Process Control - Design, Analysis and Assessment
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Controller Equations

We will continue our 13th lecture in this course. Till now we have looked at some basic terminology in control, talked about how you model a physical system. So when we model a physical system, we talked about writing conservation equations and constitute of relationships. The conservation equations can be written for mass, energy, momentum and so on. Till now we have focused largely on mass conservation equations. We will also have the other types of conservation equations and how you model such systems in some assignments that you work with.

So we saw that a physical system can be modeled using in general ordinary differential equations for this course but just keep in mind that there are other types of model equations that come out when you do this exercise. You could get what I call differential algebraic equation systems where not only they have ordinary differential equations but you also algebraic equations as part of the model. If you are looking at modeling in little bit more detail or if you want to capture both the time variation and spatial variation, then model equations can become partial differential equations.

So as an example if you take this room and then say I want to control the temperature in this room, then the immediate automatic question is where would you control this temperature, because temperature at different parts of the room might be slightly different, let you say for example. So if you said, I am going to represent the temperature of this room by a temperature at a particular point and then simply do the model equations and write equations around it, you could call it a (())(02:03) parameter model and you might get an ordinary differential equation.

So the temperature at that point as a function of time is what you are trying to model. However if you were to say that this temperature varies across this room and at different points I have different temperatures, then the temperature equation has to be represented in a partial differential equation so that you get both the variation of the temperature with respect to space and variation of temperature with respect to time. So it would mean is there is a temperature at this point and I can see how it varies with time. Or at the same time I could have three temperatures and I can see how they vary at different positions at the same time.

So those are partial differential equations and there is lot of work on control of partial differential equations and so on, so those are advanced topics that we are going to talk, not going to talk about in this course yet. So we are going to stick to ordinary differential equations. In ordinary differential equations I talked about this terminology of disturbance variables, manipulated variables and control variables. I said control variables are the variables that you want to take, want them to take certain values. And you are free to manipulate the manipulated variables to any value so that the control variables take the desired values.

When you are doing all of this, the disturbance variables are the ones which have no control over and they affect the control variable because of which you have to further manipulate the manipulated variables to reduce the effect of this disturbance. And we said there are two types of control, one is called server control. This is the type of control where we voluntarily change the set point and the want the output to follow the set point.

The other type of control is called disturbance rejection or regulatory control where we do not want to say change the set point. We want the output to be at the set point. However the disturbances perturb the output from set point and we then use the manipulated variable again to control the output variable. So these are concepts that we saw and I also showed how in the model we could have variables which are in some cases disturbance variables but if you put in a control in that line for example, they could become a manipulated variable and so on.

So it is important to understand the physical organization or the physical structure of the system that you are looking at and then tag variables as disturbance or manipulated or control variables correspondingly. So there is no fixed rules. Here it really depends on the design that you have but once you have a particular design, then you will be able to identify what are the disturbances variables and what are the manipulated variables. Then we saw that when we write these equations, in many cases in engineering systems these equations will tend to be non-linear.

However, all of this analysis as far as this course is concerned is all for linear system so either when we model the system, we model it in such a way that we get linear equations or we model the system much more accurately and then get a non-linear model and we linearize that non-linear model to get ultimate linear model. So either way what we end up with is linear model. So when we have these linear models, then the notion of linear super position works.

So if I have an equation, there will be an effect of the manipulated variable and then there will be an effect of a disturbances variable and there will be an effect of what we call as state variable. And all of these will be additive. So that is the idea of linear equation. So the upshot of all of this is that when you model the systems, if you model the effect of the manipulated variable and the disturbance variables on the output, then you would see there will be term which is for manipulated variable and then there will be an addition to that which will be the effect of disturbance variables. So these are additive.

Then we said you could solve these differential equations. However if you want to simplify your life a little bit, you could do what we called as Laplace transform and use Laplace transforms to convert the differential equations into algebraic equations. And we went through series of lectures on how this is achieved. And what happens ultimately then is that the differential equations when you solve, you get a solution in an integral form which is what I showed you which is called the convolution integral.

