

Process Control - Design, Analysis and Assessment
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Controllers and analysis of closed loop transfer functions

We will continue with the 14th lecture of this course. Now that we have seen about transfer functions and how to generate these transfer functions and convert time domain functions into Laplace domain and Laplace domain to time domain functions and so on. We are ready to start thinking about controllers and so on. Till now all the transfer function models that we have developed are either for processes which we model using ordinary differential equations or some input profiles which we assume that the input undergoes and then we convert those input profiles in time domain to Laplace domain.

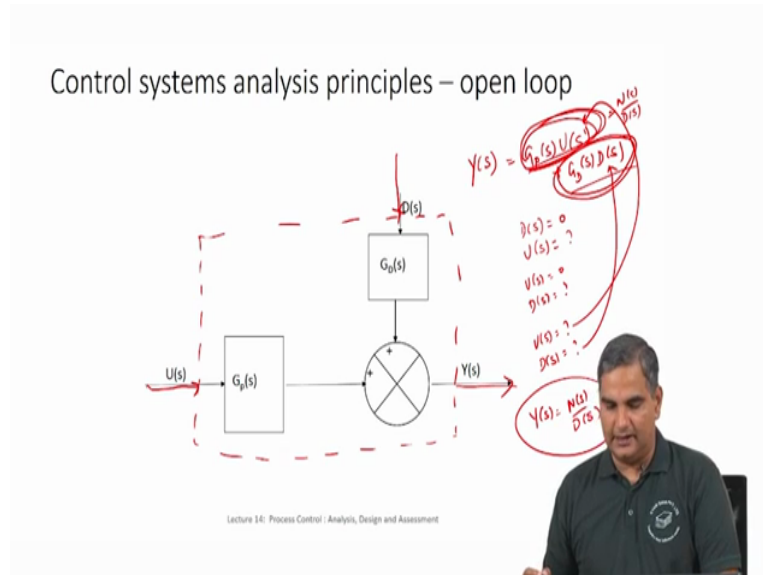
However what we are going to do is we are going to integrate all of these elements, the process, the inputs to the process, the controller and so on in Laplace domain. So we are going to design the controllers in Laplace domain and then once we have all of these in Laplace domain I told you before we do not have to solve differential equation. We are going to see that we will algebraic manipulations and then we will be able to get transfer functions between the output and the inputs of interest.

You would see that the input of interest in open loop is slightly different from the inputs of interest in closed loop. Nonetheless the analysis technique pretty much remains the same. So we are going to just do the same, GS equal to NS over DS and then basically use the same Laplace transformation ideas that we have been seeing till now in this course. So the nice thing about all of this is that once you understand how to get these transfer functions, closed loop transfer functions using algebraic manipulations, then analysis of these closed loop systems it does not need any other new concept.

Whatever concept we have learned in terms of inversion using partial fractions and the initial value theorem and final value theorem are enough for us to be able to understand these closed loop systems. So in that sense they really know difference between closed loop and open loop except that you have to get the correct transfer function to analyse. And the analysis procedure largely remains the same. In the last class we also saw that there are three different types of controllers that most process industries traditionally use. These are the proportional

integral and proportional integral derivative controller. So we are going to look at how these are integrated into the closed loop transfer.

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So let us start and then quickly go through the analysis principles that we have been talking about. So till now as it stands, we know that if I have an output Y that is going to be a transfer function times U of S and the transfer function times D of S. And this we said comes about because of linearity, the effect of the input and the effect of disturbance can be linearly superposed. So that is the reason why you have these two terms. And the way this term looks is out of this fact that when you have this linear systems, the effect of U on Y would be a convolution integral between a particular function and the input that convolution integral turns out to be a multiplication in Laplace domain.

And similarly as far as the disturbance is concerned that will also be a convolution integral. So from a time domain perspective you will have one term a convolution integral for U, convolution integral for D, both of them will become the product forms in the Laplace domain. So this is something that we saw. So now if you want to analyse this Y either for just servo kind of problems are or problems where there is really no disturbance but just you are going to vary U, then basically you will say D of S is 0 because there is no disturbance.

