Process Control - Design, Analysis and Assessment
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Lecture 12
Stability

We will continue with our 12th lecture and in this lecture I am going to introduce the notion of stability for the first time, so we can talk about the stability of control systems from both open loop and close loop perspective but as far as I am concerned once we explain the notion of stability whether it is open loop or close loop stability the ideas are pretty much the same except there will be difference in the transfer function that you are looking at.

So basically what I will try and do in this lecture is explained to you when a system is stable and will draw upon what we have learnt in terms of analysis of transfer functions using partial fraction expansion and you will see the power of that idea when we try to understand stability.

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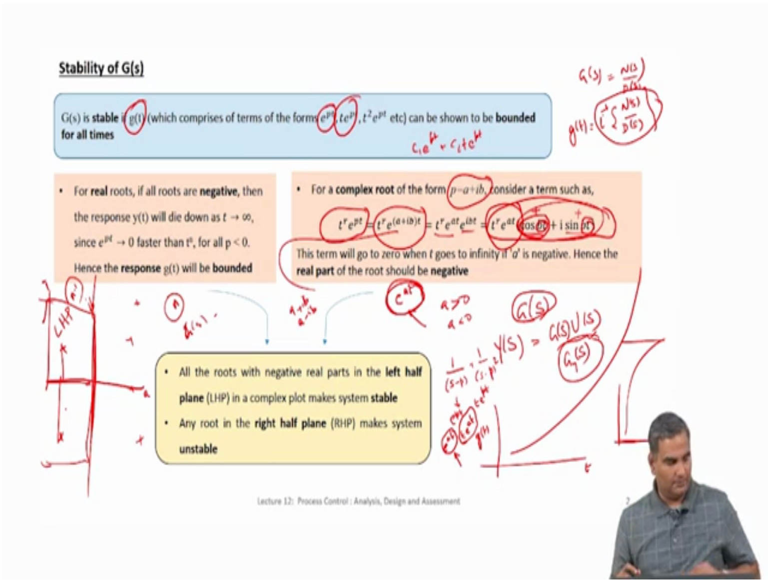

So any system now which in time domain I said would be ordinary differential equations modelled as ordinary differential equations and once we do the Laplace transforms these equations get converted into what we call as transfer functions and if you talk about stability of a system we talked about the stability properties of the ordinary differential equations that are used to model the system and consequently we talked about the stability of G of s which is the transfer function model.

So we will just talk about stability of transfer functions and you will notice that once we understand the stability of transfer functions then basically we do not really have to worry about stability of what Laplace variable we are interested in so what I mean by this is if I explain to you what the stability of this G of s is how do you understand the stability of G of s then if you define stability as output stability that is would my output be stable and we will have to explain what stable means then basically what you are looking for is a stability of Y of s which is not any different from G of s it is just that Y of s is G of s times U of s that we have seen before and I could call this as some G Y of s right.

So whatever ideas work here will also work here, so what we want to know is in general how do you understand stability for transfer functions so what I am going to do is I am going to explain stability in an intuitive fashion however it does not mean that we are going to any hand waving all the results are exact and nothing that we leave out but because of the way in which we have shown how to expand these transfer functions in terms of partial fractions it makes it very easy for us to define what stability means.

So let us start if we talk about the stability of the transfer function G of s we are going to say this G of s is stable if the corresponding g of t which you will get from inverse Laplace which is a time domain function is bounded for all times, so what do we mean by bounded? So if I have let us say simple plane and understanding of this is if I have t if I have of g of t and I plot this it should not go like this and go to infinity righ.

So I should be able to say g of t will be between you know certain values ok, so that is the idea of g of t being bounded, so as t tends to infinity g of t cannot tend to infinity it has to be bounded between certain bonds. So how do we conceptually think about stability of G of s while this definition of stability very simple English language definition of stability it is very understandable we are just saying this does not go to infinity right however how do I understand this mathematically is an important question that we should address.

So remember that then we want to get G of s we write this G of s some numerator by denominator and then g of t is Laplace inverse of this numerator by denominator and we spend enough time in trying to explain to you that this numerator by denominator I can expand it in terms of partial fractions the only type of terms that I get in the partial fraction are e power pt, so this type of term I will get if the root p is repeated only once that is it is a distinct root or I will also get this term if p is let us say repeat it twice the first term will look like this but the second term will look like this right t power pt.

So remember we had this rule where if a root is repeated k i times I will have k i terms and the last term of that sequence will be s minus p to the power k i and if I do the inverse Laplace of that then I will get time to the power k i minus 1 e power pit and so on. So if you look at this and the type of terms that you are going to get when I do this expansion right here the time terms are going to be e power ptt power ptt square e power ptt cube e power pt and so on, of course around these terms there are going to be some constant but those are constant numbers so they are not going to be really influencing whether g of t is going to be bounded or unbounded.

