Process Control - Design, Analysis and Assessment
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MATLAB Tutorial 1: Process Modelling

Hello everyone, welcome to the MATLAB tutorial for the course process control analysis design and assessment. In this tutorial we will be dealing with process modelling exercise, the in this tutorial it this will give you a flavour of how to model a real physical process using MATLAB software and process modelling is very useful in process industries because if you want to gather information about a process doing experiments and gathering data is one way, the other way is to develop a process model do the simulation understand the behaviour of the process, understand how the process response to various kinds of inputs disturbances and changes in operating conditions and it is also very useful for safety analysis purpose, training the operators in a air plant.

So for all this things process model and simulation is quite helpful and by process model what we mean is process model in general describes the chemical and physical phenomena that happens within a system and this is captured by means of set of mathematical expressions or equations and these equations are nothing but algebraic and differential equations.

So in this tutorial we will see how to model a physical process using MATLAB software and how to study the behaviour of the process to various changes in input, we will not look into it very deeply but this will give you an intuitive feeling and a flavour of modelling a process and understanding how the process behaves to various changes in inputs.

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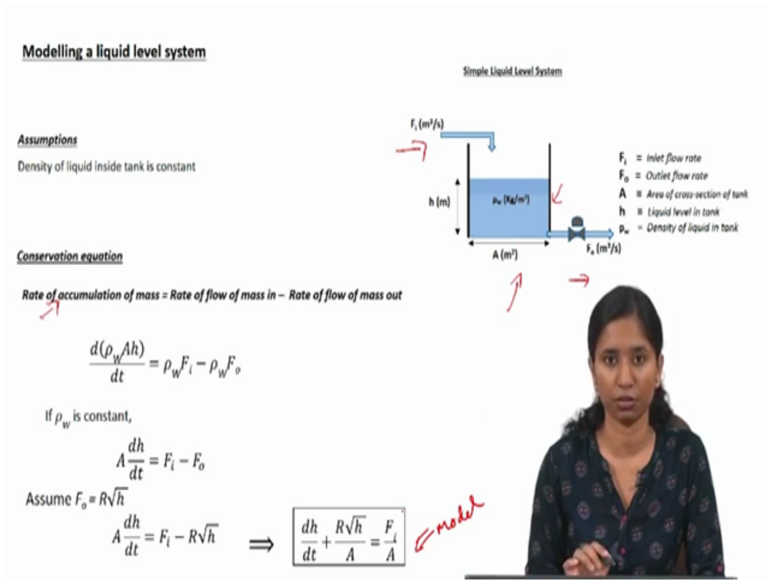

So the example that we will consider in this tutorial is a simple liquid level system, in this system we will develop a mathematical model and how to develop a mathematical model? The physical and the chemical phenomena within a process is governed by a set of conservation laws and it is using these laws that we will develop a model of the system this is a schematic of a liquid level system which is kind of a buffer system for storing liquid, it has an inflow of liquid and an outflow of liquid.

The tank is that we consider in this example is a rectangular tank and the inflow of liquid to the this tank has a volumetric flow rate F i metre cube per second and the liquid hold up within the tank as a height of H meters, the density of the liquid in the tank is Rho W and the outflow from this tank is has a flow rate F 0 meter cube per second, the cross sectional area of this tank is denoted by A.

So in this liquid level system we have placed a resistance and the outflow of the tank this resistance is a control well in this particular case. So let us see how to develop a model for this liquid level system so in process modelling, so often the physical processes are so complicated and complex that it is very difficult to develop an accurate model that depicts exactly what happens within a process.

So we have to make very simplifying assumptions so that it makes our job as a engineer easier to model a process. So in this particular example we will make the assumption that the density of the liquid inside the tank is constant and with that assumption let us look at what is the conservation equation which governs this system.

So the conservation equation is the mass balance equation, the rate of accumulation of mass within the tank is equal to the rate of (mass of) mass flow in minus the rate of mass flow out, since there is a rate coming in here it is a rate of change of mass within the system so that is nothing but density into volume so this is the expression for the rate of accumulation of mass within the system that is d by dt of Rho w into h which is a area into height is equal to rate of mass flow in which is density into volumetric flow rate minus rate of mass flow out which is density into volumetric flow rate out.

So since we have considered density is constant this express expression gets simplified to the following form that is A dh by dt is equal to F i minus F 0. Now we can assume that the flow is turbulent and this the and since the control valve is the resistance that is placed at the outlet of the tank, the outflow from the tank is a nonlinear function of the hold up or liquid level within the system and it can be assumed as F 0 is equal to R root h where R is the resistant say constant and it is a nonlinear function of height within the system.

So this expression can be finally written as A dh by dt is equal to F i minus R root h, which gives dh by dt plus R root h divided by A equal to F i by A this is the model that we get for the liquid level tank which is a differential equation and this equation describes how the height of the liquid within the tank changes to changes in the inflow rate. So if the inflow rate is in flow rate and the out flow rate is equal then the flow when the holdup within the tank that is the liquid level within the tank will be a constant.

So when there is when if the outflow rate is fixed it at a particular flow rate value and the (()) (6:10) opening is fixed, then if we change the inflow rate the height of the liquid within the tank will change, this is what is captured in this particular differential equation.

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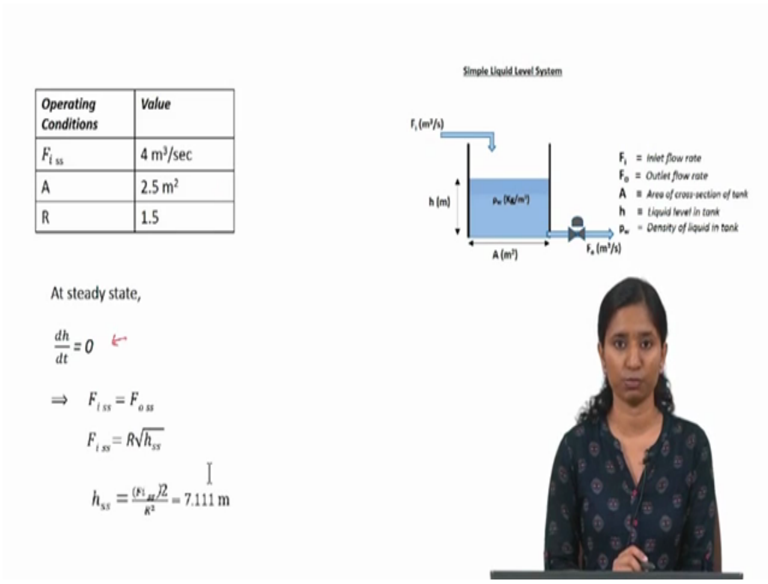

So for simulating this process we need to define the operating conditions and the parameters of this model. So the operating condition is nothing but the inflow rate and the parameters in this particular case is the area of cross section of the tank and the resistance in the outflow of the tank. So before we study how the system behaves to a change in the input that is how the liquid level varies according to changes in the inflow rate which is F i, we have to first define what the steady state conditions are, what is the state of the system before any input is applied to a system.

So at steady state the change in the of the liquid level in the system is basically zero, so that is what is written here dh by dt is equal to 0 and is in flow rate and our flow rate are equal, only then the liquid level within the tank will be constant, so and that since this is at steady state we have given a subscript ss to denote it is at steady state. So we will get an expression for in flow rate as F i s at steady state is equal to R into square root of height at steady state, from this we will get the expression for height at steady state as F i at steady state the whole square by R square and the value that will get is 7.11 meter.