U of S is whatever functional form that is, then you will have this and this will turn out to be a numerator by denominator transfer function. So you use the same trick that we have been learning till now to be able to get how the output looks in time domain. If you do not perturb the system but there is a disturbance and you want to see the effect of the disturbance on the

output, then you would say U of S is zero, D of S is something, then basically Y of S is $G D$ of S , D of S .

$G D$ of S will be a transfer function, D of S will be a transfer function, the product will give you a numerator by denominator types and then you can simply compute Y of t . So there is nothing very hard here. Now usually people get confused about what happens if there is both input variation and disturbance. It is the same thing, there is no difference at all. So in this case U of S will be some functional form, D of S will be some functional form, so you put this functional form here, this functional form here.

You know these two transfer functions, so do all the algebraic manipulations which is what I said you will do when you do Laplace transforms and do control calculations in Laplace domain. And then you will again get some Y of S after all of the manipulation is done, some N of S over D of S . Then you do the same trick and get Y of S . So pretty much once you have this form and you understand what this really means, this really means that a physical process which was modelled in time domain has been converted to Laplace domain.

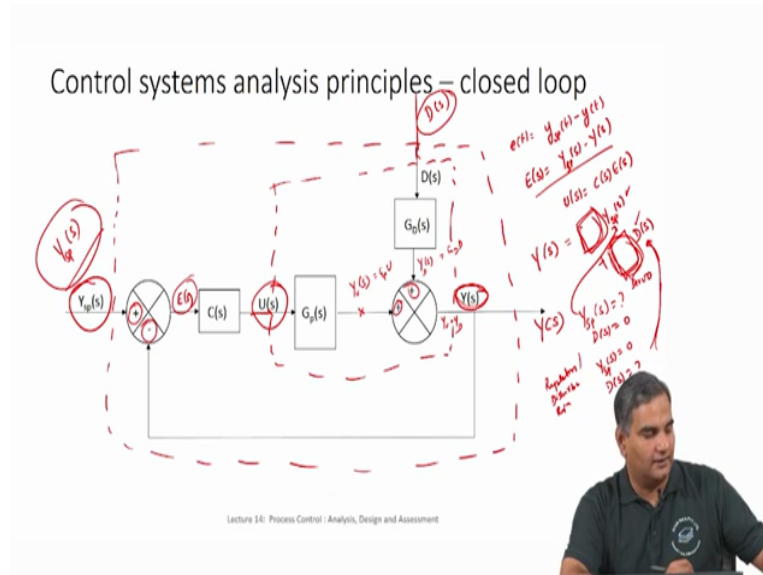
And the convolution integral becoming the product is a very important concept which allows you to do all of these. So you do all these algebraic manipulations and as long as you get this form, you know how to do the inversion for sure. So there is no confusion about how you will do. Now another view, just so that when I show you the next slide, you will see why I am doing this here. If I were to kind of do a box, dotted box of the system, and then say okay, this is my open loop system which is a dotted box.

And then if you look at the inputs that come into the system, there is an input U , there is input D and there is output Y of S . So if I think of this whole thing as an input output block, then basically what we are saying is Y is the outcome or the output from this block and the variables that affect Y are U and D , and those are the inputs to the block. So basically I have to write Y as some function of U of S and D of S . And since it is a linear assumption, linearity assumption that we make, you can say that the effect of U of S and the effect of D of S can be added by superposition and then the effect of U of S is given by this term, effect of D of S is given by this term.

So that is just another view of the same thing so that you understand how we do this open loop and closed loop. Now notice that this is completely open loop, there is no controller here

and keep this block in mind when we see the next slide and then understand what is happening.

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Now here the block is little more complicated. There are more details to the block, so let us first identify the last block that we have so that we kind of build this closed loop control step by step so that you understand exactly what is happening here. So if you look at the block from the previous slide, you will see that is basically this, so there is no change that has happened to this. This is the block that you have. Now if you look this block, you will see U of S is the input to this block, D of S is also an input to this block and Y of S is the output that comes out of this block.

