case you have to say offset equal to what is the desired value of output the desired value for output is zero it is one in the case because it is a set point change and I want the Y to follow the set point, in the other case.

I have disturbance step and I do not want the output to follow the disturbance I want output to stay where it is so that is zero, but whatever value it takes so that difference if you take as offset this will be negative number but you can basically say this is the different between what is desired and what I get so you can find this as this minus zero if you want so that is another important definition that people generally used so it would be worthwhile for you to do this competition than see what happens now while we talked about this performance in terms of final value right.

So whenever I give a step change in set point I want the final value to be one whenever there are disturbance I still want the final value to be zero and so on so these are kind of metrics, which are static or steady state matrix, but what we also want to know is dynamically what does the control do it is not only static metric that we are interested in from the dynamics view point, what is the controller doing is something that we might be interested in answering, so what we are going to do is we are going to look at this transfer function that we derive between Y and Y set point and then see whether we can make some judgments about what is happening dynamically there are reasons for control couple of them are servo and regulatory and there is a dynamic performance measure also that we talk about whenever we talk about controllers which is the following.

If I let us say have a certain time constant for my open loop process which basically says if I want to change my Y value I change my input U in an open loop and then I wait for my Y to get to its final value which depends on the time constant of the process so remember we have this equation which is K over tau PS plus one U of S, so if I change this let us say I give a step to this then this Y will follow and get to some value, but it is going to take it is time and the time it is going to take is the time constants so time constant so this is what is called an open loop and now when we close the loop really there is going to be a transfer function between Y of S and Y set point of S, remember I said the disturbance transfer function will be modified but still the disturbance will be an input whereas the input transfer function modified but the input will also be modified from U of S to Y set point of S.

So now if I have this transfer function now here what I want is let say if I have this Y set point and I suddenly change this I do not want Y to take a long time to go to the set point so I do not want it to be like a open loop time constant I want Y to very quickly go to the set point in P controller it will go very close to the set point but still I want this to be fast so when I look at this so this what is called an open loop time constant and this is what is called closed loop time constant this is standard terminology that is used but I just want you to remember that the inputs are different so when we talk about open loop time constant we are talking about you and when we talk about a close loop time constant we are talking about Y set point.

So that something to keep in mind now if this is open loop time constant typically by doing control we saw that you can get steady state performance which is how closes am I to my final set point value and how much I do not deviate from original value in the presence of disturbance and so on but if you want a transient or a dynamic performance metric then I might say look I do not want the closed loop time constant to be of the same order of the open loop time constant because open loop time constant I have no control over it is what the design of the process gives me but while i talk about a closed loop time constant and basically I have had controlled over this so because I am designing a controller can I design my controller in such a way that is closed loop time constant is very small that means whenever there is a step change it happens automatically.

So analogy that I would give you is remember the bike example we talked about let us say you are going at 30 kilometers you want increase to 50 kilometers however let say you know what the throttle position is for 50 kilometer per hour so you simply go to that position and wait right now if you just go to that position and wait your bike is going to take a certain amount of time before it gets to 50 kilometer per hour so it is going to be not instantaneous it is going to take it is own time that is what we typically call as open loop time constant as supposed to imagine a case where you are simply punching in speeds for the bike and you are really not doing throttling there is a controller which is doing throttling which is basically you are only giving the set point and you are not manipulating some controller is manipulating.

So if that happens what you want if you go and press50 as a set point you would want the bike to get to 50 as quickly as possible it cannot be the same rate at which it went to 50 when you simply got to the throttle position waited, so that is the kind of idea we are talking about let us see whether a proportional controller is able to do that so from the previous line again I am going to use this tau which have here now from if you notice from the previous slide this is a transfer function KKC divided by tau PS plus one plus KKC is Y set point of S,.

Now I am going to do some very simple algebraic manipulation I am going to take this one plus KKC outside here so the denominator will be tau P divided by one plus KKC time S plus one all I am doing is I am taking this out so tau PS by this plus one will be there and since I am taking it out it goes to the numerator, so why yes is this now this is a constant let me call it as a closed loop time constant and now if I call this as a closed loop gain and now if I call this as a closed loop time constant than I will have this equation as one plus tau CL close loop S.

Now notice something interesting here so in the open loop the transfer function was Y of S equal to K by tau PS plus one U of S and in the closed loop it is Y of S is K closed loop just closed loop here and this is tau P closed loop S plus one Y S PF S, so this is also a first order transfer function except that the gains have changed and the time constants have changed and if you look at the time constant in closed loop if you look at this equation this is an open loop time constant divided by this number here, now let us go back and think about the discussion we had we said that we want the closed loop time constants to be much smaller than the open loop time constant which is what you see here if you keep increasing K see more and more than tau closed loop will be much less than tau process.