When you convert the systems into Laplace domain, this convolution integral simply becomes multiplication. So Y of T in time domain will be convolution integral between what we called as an impulse function and the input which will in Laplace domain become Y of S is the transfer function model for G of S times U of S. So I said that what we really do is we take U of T, convert it to U of S and multiply that with G of S to get Y of S and we inverse Laplace transform Y of S to get Y of T.

So this way what we do is we basically work with algebraic manipulations instead of solving differential equations. That is the advantage of Laplace transforms. As you get more and more familiar with this topic and understand this more, there are much better uses of Laplace transform in terms of interpretation and insights where you could think the whole control in a more frequency domain. And then you will be able to make judgments about controllers and processes, much more, sometimes even more intuitively in frequency domain than time domain.

So that is something that you will get good at. Nonetheless as far as this part of this course is concerned we only focused just doing this computations and getting Y of T and we will expand on that in this lecture through this controller equations and so on. The key thing to notice here is that if you had to actually do this Laplace transform through integration and inverse Laplace transform through some complicated again integration, this whole approach will be rather pointless. But that does not happen. And what happens is that lots of these have been pre-computed and you have a table of time functions and their corresponding Laplace transforms.

And when you go from U of T to U of S, you look at the time function for U of T and then pick a corresponding U of S. And then once you get Y of S, G of S times U of S, then you look at the corresponding Y of S in the Laplace domain column and then go back to the time domain. So that is the idea that allows us to do these computations rather efficiently. Then what we saw was we said any Laplace transform is basically going to look numerator polynomial by denominator polynomial for the stress based model that we have been talking about.

And then real question is how do I invert this, then I showed you the idea of partial fractions and I showed you how you can do the inversion using simply one row in this Laplace table most of the time. You can get this inversion, so we spent a couple of lectures spending time on that. And from that idea then I introduced a notion of stability, I showed how using this partial fraction idea you can understand these concepts of stability and we came with the result that a system is stable if all the poles of that transfer function are in the left top plane and even if one pole is in the right top plane, the system is unstable.

We also addressed poles on the imaginary axis where when we talked about bounded input, bounded output stability. I showed you that if you have poles on the imaginary axis, even if you have bounded input you can get unbounded output. So you have to address the poles on the imaginary axis carefully. And I also explained the notion of resonance through the poles on the imaginary axis and so on. So as a summary we now know how to model systems, we now know how to model the effect of the manipulated variable on the control variable, effect of the disturbance variables on the control variable and so on.

So we are now ready to start talking about controllers and control equations. And what we are going to do is we are going to look at controller and control equations in time and Laplace domain and then start thinking about how would I analyze these processes if I have a controller also in place. So that is what we are going to do.

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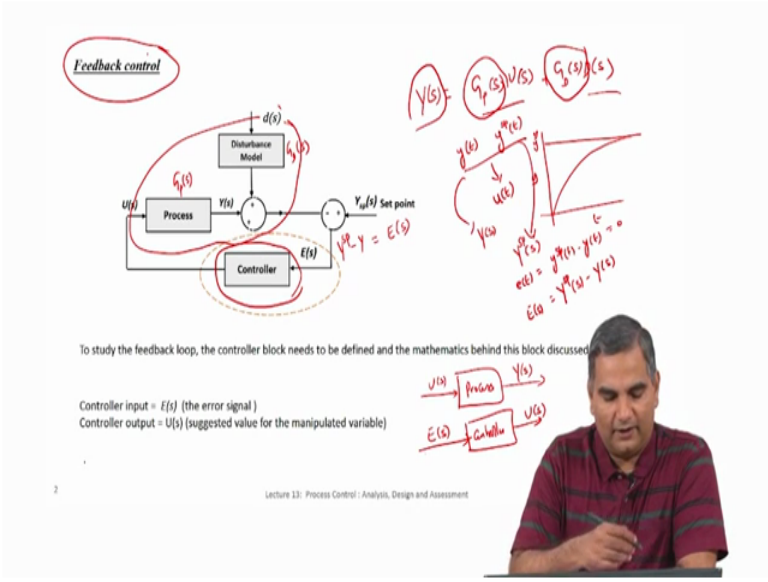

We are going to look at results in the Laplace domain, so we will first look at this part. All the summary that I did till now talks to this part, so what this picture says I have shown you this before. What this picture says is the effect of the disturbance if I do d of S, comes into the disturbance model and it affects Y of S. And I also have U of S which is a manipulated variable which can affect Y of S. So if I for example, said this process is a transfer function and there is a disturbances model transfer function, then Y of S can be written as the process transfer function times U of S plus+ the disturbance transfer function d of S.