So basically you will say Y of S is G p of S, U of S plus+ G D of S, D of S, that is the reason why you have plus+ and plus+. So how do we introduce this feedback into this open loop system is something that we have already described. So we said the feedback is introduced by computing what is called an error. So this is actually what the output value is and then our interest is really in this output value taking certain which we are going to call as the set point.

So basically error in time domain we said was y set point if t minus- y of t. So how different is the output from the set point? So this could be made into E of S, this is y set point of S minus- y of S, right? So basically the error signal has to be the difference between the time set point value and the actual output and then because of linearity and so on we can simply do the same subtraction in the Laplace domain to get E of S. So this equation is represented by

this block. So what we are doing is we are feeding back this y and then we are sending in y set point of S which is outside, so that is something that we decide.

And then we are comparing this y set point and this y . So y set point minus y of S is what we get. So this signal would then simply be E of S . Now you also notice from the last lecture where we talked about P, PI and PID controllers. The controller block takes the error and then decides what the manipulated or the input variable should be. So just like how at this point we would say it is the component of y which is due to just U which is $G_p U$ and we will say here it is a component of $d y$ which is due to d , which is $G_D D$ and then we are adding this u plus $y b$ as the actual y .

Similarly we look at this, the U of S has to be C of S times E of S . So this is something that we have seen before. I am just saying this for completeness. So U of S is C of S times E of S . And then once you have that now this whole thing is complete, what it basically says is there is a certain U . If you want to think about this in a very linear fashion, including the feedback, so we could say something like this, there is a U and there is a disturbance. Based on both of these an output is realized.

And this output is checked against a set point and if the output and the set point are the same, then there is no error signal. So there is no job for the controller, so there is no change in U . Everything is $(10:37)$ and life can go regularly. However if because of disturbance or because you actually change your y , then this y of S is fed back in. If it is different from the set point, either because the set point itself has changed or because the disturbance came in and then perturb the system and y moved from the set point, then an error will be generated here.

So once that error is generated, the controller kicks in. So the controller says okay, the error has to be zero, so I am going to manipulate the variable, input variable or manipulated variable. So this controller is going to take this error as the input and then say the manipulated of the input variable has to take a certain value. And once it takes a certain value, it is going to be different from the value it was in previously because of which this is going to change and because of which this is going to change.

And I compare this and then see whether the change that I have made in U of S is enough for me to be able to control the system. And that I find by taking a difference and if the difference is zero, the controller has nothing to do. If the difference is not zero, then

something needs to be done. So this is the basic idea of feedback control, nothing more complicated than this. Now because of all of this feedback, let us see what happens if I were to do an overall input-output block just like what I did in the last slide.

So basically I have to put an overall input-output block, so which I want only being in the input, so this block, and the output to this block. Now you will see when I put an overall input-output block, we subsume the open loop block, then you will see the output still remains to be Y of S . However the two inputs now becomes D of S and Y set point of S . So this is an important thing to note, so the two inputs have become D of S and Y set point of S .

Just recall from the last slide what happened, in the last slide we showed that the Y of S is sum of two terms. One term is for U for S , the other one D of S . So in other words, the two inputs were U of S , D of S and Y of S but when we do a closed loop control, now the inputs have D of S remains still as an input but the U of S input has vanished and I have Y SPS. So we will have to ask this question, what happens to this input? So from a closed loop viewpoint, what we could say is I have to write just like what we did in the open loop.

I have to write this Y of S as having two terms, one for each input, the first term is some transfer function which is yet to be determined, Y set point of S , plus another transfer function which is yet to be determined D of S . There is no difference between this and the previous block in the sense that once you put in this overall block and then identify that these are the inputs and this is output, then basically every time you have this, you have to basically write the output as the effect of all these inputs and because of linearity assumption you can write the Y or output as sum of the effects of each of these inputs.