So for the given operating conditions and the given value of parameters of the process the steady state value of the height of the liquid within the tank is calculated to be 7.11 meters.

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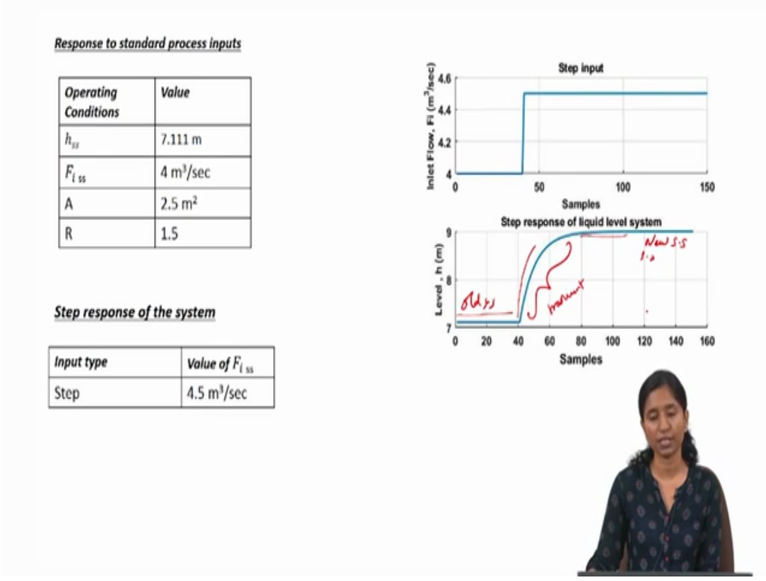

So once we have developed the expression which describes the rate of change of height to the changes in input let us look at how the height will vary if we apply different process inputs to a system. So let us assume that I have given a step change in the input that is step change in the inflow rate and when step input is given to a system the change in the output is called as a step response of the system.

So first we look at the step response of the liquid level system, so the input is step input and the value of the inflow rate is changed as 4.5 meters cube per second from 4 meter cube per second I have changed it to 4.5 and now let us look how that has affected our height of the liquid within the tank. This figure denotes how the flow rate in flow rate changed at the 40th time instant which is 40, so at the 40th time instant I stepped up my inflow rate from 4 to 4.5 and accordingly the steady state level which was at 7.11 this level of the liquid in the tank at steady state was 7.11 which is denoted by this particular line here and at the 40th time instant since I stepped it my inflow rate to 4.5 the liquid level within the tank started increasing slowly and it has reached a new steady state value which is given at 9.

So this is the new steady state value, this is the old steady state value before a change was applied. So we can say that the change in the inflow rate when it was a step change was quite instantaneously the change in the liquid level was not instantaneous, it took a little bit of time to reach a new steady state value. So this period is called the transient period and this is the steady state of the liquid level system new steady state of the liquid system.

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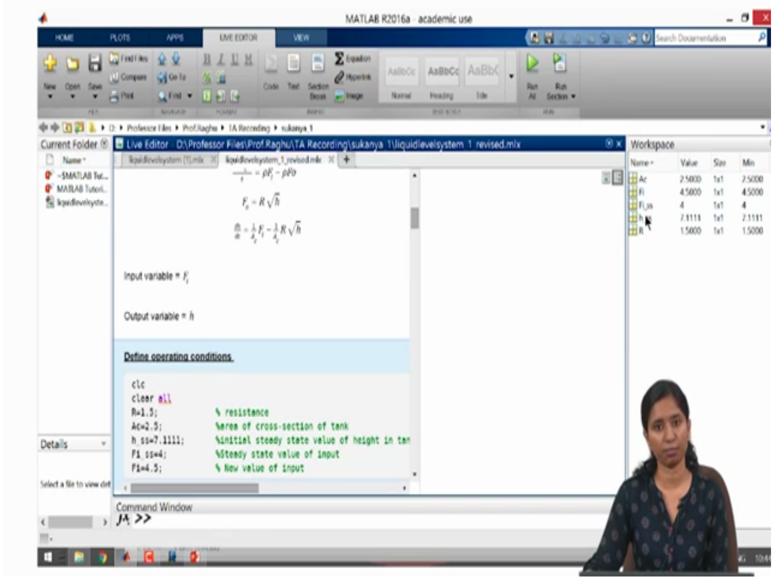