And this is the point that I have been making several times that the effect of the input and output on, the effect of input and disturbance on the output is basically additive. So I have this one here and this one here. So this is what we have talked about till now. Now also remember that we can get this Gp Gd from the physical system. So all the information about the physical system is embedded in these transfer function these are not abstract quantities but they comes from the physical system.

Now what we are going to do is we are going to do something called closing the loop. So let me quickly explain this idea and then we will go into all its detail as we go through future lectures. So basically what we are saying is in a time domain what we want to do is we want to compare Y value with a set point value. So I also say set point as a function of T and just to generalize it normally if I have Y like this as a function of time, set point might be a constant value, so this might be the value of Y set point.

But in some cases you might actually say I want to start this process slowly and then take it to this place, so this could also be the profile for Y set point. So you generalize this, I have made this as Y set point which is a function of time. But in most cases Y set point will be a flat line or a constant value. So essentially starting from the bike example that we talked about, every time our interest is in comparing these two and once we compare these two, if they are the same, then I have not job.

If they are not the same, then what I am going to do is I am going to go and manipulate, so basically the difference between this is what is going to feed into how I should manipulate this U of T. So this is the basic idea that we have been talking about. So if you think about a block diagram for this, then basically since we have made Y of T into Y of S, then Y set point should also be made into Laplace domain because we cannot mix and match Laplace and time domain in the same picture.

So everything is in Laplace domain, so since we compare Y set point times minus- Y of S, Y of T in time domain, using linearity of Laplace transform if you are comparing this, that is equivalent to comparing Y set point of S minus- YS. So the Laplace variables, so that is what is shown in this picture. So I compute this, so this is Y set point minus- Y. And this is what we call as an error in Laplace domain which will be, so if you say this is e of t is this, if you take Laplace transform of this whole equation, then the left hand side E of S equal to Y set point S minus- Y of S.

So that is what we have here. So I have this E of S which is error and if this were zero, then the error would be zero, similarly E of S will also be zero. And we take control actions only when E of S is not zero. So this is where we introduce this notion of the controller. So what the controller does, it gets information about the error and then it has to figure out what the output from the controller should be which is the instruction for the change in the manipulated variable. So in the process if you think about the way it works, if I have a manipulated variable because of changes in this, this Y of S changes. So that is the process, that is when I manipulate something, how does my output change is what the process called as.

The controller is in some sense an inverse of this where the input to this controller is the error which is the difference between Y set point and Y and the controller has to figure out how this U should change. So how should I manipulate is what the controller figures out. And when I manipulate U of S, what will happen to Y of S is what the process figures out. Now if you combine both of these, then I have what is called the feedback loop. So the feedback loop is I make some change to manipulated variable, I see the effect of that on Y output through the process, then I compare it with my set point and that error is what I am giving as information back to the controller to say okay, whatever manipulations you did either worked in which case the error is zero, the controller says do not do anything more or did not work in which case the error is still not zero, so the controller you have to further manipulate it.

And I am going to give you instructions on how to manipulate it, so that is the concept here. So that is the feedback, so that is the reason why we call this as feedback control. So it is very simple concept, you do something for some desired effect and you see whether the effect is actually the same as the desired effect. If it is the same, whatever you have done is great. If it is not the same, then you have to change your strategy and do something else.

And the changing the strategy how you change it and so on is what the controller gives and real human action that is the brain, it looks at it, it sees this and then says okay, change it this way. So that is the basic idea of feedback control. And notice how we have put all of these in Laplace domain. Now the biggest advantage of this block diagram is everything is algebraic. So you will see how we redo this.