And typically each input's effect will come in as a product which is a transfer function by that input transfer function. So that is exactly what we have here. In the open loop, Y of S is some transfer function times U of S plus+ some transfer function times D of S because U and D were the inputs this open loop. However once I close the loop, then the inputs are simply Y SPS and D of S . So this is an important thing to notice. So now you might ask what happened to this U which was there before, now it is vanished.

What has happened to this U is U is not freely changing, it is not a free input like what we had in the previous slide which was the open loop block diagram. Now what has happened really is that this U of S is now calculated as a part of this close loop control. So I can arbitrarily vary U , so if I cannot arbitrarily vary U , then I cannot call it as a real input which

can take any value. U of S is actually computed as part of this closed loop and that is the reason why U of S cannot be a separate input to Y of S .

So really if you look at this, it all seems to make sense because what can be varied without any constraints so to speak are the inputs. And in this case D of S which we have anyway no control over it is a disturbance variable. We keep varying the way it wants. So I need to figure out the impact of D of S on Y of S and the U of S has been replaced by Y set point of S because what happens here is in the previous case I could vary U of S independently but now I have made it serve the interest of Y SP of S .

So when I give a set point value, that is what drives, what U of S will be calculated as through a controller because once I give a Y set point value that is computed and compared against Y , and then I get an error and that error signal and the controller architecture that I have chosen already decide U of S . So the only thing that can vary independently is Y set point of S . So that is what is given here as these two terms, some transfer function times Y set point of S plus+ some transfer function times D of S .

So this is an important concept to remember. Now we can tie up everything that we have been talking about till now through this slide. Remember at the beginning at this course, so I said there are two types of control that we are interested in, one I called as servo control, another one I called as regulated control or disturbance rejection and you can see that here. So in a closed loop if I want to do servo control, then basic idea is I am giving set point profile which I want to follow.

So I am saying Y set point S is something and D of S is zero, so this would mean servo control because I am not trying to reject a disturbance but I am trying to basically follow the set point trajectory or profile that I am interested in. So this is servo control. Now regulatory control or disturbance rejection control would mean and I am not trying to change the set point that is still zero but this disturbance occurs and then how does my overall loop change in terms of this disturbance, this is called regulatory or disturbance rejection.

So you see how we started from some words and then we have gone through a series of mathematical steps to be now able to write the mathematics behind the words that we had at the beginning. Now again very simple if you learnt how to analyse transfer functions in general, then the analysis of transfer functions of either open loop or closed loop are the

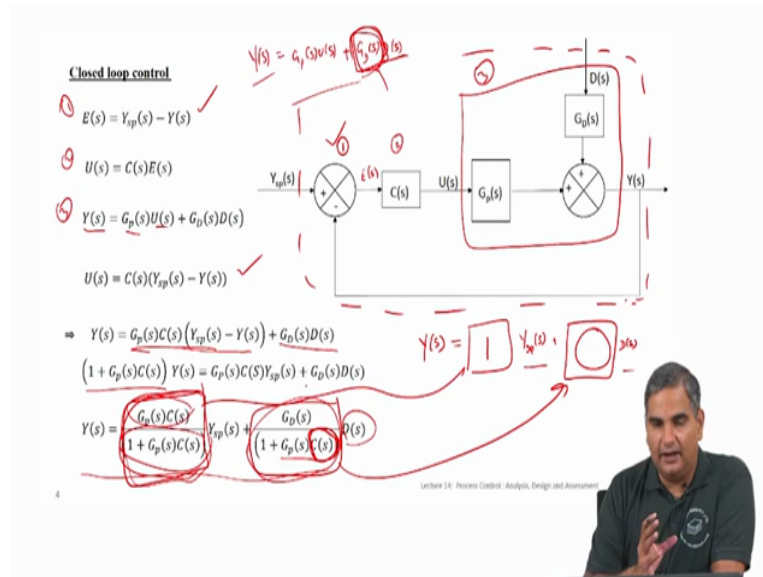
same, we just follow the same procedure. There is nothing great in analysing closed transfer functions over open loop transfer functions.