So the closed loop time constant is going to be much less than the open loop time constant which is something that is something that we desire and we see that D the controller is basically able to get us that and from a gain perspective for this closed loop transfer function you want the gain to be one because Y of us has to be equal to Y set point of S after a while but we notice that while the gain goes closer and closer to one but it is not exactly one that is where the offset comes in but you can also see that from this first order forms here quite easily that I have gain which is closed to one but not really one so the upshot of all of this is the P controller works nicely in terms of reducing the closed loop time constant making closed loop time consume much less than the open loop time constant it is also getting the gain closed to one but not exactly and that is the reason why you have an offset so again the advantageous speed of response simplicity however the disadvantages is an offset.

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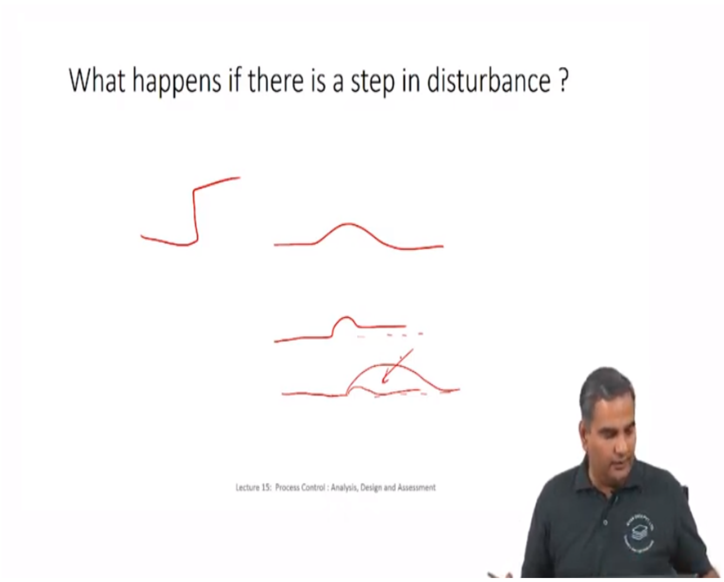

So what happens if there is step disturbance let us quickly summarize so if I have a step disturbance here then I want my output to do this but is a P controller what will happen is the output might do this, so that will always be an offset between what did original was and what happened to this and similarly the time constant part of the disturbances the output could do something like this or it could do something like this, so the offset cannot be got rid of but how quickly it comes back to the close to it is zero value will be given by a corresponding and time constant for the disturbance transfer function and close loop, so which is something that you can also work out for service, so that she get comfortable with this material.

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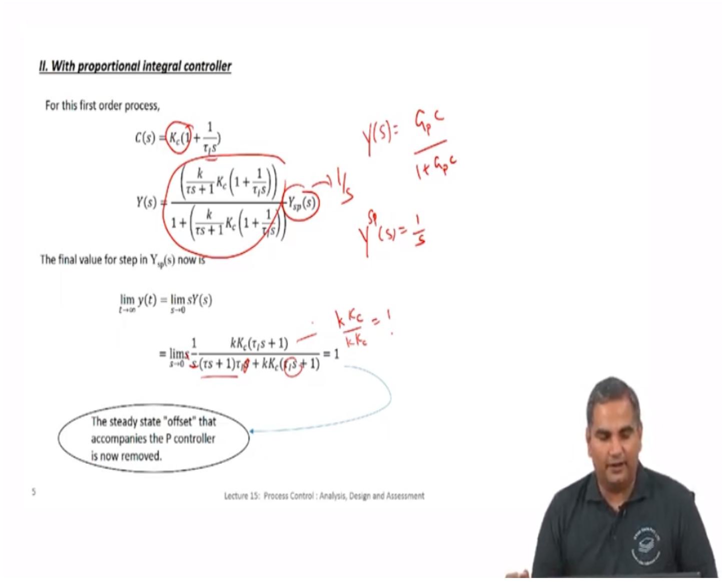

Now let repeat the same exercise for proportional integral controller and the reason why you use this proportional integral controller is to get rid of one disadvantage of the proportional controller which is the fact that a proportional controller has an offset so in this case what we are going to try and do is see weather introducing and integration action removes this upset, so let us see what happens so the procedure is the same there is nothing that is complicated we simply follow the same equations except that we are going to use the correct equation for a controller when we use a proportional integral controller.

So the proportional controller had only one term KC in previous lecture at shown you that if I have a proportional integral controller I am going to do integration of E of DT, which basically give me S over so I have this extra term and we put it in this KCE times one plus one over tau S form, now this basically is the same I am going to do Y S equal to GPC divided by one plus GPC except that the C is KC when it is a proportional controller and when it is a proportional integral controller it is KC times one plus one over tau S.