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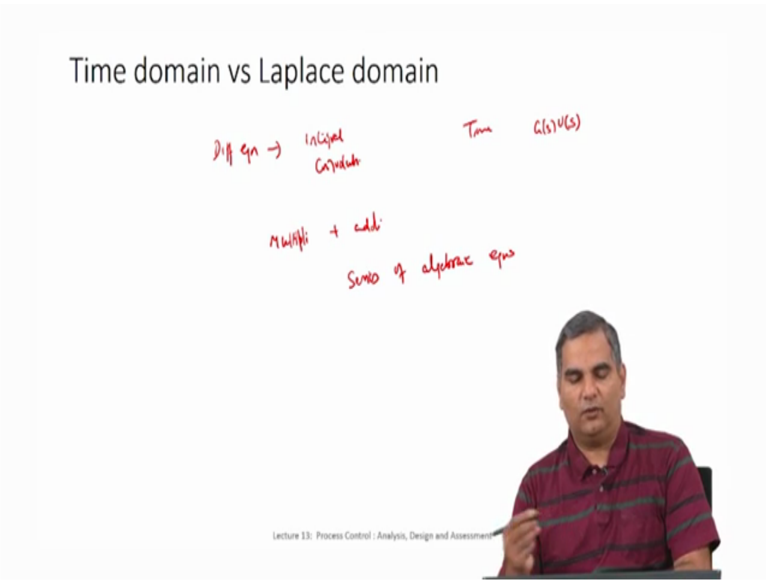

So in time domain, basically I am having differential equation which when the solution is an integral solution you get and which is a convolution integral. In time domain, this simply becomes multiplication G of S times U of S. So when you have these blocks that I had before, so you can simply, the block uses only multiplication and addition. So what you will have is you will have a series of algebraic equations and we will see this in the next lecture.

And once you have the series of algebraic equations, basically you collect terms to one side and solve for it or use Gaussian elimination kind of things to solve for it and so on. So ultimately all the variables have become part of these algebraic equations and you can quite easily solve for them.

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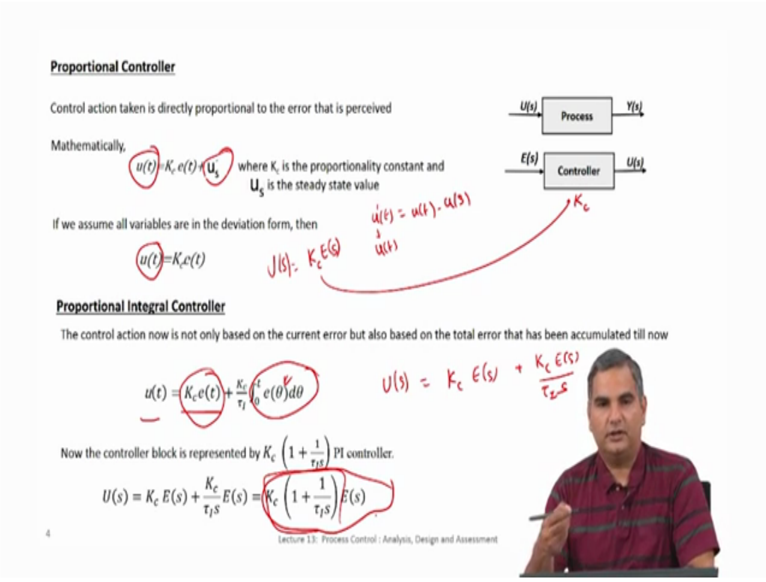

So now let us look at different types of controller that we will have. So first is what is called a proportional controller, we will come back to all of this in more detail. In this first lecture I am going to quickly introduce this idea so you get a feel for how these things work. So a typical U of T in time domain for example will be the way we have written is usually U of S plus+ this and if you do this in deviation variable form, so we are going to write U of t or U prime of t as U of t minus- U of S and we are going to say no point writing this prime every time.

So I am going to call this itself as U of t with the idea that at steady state it is 0. So a proportional controller is going to set the value of U of t and remember this is in the deviation variable. So once you compute this, the actual manipulated variable value this plus+ the steady state value. So the proportional controller is a simplest controller. What it says is if I see a mismatch, I will take an action proportional to the mismatch. So if I want to do something, and I do something, and I see the result is slightly different from the desired value, so the error is small, so the next time I want to take an effort to correct this error.