So if this is the case, then you will have this transfer function here. Once we determine what this block is, then Y of S will be this block times Y set point of S which is some profile we are giving. So we can find the Laplace transform of this, then we will basically write that NS or DS , do the partial fraction, do the inversion, same procedure. Now if this is the case, if that is the case, it is again the same thing. You have D of S is a particular functional form, you have the block that you have there and once you determine what this block says, then again Y of S is this times D of S , do the NS of DS , do the partial fractions, do the inversion.

So that is really nothing that is different that is going on here. Now if there is both servo and disturbance rejection at the same time, there is conceptually nothing complicated. All you need to do is you give this profile exactly, you give profile for disturbance that you want to analyse, do this transfer function times this multiplication, this times this multiplication, add them up, do the algebra, get it in the NS or DS form.

And once you get it in the NS or DS form, do partial fraction inversion. So basically once we get these blocks, that is our key aim. Once we get these blocks, all of these are straightforward. You have all the mathematical machinery that you need to be able to understand, analyse and so on. So the only question that remains is how are we going to derive these blocks. So you will see that it is a very very simple derivation. And the reason why it becomes very simple is because we are going to simply do algebraic manipulations, nothing else. So it is going to be a very simple algebraic manipulation to do this and that is the power of using Laplace transforms and control.

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So let us do this derivation. Let us first look at each of these and see how this calculation flows. So you start from the left, so the first block is this, so what is the calculation that this block is doing? So it is comparing Y set point of S and Y of S and it is giving an output. Let us call this output as error. So the first block computation is simply the E of S is Y set point of S minus Y of S .

Then we come to the next block which is the controller block. So the controller block gets as input E of S and output is U of S . So U of S is C of S times E of S . So this is the second block computation that we do. Now the third block, what we are going to do is we are going to do our original open loop block, this the whole thing I am going to call it as the third block. And if you write the equation for this third block, you will quite easily see that Y of S is G_p of S times U of S plus G_D of S times C of S .

So these are the three blocks. That is all you need. So once you do these computations, then it is reasonably straightforward to do the algebra after that to find The, other two blocks that we saw in the previous slide. So what we are going to do is remember simple thing is ultimately our interest is in writing Y of S as, so because this is simply big block write here, so we have already talked about this quite a bit in the last slide.

So our interest is in writing Y of S as some block times Y set point of S plus some block times D of S because these are the two real inputs. U of S has become a part of the computation in this case and Y of S is the output. So when we look at this equation, we find that we want to get to this form but here we have U of S and D of S . So what we are going to

do is we are going to U of S in terms of Y SP of S . So the way you do that is U of S is C of times E of S .

But E of S is simply Y set point S minus- Y of S . So I am going to replace this U of S by C of S , Y set point S minus- Y of S , very very simple. So U is controller transfer function times the error, however the error is Y set point minus- Y . So that is what we are writing here and I substitute this back here. So G_p of S which is there, and U of S which is this, and the second term remains the same. Now if you notice, this is where the beauty of Laplace transform is and I am just going to do algebraic manipulations.

If you notice that there is Y of S here and Y of S here, so I simply take the Y of S term to the other side. So this is Y of S plus+ G_p S Y of S . So this is 1 plus+ G_p C S Y of S you have on the left hand side. Now this term is left behind here which is G_p C of S Y set point of S . And this term is left behind, G_d D of S . Now collect all of these and then bring it to the other side, you will get Y of S as G_p of S C of S divided by 1 plus+ G_p of S , C of S , Y set point of S plus+ G_d of S divided by 1 plus+ G_p C .

Now notice this, this is one block here and this is the other block here. So this is how you derive the closed loop transfer functions, very simple. Just simple algebra that we are doing here, so the open loop transfer function where Y is a function of D of S and E of S has been converted to a closed loop transfer function where Y is a function of D of S and Y set point S . Now notice some nice interpretations that we can have. If you have for example, disturbance that is affecting an input, so Y of S is G_p U plus+ this I am writing in open loop.