So I substitute all of this and then I will simplify this and then now again what I am going to do is I am going to see what happens when I have set point change which is a step so I am going to again do this Y set point S equal to one over S that will give me this times this will be one over S as Y of S and then now that I have a proportional integral controller, I want to see what happens to the offset so it does not really matter in terms of offset so what I am going to do is I am going to use the final value theorem again which is limit T tends to infinity Y is limit S tends to zero SY of S when I multiply this term by S this S and S will get cancelled then what will remain is the simplification so this simplification will turn out to be this so this S and S will get cancelled from what I shown you now let us substitute S equal to zero.

So when you substitute S equal to zero the top term will just be one and when you substitute S equal to zero here this term will vanish because S is zero and this term inside this is S is zero is going to vanish, so this is going to be since this is one in the numerator I have left with KKC in the denominator this is zero this is zero so I am only left with KKC, so surprisingly or interestingly I get one so what this basically says is I gave a step input in set point and then the output also takes final value of one that means the offset that was there in the P controller got removed in a PA control.

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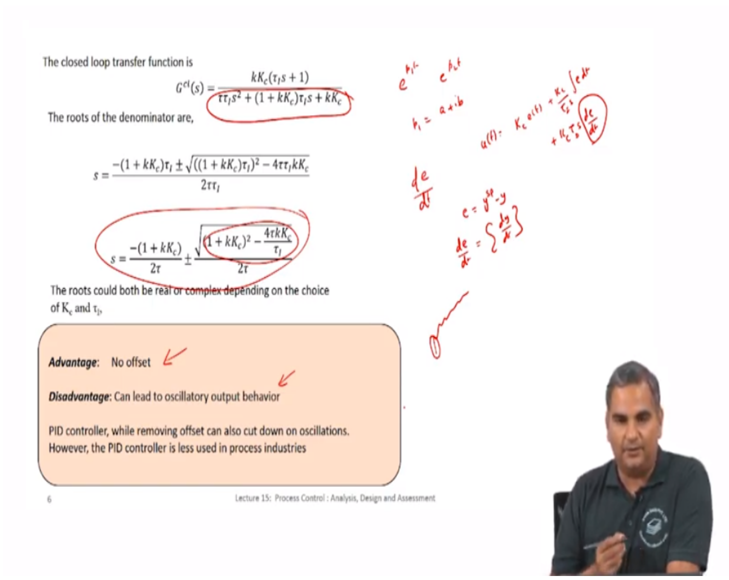

So this is one of the most important reasons for using P controller now this mathematics is very easy to understand there is nothing other than simple algebra to see why PA controller remove offset but it takes a little bit effort to really understand in a time domain, why there would be an offset for P controller and no offset for PA controller so I am not going to explain that here but I would leave that as an exercise for you to really think about why from a time domain view point have an offset when you have P controller but not so when you have PA control.

So this is advantage of PA controller that it gets rid of the offset however if you look closed loop transfer function for let say this example you will see that the denominator now for the closed loop transfer becomes a quadratic equation and you can write a solution for a quadratic equation and depending on the values of tau K, KC tau Y and so on that we choose it could happen that the term inside the root become negative in which case we get complex roots for this closed loop transfer function and remember the minute we have complex roots for this transfer function which we has seen before then you could have options or you will have oscillations because when we do this Laplace inversion we saw that you will get this term C power P1TE power P2P and if P1 is a plus IB then I will get the cause BT and sine T term.

So which is why you will have oscillations that come in so basically while P controller removes the offset the disadvantages is it can lead to oscillatory output behavior but it is not that the oscillatory behavior is always going to result by careful tuning and careful choice of the parameters of the integral action you will always any case minimize or get rid offset but by carefully choosing the tuning parameters you can reduce the oscillatory behavior to a large extent nonetheless you just want to remember that this can lead to more oscillatory behavior then and then the P controller.

Now it is also important to understand that it is not necessary that P controller will never introduce oscillation that is not what we are claiming here P controllers can also introduce oscillations the way you have to figure out whether oscillations will be there or not is very simple something that we have already discussed before in a previous lecture all you need to do is really look at the roots of the denominator polynomial of the closed loop transfer function and if the roots are real then you are not going to have oscillator behavior but the roots are complex then you will have oscillator behavior and roots can become complex even with P controller if your transfer function is of higher order so in the previous example that she had we did not see that effect because we took a first order transfer function and then we implement a P controller supposing I have I were to have taken a higher order transfer function I can see that even P controllers introduced oscillation nonetheless there is more oscillatory behavior in general in layman terms that is introduced when you have integral action.

Now we talked about P and PI the third type of controller PID controller now most often you will see in industrial processes either P or PA controller being used for system where you are looking at getting a response which is very fast ease of implemented of a controller and you really do not care too much about offset you would implement P controller but in cases where you cannot tolerate an offset if it is related to quality let us say you want to produce certain product of certain level of purity in that case you might not be able to afford offset in quality parameters so those cases you would really ensure that you introduce a PA controller and you will tune in such a way that the oscillations are minimum one controller which retains the removal of offset and makes a process closed loop behavior faster and removes or cuts down on oscillation is the PID controller so if you add another derivative action you seem to get no offset oscillations cut down and so on.