I will take a small effort. Now if the error is very large, I will take a large effort. So that idea mathematically is very very simply written as U of t is K c. So this is what is called the proportional controller. So this operates on very very simple principle saying my new effort is going to be basically directly proportional to how much error whereas how much is the distance between what I desired and what I actually got. So that is this, so if you do this in the Laplace domain, it is very simple, U of S is K c times E of S.

So if you think about this controller, then the controller is simply K c, just one parameter and nothing else. So if you think about this in the Laplace domain block diagram, this controller is simply K c. So if you give me E of S is simply K c times E of S. Now a slightly more sophisticated way of thinking about this would be to say look, I want to take a control action not just based on the current error that I had but I have seen errors before. So that information is useful.

So what I am going to do is I am going to integrate all the errors that I have seen till now and then I am going to write my controller or the input, manipulated variable as not only proportional to the error that I see currently but also I am going to account for all the past errors that I have seen and then consolidate it with an integrated error and then I am going to take an action both on based on the current error and the integrated error that I have seen.

So it is like whatever past actions that you have taken, they are all accumulated to some error, so you are going to take that also into account. So a form in which this is written is K c times e of t which is the action based on the current error. And K c by tau i. This is to make this in standard form, we could have also called it as K c 1. But this is standard form. K c by tau i, 0 to t, e theta, the theta and this is the theta because I want the running variable to be different from the time variable. So this is proportional integral controller.

In this case if you take the Laplace transform, you will have U of S is K c E of S and we know this from before. If you do this integration, the Laplace transform of this will be E of S by S. So I have K c, E of S by tau i S. Now if you simplify this and so on, you will get this form which is K c into 1 plus+ 1 by tau i S, E of S. So if you think about the controller itself, because E of S is some multiplication times E of S, so the controller is defined by this multiplying factor which is K c times 1 plus+ 1 over tau i S. So that is called the proportional integral controller.

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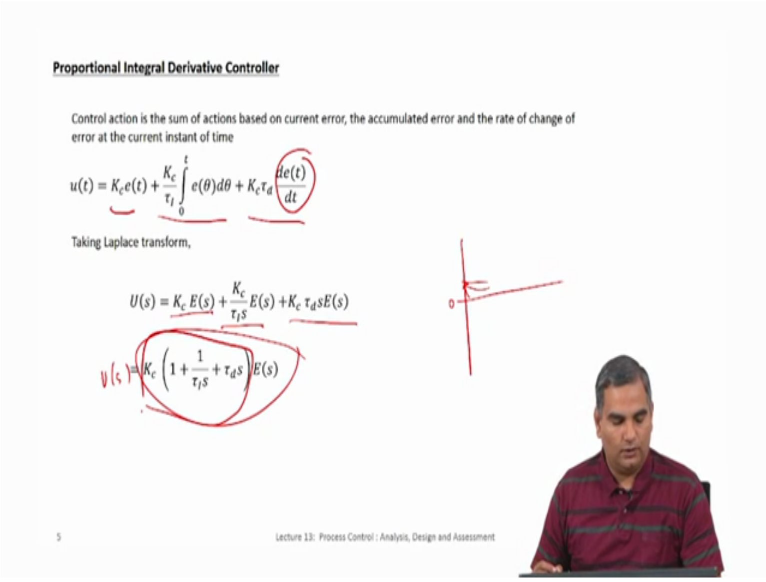

The third type of controller we are going to see in this course is what is called proportional integral derivative control. So here the idea is I am going to take a control action or I am going to set my manipulated variable based on first the current error, based on integrated error and also based on the rate at which the error is changing. So if let us say, so this is kind of a zero line and supposing I am here which is error or near zero, how fast I am dropping also matters. So this is the rate at which this is dropping.

So I am going to make this proportional to the rate at which this error is increasing or decreasing. So that is given by de by dt and I used K c over tau i here, K c tau d. This is again the standard form, so you could have called this K c 1, K c 2, K c 3. But this is the standard form in which we are going to look at this whole course material. So the very first controller just looks at the current situation, the second type of controller is slightly more sophisticated, it looks at the current situation, and all the previous errors that have been seen.