Now if I want to control the effect of this disturbance, or control or change the way Y behaves to this disturbance, I have no control here because this I have no control over and G_d of S is a transfer function which is actually defined by the process itself. So I have really no control over what I can do here. However what we really are interested in is somehow reducing the effect of the disturbance on the output.

So if I want to reduce the disturbance, effect of the disturbance on the output, somehow I have to modify this transfer function that affects my output. There are only two ways of doing it, one is actually go and redesign your process and then say I am going to add something here, I am going to add some active and passive and whatever that is and then you say the new process I model. And when I model that new process, then I get a new G of D and then I can see whether this G of D is better in terms of how the output reacts to disturbances.

So that is one way of doing it, that is actually redesigning the process so that it is inherently more controllable and so on. Feedback control takes a slightly different approach, what it says is I am going to feedback the error and then I am going to manipulate something. So I am going to add again, it is equivalent to redesign. I am adding some other elements to the process but the element that I am adding is the feedback control.

And once I add this feedback control, now I want to see when I see the effect of this disturbance on the output in closed loop, is there some leverage for me? Is there something that I can change so that I can shape or modify this G_d of S ? And if you notice by doing feedback, you look at this equation now very interesting. Y of S , the effect of the disturbance on Y in closed loop is different from the effect of disturbance on the open loop.

Here I had no control, here now I have introduced this C of S . Notice I cannot still do anything with G_p of S because G_p of S is process related. I cannot change it by control. However this is completely something that I can give any value for, because this is something I am going to define. So since I am going to define this, now I have got a hook onto the transfer function which says how the disturbance is going to affect the Y . So since I have hook on the transfer function, now I can do whatever I want to see how much I can minimize the effect of the disturbance on the output.

So that is the critical idea, key idea of feedback control. Similarly if I want to control or have an effect on how this set point change here, so there is no equivalent here because there is no set point here at all. But when I talk about closed loop transfer function, basically what this say I can manipulate or shape or modify this transfer function. Then I can actually have some control over how Y varies as function of set point. So I assume that this notion is clear and how a very simple idea of feedback control allows us to have a hook on both the effect of Y set point on Y and D on Y and how we can shape or modify this.

Now just one last concept for this slide so that we understand the whole idea before getting buried into the calculations, I just want to make sure that the major thematic ideas are clear right at this point. Now if you notice and then say okay, let me understand what is the ideal transfer functions that I would like to have for each of these blocks, so forget all of these Laplace domain, forget all these transfer functions and so on.

And just say that I have this block and this block here, what is the best value for this block? It is a time function, it is a Laplace domain, all of that is true but just conceptually for us to

understand we will say pick a number and then say what number would be good for this block. Now first let us look at disturbance. So what is the idea of disturbance rejection? What we want to say is if I am at my set point value and nothing else has changed, I am not deliberately changing the set point.

If there is a disturbance that occurs, then basically what I want to say is my Y should not change. Y should be at its steady value. And since I have written Y already in the deviation variable form, it is already at its steady state. So what we really want is whenever there is a disturbance process, I do not want my Y to change at all. And the steady state value of Y is 0 because we have written it in deviation variable form which we discussed right at the beginning.

So if that is the case, then what you want is this block should have value as close to 0 as possible because if it is zero, irrespective of whatever happens to D of S , Y of S will be zero, that means it is at its steady state at all times. So this is the best value for this transfer function. Now if you make similar arguments for the other block, then you will say look, what is the idea of control itself? The idea of control is Y should be equal to Y set point. So that is the idea of control, set point is the desire for us that the output should follow.

So if Y should be equal to Y set point, then the ideal value for this is 1. So in fact if I can design a system where there is always 1, there is always 0, that is the perfect system because the minute I give you a set point change, Y immediately becomes set point value irrespective of whatever disturbance that is there. So this is the ideal case. But in reality you never get this, so just to project that because we are going to use this in stability and when we do tuning of controllers, to project this back if we look at this and then say I have no control over G_p in the first block and similarly I have no control over G_d and G_p in the first block, so we just think of these as numbers and say what would give me really good control.