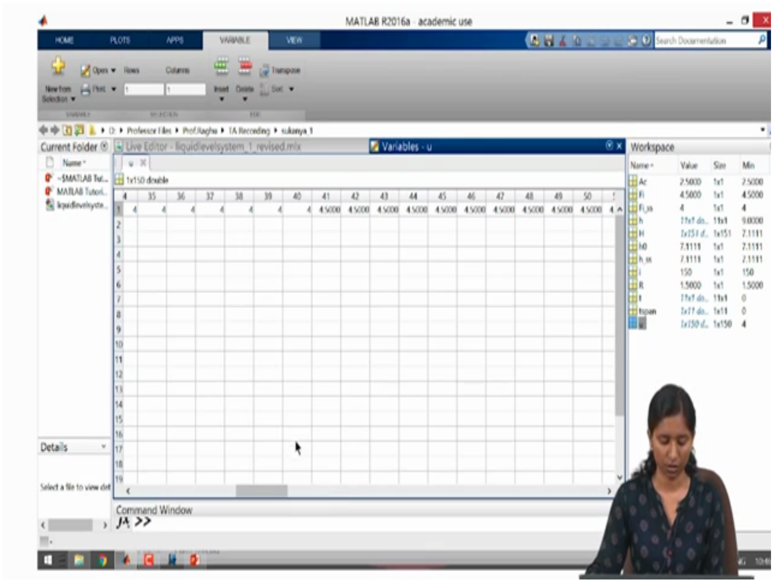

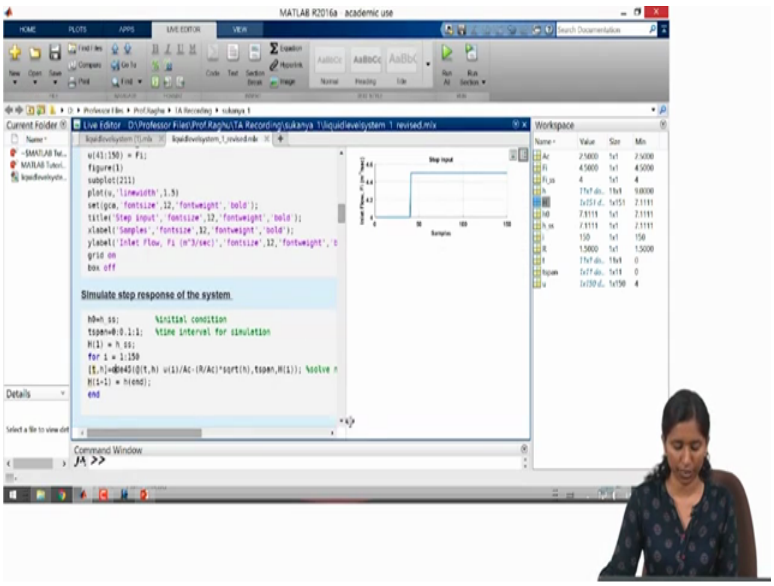

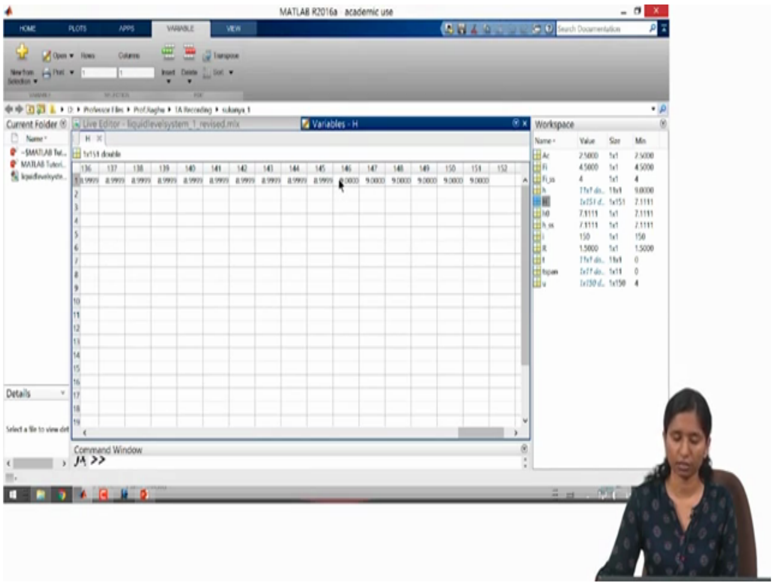

So let us see how to model the process in MATLAB and how to simulate the step response of the system in MATLAB. This is the live editor in MATLAB and so first we have defined that what are operating conditions where we know that the resistance where we have assumed is 1.5, the area of cross section denoted by AC is 2.5, the steady state value of the height of the liquid and angle 7.111 and the steady state value of the input was 4 and the new value of the input that is the new value that we stepped the input to is 4.5.

So first let us run this particular section of code, so now my operating condition the parameter values have been stored. Now let us simulate the step input to the process, I am giving the step input at the 41st time instant, so till 40 the step input value is my steady state value of the input which is nothing but 4, at the 41st time instant my input this is the at the 41st time instant my input value got changed to 4.5 and it remains so for the next for all examples, let me plot the input signal.

So this is my step input as you can see on the right side at the 41st time instant I (())(11:29) to 4.5 and it remains so for next till the 158 sample, I have generated only 150 samples of the input so it remains so till the next Nick till the 150th samples. So all these commands are which for changing the appearance of your plot like a plot command is for plotting the signal but line width is for adjusting the width of the line here, similarly I can change my access properties like font size, font weight and so on, I can give I gave a title like step input and the way I gave x label and y label so that is all and there is in this section of code.

Now my input is stored in this variable called u, so you can see it is all 4 up to 48 sample, but at the 41st sample it is 4.5, so till 158 sample. Now let us simulate the step response of a system and by simulating a step response system what do you mean is that when we give a step input or a step change in the input value what is the value of the height of the liquid within the tank?

So we know that the equation is a differential equation and in order to determine the value of height at different time instants we basically have to solve the differential equation that is we have to integrate the differential equation and in MATLAB we can integrate the we can do so we can solve the ordinary differential equation using inbuilt commands and since it is a initial value problem for a given initial value of the height and a step or a change in input that is given or input value how will the height of the liquid change as time progresses?

So we need to find this out and for that first we need to specify what our initial condition is, since our initial condition here is the steady state value of the height itself, we want to know how the height changed from its original steady state value to when a input change is given to a system. So the initial value is nothing with the steady state value and we have to give the time interval for integration that is the (limits) time limits for integration lower limit and upper limit.

So the MATLAB command for giving the time vector is as follows 0 to 1 with 0.1 increment, the MATLAB solver that I had I chose for solving the differential equation here is ODE45 which is very quiet commonly used solver and in this ODE45 and I have mentioned the function which needs to be integrated here itself in this particular line itself, then you have to specify the limits of integration which is which I have stored in T span and this is the initial value of the height.

So one usually when we use this when we solve differential equation it is usually from starts at 0th time instant and progresses but since I want to show the previous value of the height which is steady state value and the change of the height from the steady state value to the new steady state value as the time progresses I have given used a very small for loop in which every final output of the integration is stored in the (val) variable called H and it will give the value of height as time proceeds.

So the first value H of 1 is the steady state value itself and inside this for loop I have mentioned the time total number of samples that is 150 samples are there total and for 150 input samples are also there, so once I solved this particular section of code I will get value of height for a step change in input.

So I ran this code and my height is stored in my H value, value of height (for the) for all the 150 samples of input is stored in this vector H. So you can see that all the initial values are the steady state values of the height there is 7.111 and at the at the 41st time instant also it was 7.111, but at the 42nd from the 42nd time instant the value of the height started increasing slowly and it reached the steady state value of 9 at the end and the steady state value was achieved at the 146 time instant.

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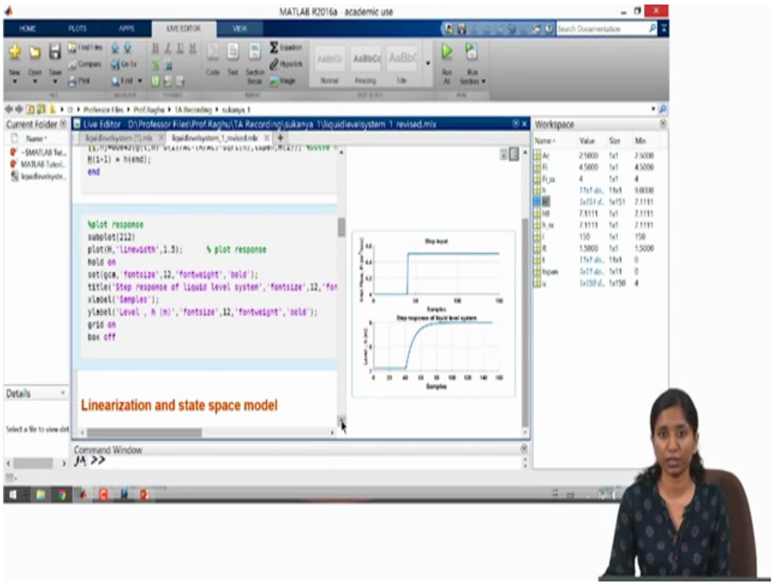

So now let us plot the response as you can see in this plot when we gave a step input here it was quite instantaneous and it remains so but when the liquid level within the tank took some time to reflect that change in height, like it was not instantaneous height did not change like this instantaneously, it slowly increased like in an exponential fashion and finally reached the new steady state value. And how fast the system reaches that new steady state value is defined by something called the time constant of the system and what is the final value that is achieved, how much change is there (from its pre) from the input value that is given to a system is governed by something called the gain of the system.

So we can see that we gave a change of 0.5 magnitude in the inflow rate and in the height of the liquid the change was like from 7, the change was around 2 that is for 7.11 and the final steady state was 9, so around 0.2. So all these characteristics will be more defined and explained in another tutorial related to the dynamic behaviour of the process in this particular tutorial we will consider only the modelling aspects.