So basically in general you can think of this as having c 1 e power pt c 2 t e power pt kind of terms right, c 3 t square e power pt and so on. Now we can look at each of these terms individually and then look at these terms and ask the question when would these terms go to infinity as time tends to infinity. So let us take a generic term which is t power r e power pt and let us also assume in general the route can be complex we will write the expansion here and then really talk about each one of these things carefully and we can talk about what happens if the route is real? What happens if there is this pole is distinct and so on by looking at this generic term?

So as I said before the expansion of g t or the time function of g t is going to have several terms which are of the form t power r e pt r is 0 means you will get e power pt, r is 1 you will get t power pt, r is 2 t square e power pt and so on. So you are not going to get any term outside of this form right because remember we are doing this for N s or D s, if your transfer function is not of the form N s or D s where N s and D s are polynomials there the order of D s is greater than the order of N s then you have to think about other things but as far as we are concerned till now and I have shown you that most of the transfer function models still now that we are interested in are always having this form that basically means that the some of these terms can only have functional forms like this.

So let us take a generic functional form like this and then ask the question as t tends to infinity what will happen to this term, now if every one of these terms as t tends to infinity and does not go unbounded then we are in a good shape right but if there is some reason why if a (())(07:08) tends to infinity some of these terms become unbounded then we say the system is not stable.

So let us expand this now let us say T power r e power a plus ibt, now if this particular pole is distinct ok and also real then that basically means that since it is distinct I will never have the t term because if it is distinct I will have only 1 by s minus p which will be e power pt, so r equal to 1 for this root. Now if you also assume that it is real then this term will simply boil down to e power at, so this is for poles that are real and distinct.

So poles that are real and distinct will only be able to generate terms like this ok, so that is an important thing to remember the notion of distinct is it is not repeated so it can have only one term which is 1 by s minus p so that will just be e power pt so r is 1 and here since it is real I am setting b equals 0. So if your pole is distinct and real then we can ask the question this is one term in the sum and what will happen to this term e power at ok, now if a is greater than 0 then this will become unbounded as t tends to infinity right, so if it is e power 2 t or e power point 5 t and so on, so this will go to infinity however if a is less than zero then this e power at term is going to be going to 0 as t tends to infinity.

So as long as the real part is less than 0, so we will come to the e equal to line later, so right now let us just talk about less or greater e equal to if you look at this if you put a equal to 0 this will be 1 so it is not unbounded, it is a bounded number but for now we will focus on just the greater than 0 and less than 0. So if I plot these roots in a complex plane because the solution can be complex also, so the real part will be here the complex part ib will be here.

So what this says is if the root is real and if you want that term not to go to infinity then basically it has to be on this side of this line and I am drawing this line because we have assumed it is real, so this ib does not come into picture. So for a root that is real and distinct if a is less than 0 that term will never blow up to infinity ok, so that is something that you should keep in mind. Now we are going to do this step by step and then we will write the final result which is in this form.

Now if the root is real but it is not distinct let us say for example it is repeated twice ok then the two terms I am going to get is I am going to get s minus p plus 1 by s minus p whole square because it is repeated twice I will get s minus p and s minus p square and we already have said many times this will give me a term e power t and this will give me a term e power t e power t and if p is real then it will become e power at and t e power at, so these are two terms in the g t expansion that you will get.

We have already discussed this term as long as a is in the less than 0 or to this side of this line then we know that this term will tend to 0 as t tends to infinity mathematically it can also be shown that if you have a term like t e power at if a is less than 0 this t e power at will also tend to 0 as t tends to infinity this can be mathematically shown, not only this you can also show t square e power at will tend to 0 as t tends to infinity if a is less than 0 and so on.

So as long as you have a finite power of t and right e t to the power e power at and if a is less than 0 then all of those terms will go to 0, so what this basically means is essentially that it does not matter how many times the real route repeats as long as the real part or the real route is less than zero this the terms that come out of these routes can never make g t go to infinity, so these will still make g t stable, so that is a key idea that that you have to remember.

So let me repeat so if a route is real it does not matter whether it is distinct or repeated many times as long as a is less than 0 then you will have stability as t tends to infinity all of the is terms that come out of these expansions will all go to 0. Now let us take the case where I have a complex root it is distinct ok, so if I have a complex root that is distinct that means I am saying I have a plus ib as one root but remember if a plus ib is one root then a minus ib also has to be another root a complex conjugate, so whatever analysis I show for a plus ib will also work for a minus ib.