Then basically we will say C of S is very large, then $G_p C$ of S divided by $1 + G_p C$ of S will be close to 1. So large C of S which is kind of large gain control in simple terms and as a first idea you will understand that as I keep making CSK larger and larger and larger, I will get better and better servo response. So remember the last lecture, we talked about proportional controller. The form of the proportional controller was simply K . Supposing I were to just put K here, and I keep making K become larger and larger, if K tends to let us say very very large value, this divided by this will be very close to 1.

So from a proportional controller viewpoint larger and larger gains give you better and between results, just purely from the simple analysis. Let us see whether that has something, that is something which is consistent with what we want for the second block or is there is a trade-off, it is inconsistent with what we want for the second block is something that we can see. So it could happen that larger K makes this 1 whereas larger K takes this away from 0. It is possible but let us just check what happens here.

Now when I come here, if I make this K larger and larger, here luckily for me that this also becomes smaller and smaller because this is fixed, this is fixed and I starting to divide it by a larger and larger number in the denominator. So this will go close to zero, so looks like if just think about a proportional gain. As long as I keep increasing the gain, it looks like will serve both my servo ambitions and my regulatory control ambitions which is a nice thing.

If there were trade-offs, then I have to really worry about that also. But right now it simply says the performance keeps increasing as I keep increasing the gain. Clearly you will not increase the gain arbitrarily. There are several reasons why you would not do that. Number one, it might physically it might lead to physically unrealistic expectations for control modes in terms of the change that I can affect in an input variable or manipulated variable or rate of change that I can affect in an input or manipulated variable. So that is really a physical constraint and that cannot be violated.

So that is one thing that can limit. The other one which is more subtle and inherent to the way the systems work is that at some point you will see that you can make your system unstable. So you might take a stable system and in pursuit of excellent performance, you might make the system unstable. So these are reasons why you cannot arbitrarily increase this value. And we will see these things as we go forward. And this form the basis for really how we are going to tune controllers and so on.

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Transfer function between input and output = $\frac{\text{Product of all the transfer function blocks between the input and output}}{1 + (\text{Product of all the transfer function blocks in the negative feedback loop})}$

If one were to study servo control, then $D(s) = 0$,

$$Y(s) = \frac{G_p(s)C(s)}{1 + G_p(s)C(s)} Y_{sp}(s)$$

If one wants to study disturbance rejection, $Y_{sp}(s) = 0$,

$$Y(s) = \frac{G_D(s)}{1 + G_p(s)C(s)} D(s)$$

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So another way to write the same result, the algebra is a standard rule that is given. I am just adding this here for sake of completeness. In general, you can always do this algebra, it is very very simple. Whenever you are in doubt, you do not have to follow any of these rules. You simply do the algebra and you will get your result. Typically if there is a feedback loop like this, what this rule says is if I want to find the transfer function between a particular input and any output in a feedback loop, so all I do is I go from here to here in a forward path and then I put all the transfer functions on the numerator.

And then the denominator is 1 plus+ all the functions or the blocks in the feedback loop. So that is what this rule is. So let us apply this rule for the transfer function between Y of S and D of S. So if start from D of S which is the input and go to Y of S, I encounter G or Gd which is in the numerator. And if I multiply all the blocks in the feedback loop, then that is simply Cp and CS, G p and CS sorry. And then add 1 to that, that is in the denominator. So the transfer function between D of S and Y of S in the closed loop is the following.

Let us try and see whether the same rule works for Y set point and Y of S. So if I go from Y set point to Y of S, I encounter two transfer functions, G p of S and C of S that is in the numerator. And denominator is the same as this and you see that this is consistent with what we have seen before.

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Block diagram manipulations

Using the block simplification rule,

$$Y(s) = \frac{G_p(s)C(s)G_v(s)}{1 + G_p(s)C(s)G_v(s)H(s)} Y_{sp}(s) + \frac{G_D(s)}{1 + G_p(s)C(s)G_v(s)H(s)} D(s)$$

$G_v(s)$ representing the valve transfer function ✓
 $H(s)$ representing the measurement transfer function ✓

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Now you can flush this out a little more, so instead of just having C and G_p, so this is also something that is seen in control textbooks, so I am just mentioning this for the sake for completeness. Typically you might say this is actually a process transfer function but we go forward with these lectures. You will see that control is always affected using control valves. And you cannot assume the control valves act instantaneously, that is you could not assume the control valve transfer function as basically 1.