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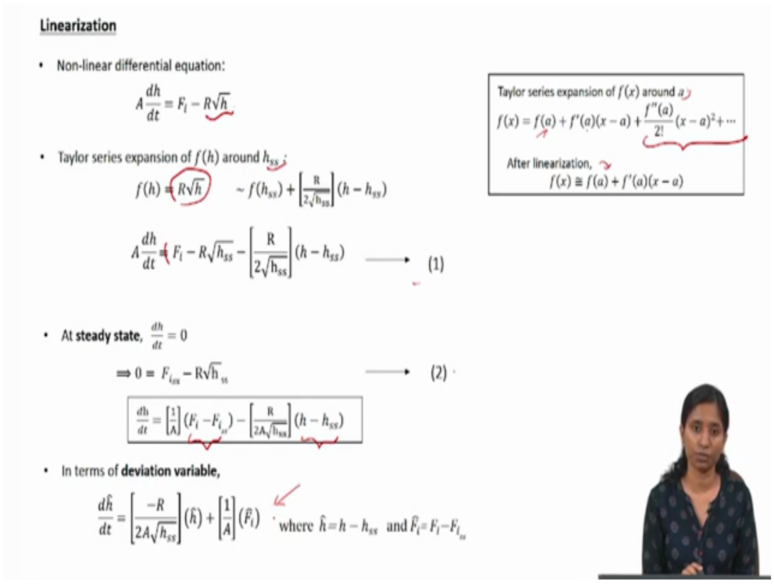

Till now we have simulated the liquid level system, which was described by nonlinear differential equation so the it was a nonlinear system and most of the chemical processes are nonlinear in nature but most control system, design analysis techniques all are (())(18:06) around linear systems. So what is the usual norm in modelling and control engineering field is, we will linearize a nonlinear system around a specific operating point and it will be linear only within a very small range of those operating conditions so usually will linearize around the steady state operating conditions.

So let us look at how to linearize the system, linearization is usually carried out by expanding the non-linear function or non-linear term in the nonlinear model by Taylor series expansion and neglecting the higher order terms. So that just for a quick revision the Taylor series expansion of a function f of x (around a steady) around an operating point a is given by this expression that is f of x equal to f of a plus f dash of a in the x minus a plus f double dash of a by 2 factorial in the x minus a whole square and so on, (f of a) f of a is the value of the function at a and f dash of a is the first derivative of the function calculated at a into x minus a and after blush a second derivative and so on.

So in during linearization what we do is we will neglect all this higher order terms and what we get is this expression which is an approximation so f of a plus f dash of a into x minus a. So let us see how we can perform linearization for our particular example of liquid system. So this we know that the liquid system is governed by this nonlinear differential equation that is A dh by dt is equal to F i minus R root h.

Now the nonlinear term here is our root h, R is a constant so root h is the nonlinear term but still we will take our root h here and let us expand this our root h around the steady state value (h s) h ss, ss is the steady state so if a function of height R h is here f of x it is f of h is equal to our root h, which we can approximate as f of h ss we are we are performing the Taylor series expansion around the steady state value, so first one is the function calculated at h ss plus the first derivative of this particular term which is nothing but (R by 2) R by 2 times square root of h and if it is since it is calculated at a or h ss h is replaced by h ss, so R by 2 root h ss into h minus h ss.

And if we can substitute and this is the linearized version of this nonlinear term and we substitute for this R root h in this expression and we get this expression as A into dh by dt is equal to F i minus R root h ss minus R by 2 root h ss into h minus h ss. So this is a linearized expression or linearized differential equation which is governing the system and it is linearized around a steady state operating point which is nothing but the steady state value of height.

Now when in (control) process control we are more interested in general in developing generalized models that is models which could explain the changes in the variable to any change in input. So basically we are interested in change in the output to a particular change in input and for this we need to remove the explicit dependency of the model on the steady state operating conditions.

So for that we will subtract the steady state equation from the non-steady state equation, so at steady state we have dh by dt is equal to 0, so this is the expression, this is the equation governed in the system at steady so that is 0 is equal to F i flow in flow at steady state minus R root h ss. So to remove the dependency on the steady state condition we will subtract this 2 from 1 and what we get is a differential equation that is dh by dt is equal to 1 by A F i minus F i ss minus R by 2 A root h ss into h minus h ss.

So these two (())(22:47) this is this variable and this variable this denote the deviation or the change from the steady state values and hence they are called as deviation variables, so these are new variables and the new equation that we got is in terms of new variables called deviation variables. So we can denote the deviation variables by cap so in terms of the deviation variables we can write the equation as dh cap by dt or h hat by dt is equal to minus 2 by A root of h ss h cap plus 1 by A into F i cap.

So for the further analysis of the system and further modelling we will consider this equation which is written in terms of deviation variables.

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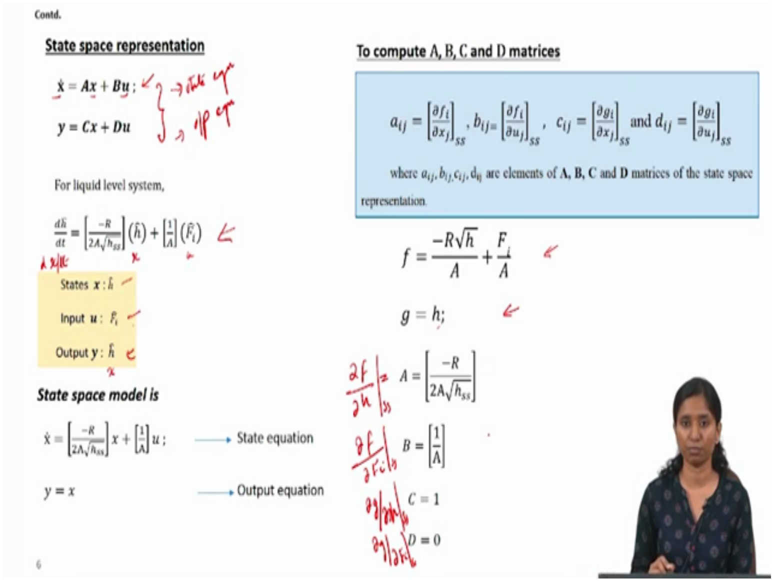

And differential equation moment can be more compactly written in another form which is very quiet popular in the control engineering field is the state space model. So the state space model is given or denoted by these two equations, so this equation is nothing but x dot is equal to Ax plus Bu, y equal to Cx plus Du. So this equation is the state equation this equation is called the output equation, in this static state space representation the x is called the state variable which basically lead denotes the state of the system in a way that it describes the energy, mass, momentum which are the fundamental quantities of the system.

And once we know the state of the system we can find or we can describe the system effectively. So state variables are nothing but those variables which has capability of describing the entire chemical or physical phenomena which occurs within a system and u is nothing but the input variables, these input variables are the variables which can be independently changed which will affect the state of the system and x dot is the derivative or the change in the state variables.

So this state equation shows for a change in input how the state variables change in time. So that is what is explained in this equation and this is the output equation which in the output equation it is also known as a measurement the equation measurement model which gives the output variables.