However PID controllers are not implemented too much in process industries at all the reason is that so whenever you take at the derivative which is D by DT which is what the portion of the controller required remember we said U of T is KC ET and KC over tau S integral EDT and KC tau DS DDT so this is a form so for the derivative action you need DDT and you know E is basically Y set point minus Y so DDT basically also requires you to calculate DY DT now DY DT is basically computed by measurement and typically whenever you have a sensor in a process industry the measurements are likely to be noisy.

So if you have some noisy measurement like this so in these case D by DT might kind of create all kind of robustness problem so that is part of the reason why PID controller are not use too much in process industry you might argue that you know one could do filtering and remove the noise and still use D action but if you are filtering then basically you are anyway slowing down the rate at which each changes so whenever you are filtering you kind of smoothing the curve so you are losing some aspects of the speed, so unless there is a very special need very specific process where you need PID controller typically industrial processes either P or PA.

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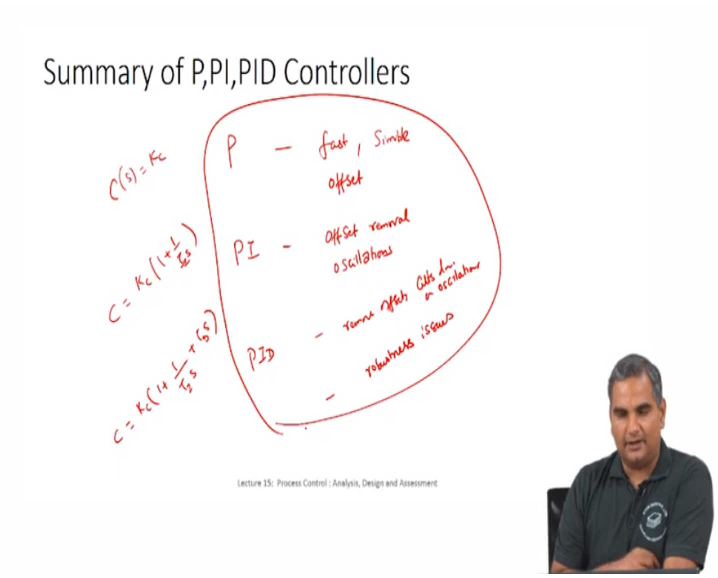

So P controller summary it is fast acting controller simple but the disadvantage is offset PA it is offset removal is one of the big advantages of PA   controller however the disadvantages is that introduces oscillation PID controller seems to remove offset cuts down on oscillation however it is not very regularly used in industrial control because of it is robustness issues robustness issues means whenever you have measurement noise all the differentiation can create problems for the overall control.

So this the summary of the three types of controllers and in this lecture I showed some computation with first order transfer function the procedure is just the same I said there is no difference at all you can take a second order transfer function and then play around with PPI PID controller computer calculations the idea is the same the transfer function for Y set point is DPC divided by one plus DPC the transfer function for the disturbance is GD divided by one plus GPC and once you have GP and G you can choose any controller form if choose P controller then C of S is simply KC if you choose PA this is KC one plus one over tau Y and if you choose PID so this is KC one plus one over tau Y plus tau DS.

So all of these are transfer function form you just put them into those equation and then you do your basic numerator by denominator polynomial partial fraction inversion you can analyze all kinds of controllers and this is an important set of idea when someone talks about these three types of controllers so we end this lecture this is the next lecture we will talk about t how to analyze the stability of closed loop systems the way I have been teaching is I have talked about how you analyze open loop system using the partial fraction idea and then I talked about stability of open loop system by basically saying all you need to do is look at the roots of the denominator and then see what happens and if every root is in the left half plane then we said the open loop system is stable and then I showed you that when you go to closed loop and you want to understand the performance of loop systems you do not have to do anything and differently at all.

All you need to do is derive the corresponding transfer function and then all the open loop idea are directly applicable to these and from a stability view point again it is going to be just the same because once you get this closed loop transfer function again you are going to look at the denominator polynomial and then see whether they are in the right half plane or left half plane and then you are going to analyze the stability of close loop system so from a stability of viewpoint the idea are the same only thing is the transfer function you are looking at is different and the key connection is we have shown you how to derive the transfer function for closed loop from open loop transfer function and the controller transfer function.

So once you have this logical third process then analyzing stability of closed loop transfer function or closed loop systems become very simple so will pick up on this idea in the next lecture and then show you some interesting concepts in terms of stability of closed loop system I will see you in the next lecture thanks.