So you might explicitly say this is the valve transfer function, so basically what it says is there is an error that comes out. The computation that is done from the controller goes to the valve and the valve has to move before the manipulated variable value changes. So that affect is brought by this transfer function. And on the flip side you might have some measurement, so which will be measured by some device, so let us say it is temperature.

I am just saying I gave you a set point and temperature minus- temperature set point, but if I measure a certain temperature, I might not be able to immediately measure it. There might be a slight time lag, there might be a slight bias between what is measured and what the truth is. All of that is modelled using this transfer function for the sensor or the measurement device which is H of S. So this is called the measurement transfer function and the valve transfer function. So this will make the loop a little more realistic that you can basically use the same ideas as nothing that changes in any of these.

Once you understand this NS or DS and partial fractions and inversion, so we use the rule that we talked about in the last slide. So if I want to find the transfer function between D and

Y, basically first I do the denominator which is 1 plus+ the multiplication of all the blocks in the feedback loop, which is 1 plus+ $C S G_p S H$ of S. And when I go from $D S$ to Y of S, the only block I encounter is G_d of S. So I put it in the numerator. Now when I look at the transfer function between Y set point and Y , then the blocks that I encounter along the way are $C S$, G_v of S, G_p of S. So all these three blocks that I encounter, so I put them in the numerator and the denominator is the same as this.

So this is a very simple idea of doing the same thing when you have more blocks. You do not have to particularly worry about any of these discrete, any unnecessary problems. Now once you have this, once you have all of these transfer functions, then you can do the same thing. If you want to just study the effect the disturbance rejection, you just set Y set point as zero and then whatever disturbance profile you want to study the effect for, then simply use that D of S and then you will get the $N S$ or $D S$ formula.

Now if you want to find how your controller will do when you give a steep set point or ramps like this, or some sinusoidal, correspondingly you find the Y set point and then compute Y of S and then do the partial fractions, compute Y of t , then you will see from this time profile how we get the corresponding time profile for Y . Now notice beautifully the Laplace transforms helps us do all of these.

Because I am drawing here time profiles, the notion is the minute I draw the time profiles here, I can correspondingly get the Y set point or disturbance profile in Laplace transform. And once I get that in Laplace transform, once I put all of these into picture, I will get Y of S in Laplace transform, and once I get Y of S in Laplace transform, I simply do a inversion to get Y of t . So I go from here, so you might say my Y set point is like this, but my Y does not follow it immediately but maybe a lag and so on.

So all of these you can actually quite easily study using these block manipulations. So this is an important lecture where I transition from open loop to closed loop, showed you that two of the important ideas that we have been talking about for a while now which is that this Laplace transforms will make all the computations algebraic. I show you how that works and then once I show you how that works, then that leads to this derivation of these transfer functions in closed loop which is given by the simple rule if you want to follow that, multiplication of all the blocks between the two input and output divided by 1 plus+ multiplication of all the blocks in the feedback loop.

You can use that or you can simply derive these cases quite easily by simple algebraic equations and get your output of interest in terms of the real inputs that affect the output. And once you do that, then it is standard, find the transfer function and then simply note it. So now that we have set the stage for how to analyse closed loop transfer functions, in the next lecture I will actually take P, PI controllers and their forms and put that into these blocks. Take a particular transfer function and then see how all of these comes together and that will also help me explain to you what are the advantages and disadvantages of the controller and what are the advantages and disadvantages of PI controller and PID controller.

So once we do that, then we have very good intuitive idea of how all of these is done, the advantages and disadvantages of these controllers, then we can go on to actually how to assume these controls so that I get performance that I desire. Okay, I will see you in the next lecture. Thank you.