So for our particular it we can develop this we can develop the state space representation of our liquid level system in the following manner, we know that in the terms of the deviation variables the differential equation which governs the system is this expression and for our system the state is the liquid level within the tank and it is given by h cap, the input (which is) which can be independently changed to affect the state of the system in this case is F i and the output which is the measured variable in this case is also h cap, so this is now this is so dh by dt is dh by dt so it becomes x naught and the coefficient of h, h is x the coefficient of x is A.

So here A is minus R by 2 a root h ss and then we can see that the F i is the u this is the so I will write it down this is dx by dt this is x and this is you, so comparing these two we can see that A is minus R by 2 a root h ss, B is 1 by A so this is what this is how we arrived at the state equation for the simple liquid level system, but this is not the usual case in this case there is only one state variable (one unit) one input variable and one output variable, but in many cases system but in usual systems there will be maybe hundreds of state variable, many number of input variables and so on, so these are vectors and ABCD are basically matrices.

So the state space model for the state equation is written like this and since y is the h cap so y is nothing but x, so that is how we arrive at the output equation. So this is a very intuitive and easier easy way to compare the state matrices but the actual way for computing is ABC and D matrices from the nonlinear differential equation is by computing the Jacobian, so we have our non-linear differential equation denoted by F and this is the output equation or the measurement equation since we are measuring h, g is equal to h.

So how to compute the Jacobian a ij denotes the i jth element of matrix A, b ij denotes i jth element of matrix B and so on, so for computing the elements of matrix A we calculate dou f i by dou x j, so since there is only one element there and we can easily compute dou f i this is dou F by dou h so if we compute this we will get as minus R by 2 a root h ss at steady since this is at steady state at steady state we will get this value, this is dou F by dou u and u in this case is F I, so we will get this expression that is 1 by A, then we have calculating C, we have dou x by dou x which is h at steady state this will steady state and we will find it as 1 and similarly dou g by dou F i at steady state, we will find it as D equal to 0, so this is one way to compute the state matrices.

Now let us see how we can do the same thing in MATLAB and how we can study the response of the system to various inputs once we have the state space model of the system.

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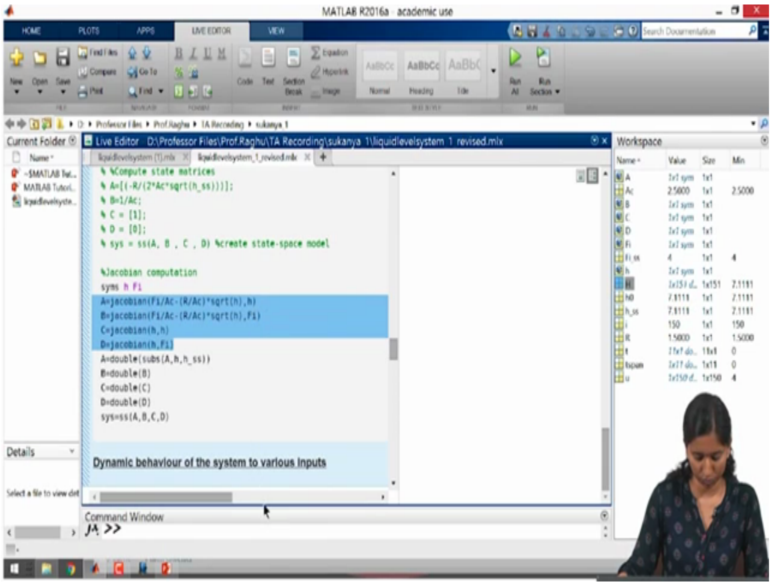

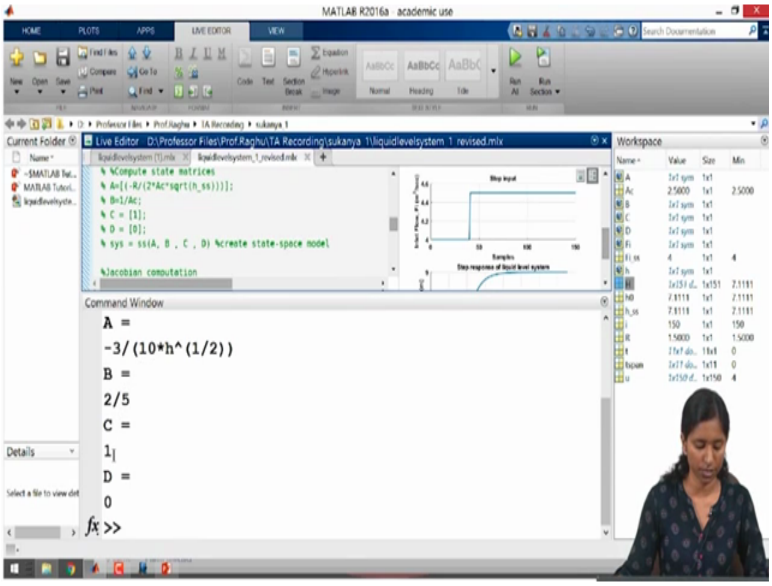

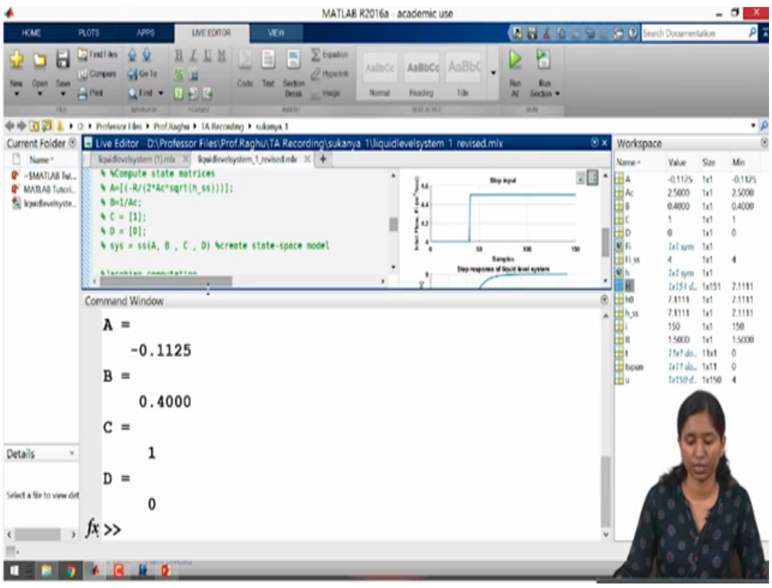

One way to do this is took write the expression for every matrix A B C and D write it down, right down to linearize perform the linearization by hand, write down the expressions for A B C and D and then compute or other way is we can compute the Jacobian using MATLAB command and then find the state space model. Let us see how we can use MATLAB determine the Jacobian so we can define the variables symbolically h and F i, F i is the input here so we can define them symbolically and then we can write down the expression only the non-linear expression and we can define the variable with respect to which we have to find the partial derivative and then we can find (())(29:57) Jacobian to compute it.

So what symbolic will do is it will give expressions it will give us mathematical expressions instead of numerical values. So let us see let us first define the variables as symbolic variables evaluate and let us perform these commands Jacobian calculating the Jacobian. So in the command window you can see that the expressions are calculated since I have already defined what my AC and R and all is it has taken these values but since I defined my h as a symbolic variable h came as h itself, so minus 3 by 10 root h that is what is obtained here, B is 2 by 5, C is 1 and D is 0.