The third type of controller looks at the current error, all the errors that were seen and what is the rate. So future, how the error is likely to change. So this takes into account current, past and future. Proportional integral takes into account only current and past and proportional controller just takes into account current. So that is how you want to think about this. Now if you do the Laplace transform of this, the first term is K c S, the second term we did in the last slide. This also you know, if you assume the error is initially zero and then this is K c tau dS E S which is also something we have seen.

And if you simplify this, you will get this equation. And again since I have U of S, is some controller function times E of S, the controller function is the following here. So K c times 1 plus+ 1 over tau i S plus+ tau d S.

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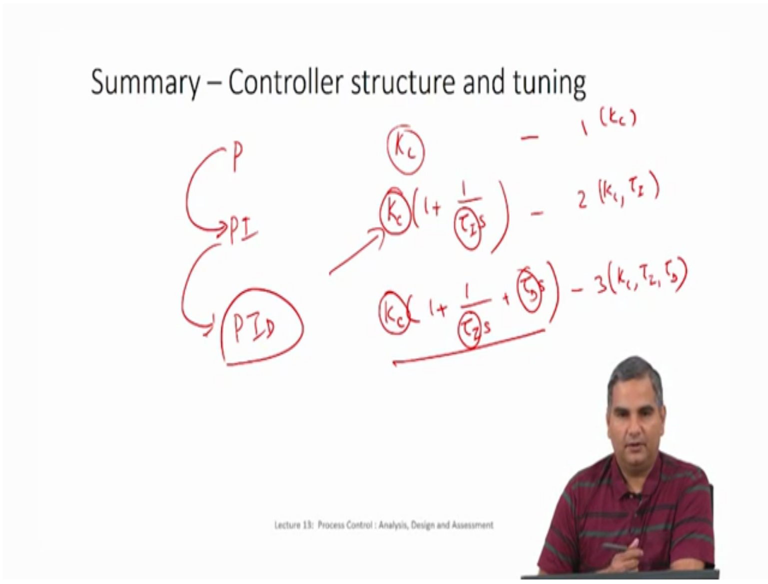

So in summary the P controller is simply K c, P I K c into 1 by 1 plus+ tau I S. And PID is K c into 1 plus+ 1 over tau I S plus+ tau D S. So this has one parameter K c, this controller has two parameters K c tau I and this controller has three parameters K c tau I and tau D. So when we talk about choosing a control structure, we talk about whether we are going to choose a P, PI or PID controller. That is what we talk about when we talk about a controller structure. And when we talk about tuning, then we talk about what values do we keep for K c.

Supposing I say I am going to choose a control structure P I, then that means the controller form is this and later when I say I am going to tune this PI controller, basically I am going to find values for K c and tau I. Similarly if I said I am going to choose a control structure as PID, then this is the controller equation. And when I say I am going to tune this controller, I am going to choose values for K c tau I and tau D. So that is what controller structure and tuning means.

Now what are the advantages and disadvantages of each type of these controllers? Nothing is perfect, so in some cases you just want to use a P controller, some cases you want to use PI, some cases you want to use PID and so on. Because this is more sophisticated, does not mean this is always the best controller. So there are trade-offs for each of these. There is a reason why we move from P to PI. And there is a reason why we move from PI to PID.

But because of certain reason we move from here to here but that will introduce some disadvantages for this controller also. So it is not always that PI is better than P. Similarly when we move from here to here, we get certain advantages and certain disadvantages. So starting from now, we are going to look at these and then how we are going to tune this, why do we go from P, PI, PID and what are the advantages and disadvantages and so on, are things that we are going to see.

Before we do that, what we are going to do is because we have looked at how to analyse this transfer functions, and we never analyse transfer functions with a controller in place, I am going to show you that whatever you have leant till now is enough for us to analyse transfer functions even with a controller in place. So without actually going into this tuning and so on, first I am going to assume that I have some controller transfer function and I am going to integrate that into my block diagram and show you how you can do analysis of these so called feedback control systems also in the same manner in which we analyse the other systems till now.

So that will be the first part, then we will come to P, PI, PID. Then we go to tuning ideas and that basically will cover what most universities teach us basic undergraduate control plus, so I will come back, pick up from here in the next lecture. Thank you.