Now we can we have to substitute the value of we have to compute this at the steady state value so we have to have absolute the steady state value of height in the expression for A. So to use to do that we use the command subs and we define in which expression we want to substitute will define which variable we want to substitute the value to and finally will give the value of the variable.

So and since all this is in symbolic format we want to convert into a numerical way so that one way to do that is used to command double. So let us you do that and convert every variable into double (evaluate) so as you can see A B C and D turned came as numerical values, we found out the matrices for state space model using this command.

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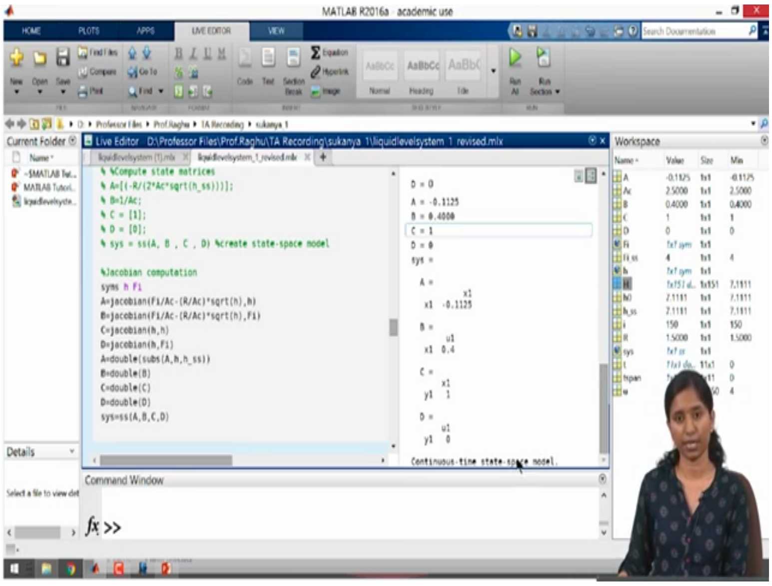

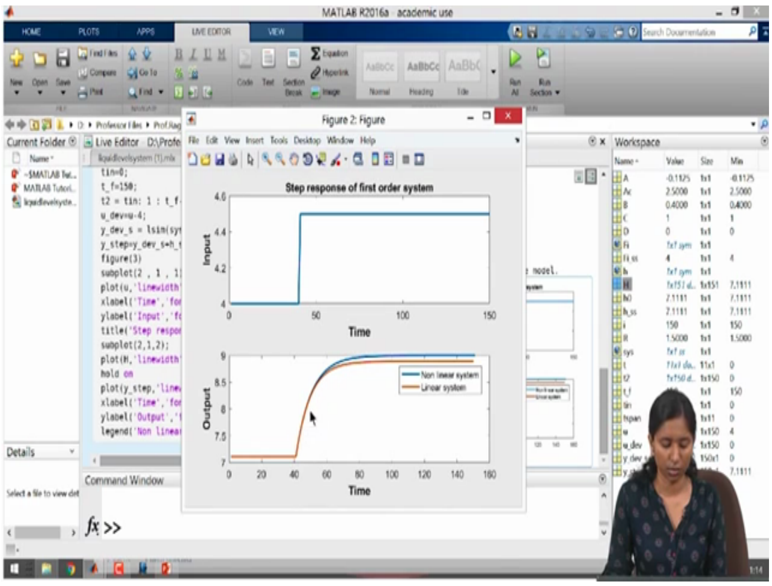

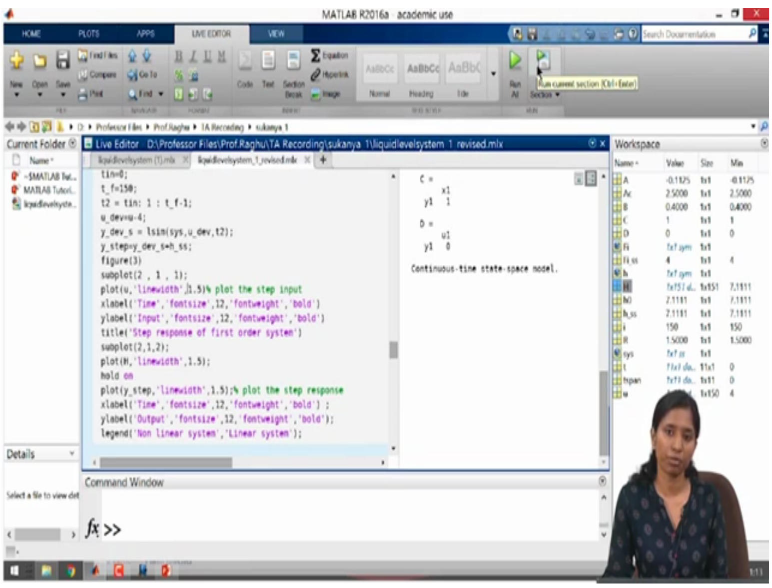

Now how to use this more computer values of the matrices to develop the state space model? There is an inbuilt command in MATLAB called ss which if we run we will get the state space model and we will save it into the variable called system. So when we want to simulate the response and study the dynamic behaviour of the model we can just call this variable use this variable ss.

So this is how it came A is minus 3 by 10 root h, B is 2.5, C is 1, D is 0 then we calculate we converted them in numerical values and found out their values so that is ABC and D and then this is our system, system is defined by the state space matrices that is ABC and D, so we found out continuous time state space model. Now let us just have a look at how the system response will be to various types of process inputs, one process input that we already know is a step input that we have already seen before but that was given to a nonlinear system.

Now let us see how the response will be to when we give a step input to the linear model of the liquid level system, we will compare those two, we have to define the time vector for simulation that is start initially time and the final time so 150, so T 2 is the time vector with the initial time of 0 and final time 150, since our state space model is in terms of deviation variables we have to specify the inputs also in terms of deviation variables. So that is the reason we are subtracting our steady state value of the input which is here (())(33:33) from the input that we have created before the step input, so we had defined the deviation input deviation variable (UD) u underscore dev.

Now let us simulate now lsim this particular command lsim is used to simulate the response of the system to arbitrary inputs and here we have specified the input as the step input so this is our system defined here, next we have defined that what kind of input that we are giving, we have already generated the step input and we have subtracted the steady state value from it so this is the deviation variable and we have specified it here and we will we have to specify the time vector also for simulation.

So we have defined so here and the response of the system will be saved in this valid command in this variable y dev underscore s, but this is also in terms of deviation variables if you really want to see what is a new value of the height we have to add the steady state value to this deviation variable and then plot the figure, so (this is the command) these are the commands to plot the figure have used the command subplot here subplot what it will do is it will plot two plots in the same figure.

So let us run this section and see how the step response of the linear model is with command compared to the non-linear model so this is the figure as we can see I gave a step change at 40th time instant and new value is new 4.5 and the orange line here denotes the linear system response a linear system and the blue line denotes a response to the non-linear system.

So you can see that there is a varying slight difference in the steady state values final steady state values of the linear and the nonlinear system, which means that our approximation of the non-linear system as a linear system are linearize around the steady state operating point is okay we can live with it, it is not like it is wrong or anything, it is quite close to the actual performance of the non-linear system so the linear system is quite close to the non-linear system the response when be linearized around the steady-state operating point and here the time behaviour asked time proceeds the way that both linear system, non-linear system behaves is very similar in this particular section.

Now let us see how the process response will mean if we linearize a nonlinear model around some other different operating points. So I have I have defined my model here so first I have to change the operating point so I have changed my steady state value as new steady state value h ss underscore 2 is equal to 15 and I have computed this here my state matrix which I have to compute for the newest operating condition, then I have to simulate the step response which is quite similar to what we did before only thing is that the operating conditions change and the model is linearized around this particular different operating point.

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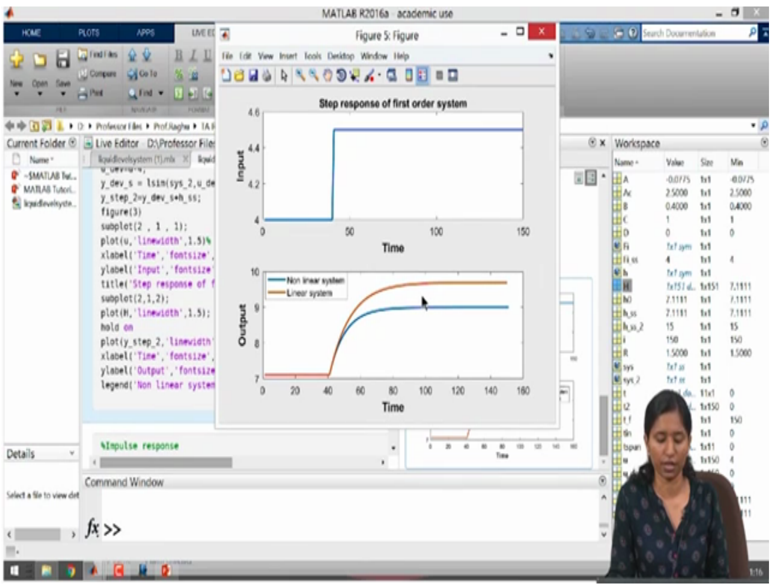

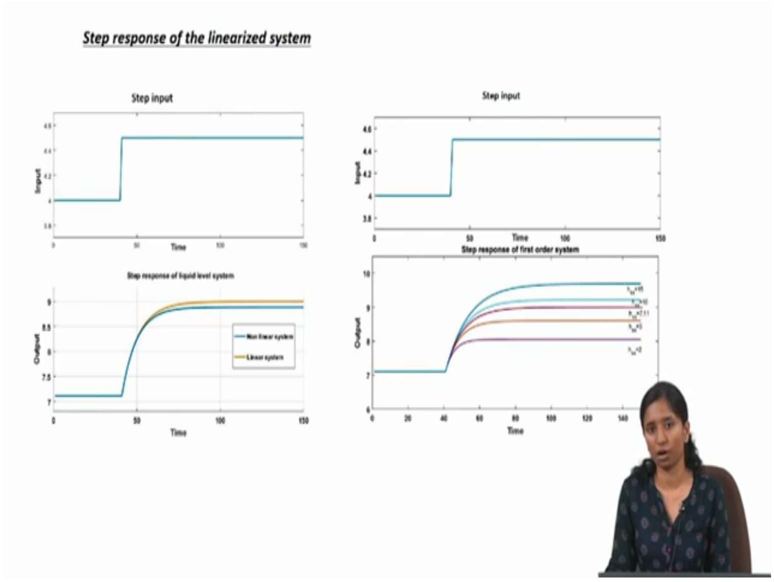

So once we do that let us see how the response will be as you can see this blue line here denotes a nonlinear system which was which hosts a new steady state value is close to 9, but since I changed my since I changed my steady state operating point and linearize around this new operating point the behaviour is very far away from the actual steady state of a nonlinear system.

So this is how the response of the first order system liquid level system is to different steady state values and a steady state value was 2 this is the response, when it was 5 this is a response, when it was 7.11 the response is this and as the as the operating conditions are changed we can see that the approximation of the nonlinear model as a linear model is quite far away from the actual model. So we cannot then approximate the nonlinear model as linear model as a behaviour is very very different from how a nonlinear model would behave.

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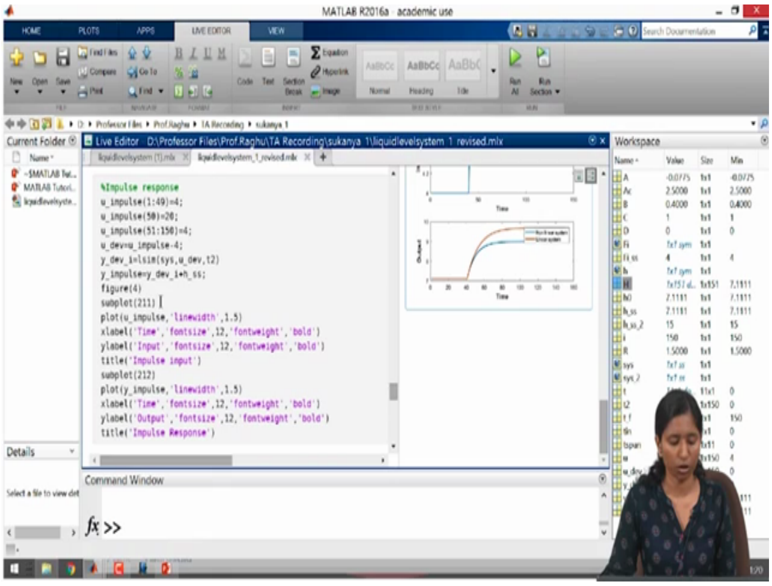

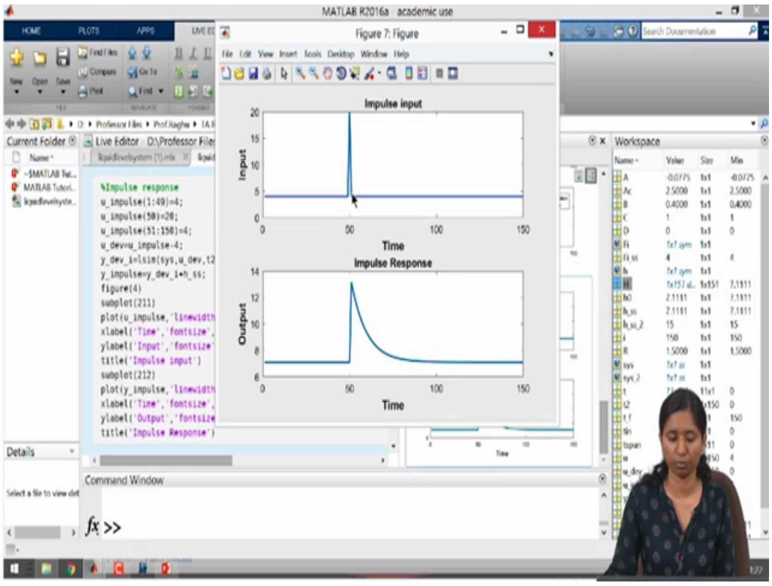

Now let's look at another kind of input called the impulse input and how the system responds when an impulse input is given to a system. So usually in industries impulse inputs are very short energy inputs given to the process, which is like inserting dye in a pipeline which is a very infinitesimally small duration. So first we have to simulate the impulse input, so I have put my impulse input till 49th sample as 4 which is my steady state value of the input that is in flow rate and the 50th sample I have changed it to 20 which is the impulse input magnitude and from for the rest of the samples I have removed that change and have kept it as my as 4 which is the inflow rate itself.

Now again here I have to give the input in terms of deviation variables so I have to subtract 4 from the impulse input that I have created then I using this else in command I have given this impulse input into the system and generated the output and since its terms of deviation variables I have added the steady state value to the output that I have obtained out of simulation to generate me output. So let us plot this and see how the impulse input and the impulse response looks like.

So this is how our impulse response looks like at the 50th sample I gave a impulse input of magnitude 20 and then I removed it to that instant itself, but as we can see the response the liquid level shot up but it shows a exponential decrease instead of an instantaneous decrease, so this is how the impulse response looks like. So when the impulse input was given the liquid level got increased as a burst for a short duration of time and then it started decreasing in an exponential manner, rather than the instantaneous manner in which the input was removed.

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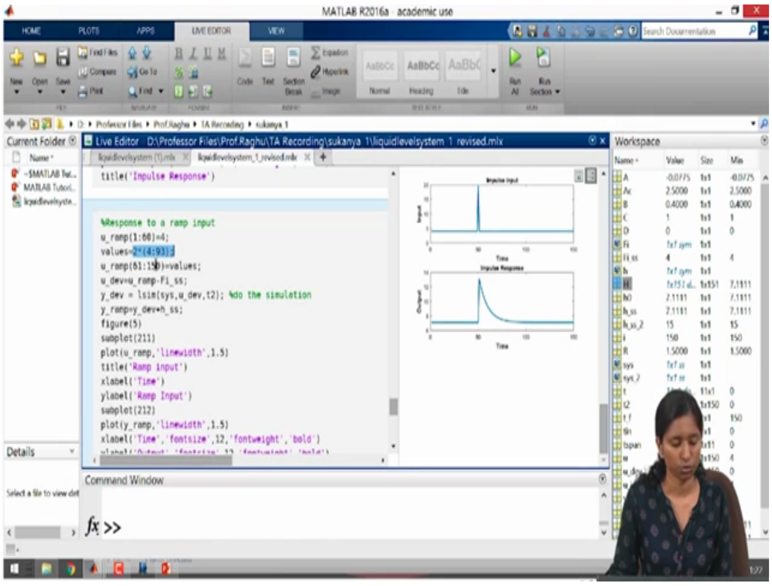

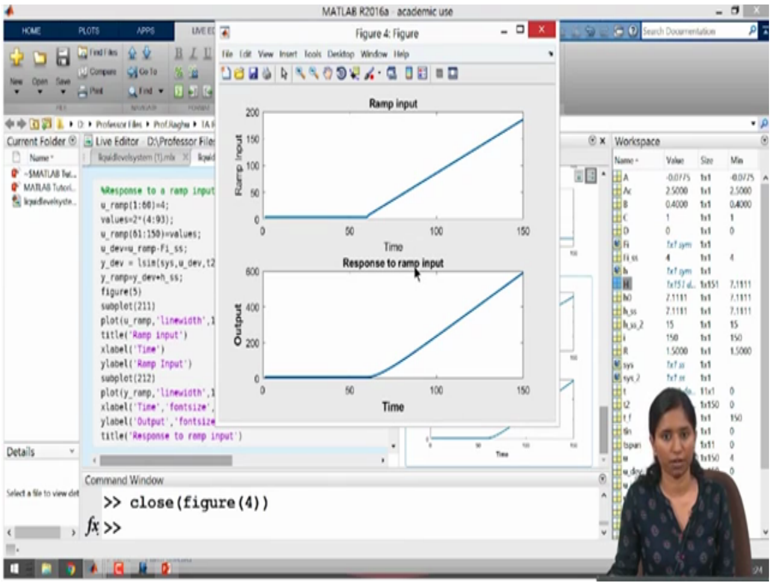

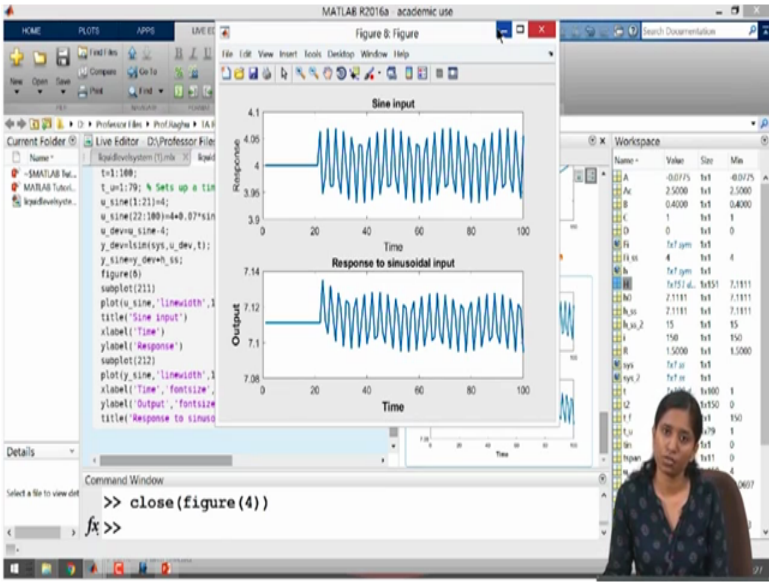

Another kind of input which is quite commonly found in industry is the ramp input, the ramp is nothing but a kind of a drift in the input signal it is slow varying drift in the input signal and now let us see how we can generate a ramp signal so till my 60th sample the value of the input is steady state value itself and then from 61st sample they started increasing and it is given by this expression 2 times samples from 4 to 93. So I have ramped up the values from 60st sample and I have simulated the response.

So let us look at how the system behaves when a ramp signal is given, you can see that at the 60th sample I have ramped up my input like this the response were ramp input the level liquid level slatted slowly increasing and the magnitude is also very different. So when a ramp input is given when a slow drift in input is given the output also increases in similar fashion.

Finally let us see how the system behaves when a sinusoidal signal is given as an input and we can define the sine wave in this following manner till 21st sample it is the steady state value, at the 22nd sample I have given a sine wave and this sine wave in the sine change sinusoidal change in the input. So the amplitude of sinusoidal change is 0.7 and the angular frequency is 2, I have given the time I have defined the time vector in this step.

So again I have to subtract 4 to put it in my in the deviation variable form then simulate it and then finally add the steady state value of height to the response to get it in terms of the original variables. So let us see how the height of the liquid varies when the inflow rate varies in a sinusoidal fashion, as you can see here at the 21st sample I started changing if I start changing my inflow rate in a sinusoidal fashion liquid level within the tank varies in a sinusoidal fashion, but change the amplitude as well as a phase shift is can be observed in the sinusoidal response of the system where it is not quite evident here and this particular response is very important and quite widely studied in control system control design and analysis of control systems and it is called frequency response analysis.

We look into it in much more detail in the upcoming tutorials for now what we have seen in this tutorial is how to model a system using differential equations, how to solve the differential equation and simulate the response, how to model system, how to linearize the system, how to model a system using state space model in MATLAB, then how to simulate the response of the system to various kinds of inputs, there are much more implications to the response of the system to all these inputs, step response, impulse response, sinusoidal response all these are quite important and will be study it in detail upcoming tutorials, I hope this tutorial is quite informative to you thank you.