So let us take this t r e power a plus ib times t because now the pole is imaginary now if you take this t power r let us say this pole is repeated r times and now we can do this analysis without talking about distinct and non-distinct and so on, you will see why quickly then I have t power r e power a plus ibt which I can expand as t power r e power at e power ibt and you know e power i theta is Cos theta plus i sine theta, so e power ibt can be expanded as Cos bt plus i sine bt.

Now if you look at this term that has come out of an imaginary or a complex root which is repeated it is not repeated r will be 1 but we can address the whole case together because we have already talked about repeated roots when we talked about real roots. Now so there will be terms like this right and what we are doing is we are going term by term right and then seeing whether any of these terms can actually go to infinity, so we are looking at it carefully in terms of term by term and when I look at this then I say ok let us look at this as t tends to infinity.

So though the argument inside Cos goes to infinity Cos itself is a bounded function, so this cannot go unbounded similarly while the argument here to can keep going to infinity sine itself is a bounded function so this cannot be unbounded, so this whole thing can never be unbounded it can be oscillatory which is what we will see later but it cannot be unbounded.

So really whatever b the be it does not matter so what it says is irrespective of the b right whether b is positive, negative does not matter and whether t tends to infinity does not matter because these two terms are going to be bounded the stability of g of t depends only on this and this is like the real root case that we have already talked about as long as a is negative this term can never be bounded so this term will go to 0 as t tends to infinity, so because of this irrespective of b being positive or negative anytime you have a negative you will get the system to be stable.

So as long as all your roots are here ok, which is always negative and irrespective of the value of b, so b can be positive negative does not matter so this whole side is what is called left half plane LHP, so this is a thing that people use in control so as long as all your roots are in the LHP, so it does not matter whether it is here, here, here and so on because for all of this if you pull this down the real part is always negative.

So as long as the real part is negative those terms cannot create any problems so g of t will be stable however if even one root of G of s is in the right half plane it does not matter where it is it is here, here it does not matter even if one root is on the right half plane the system will become stable because I might have n root n minus one of these could be on this side only one root is on this side however when I expand this G of s I will have a term for each of these n minus 1 and I will have one term for this all of which are some so n minus 1 terms will go to 0 but the nth term because this a becomes positive will necessarily go to infinity as t tends to infinity.

So that is what this thing says here all the roots with negative real parts in the left half plane in a complex plot makes system stable that is if you have all your roots only in left half plane it makes it stable however even if you have one root in the right half plane then it makes the system unstable, so this is a result that you might have seen before and this is the logic behind this result and notice how I do not have to do anything more this is a very general result right because I am assuming my function is of the form N s or D as I have done partial fractions there is no error in the partial fractions, there is no approximation in the partial fraction so if these are true then it has to be necessarily true so this is the main result of stability.

So if I have a transfer function G of s which can be written as a numerator transfer function by denominator transfer function if I find the roots of the denominator transfer function and if I find all the roots of this denominator transfer function are in the left half plane which basically means the real part of that root is strictly negative then I call the system stable.

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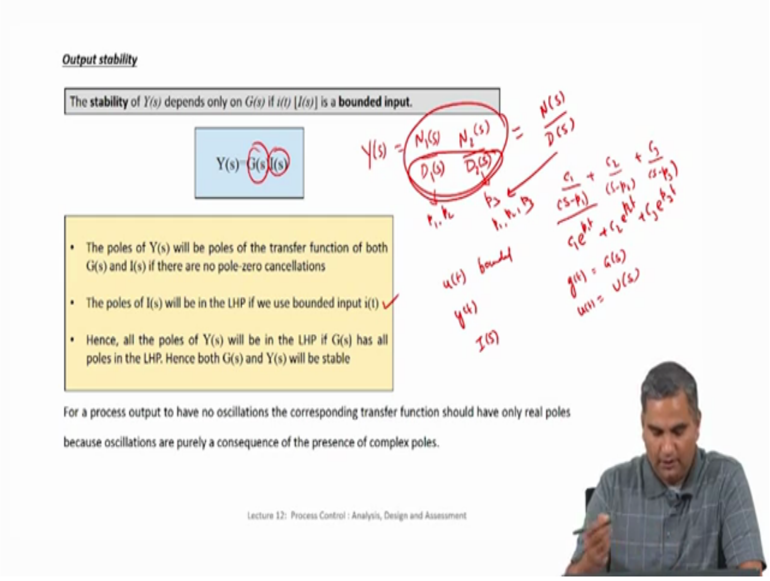

Now as I said before you can also talk about stability of Y of s, now when you talk about stability of Y of s we have two terms here, so initially we only talked about stability of G of s right now I have Y of s which is G of s times I of s, so while we can talk about stability of G of s it is not necessary that Y of s stability depends only on G of s right because Y of s stability will also depend on I of s ok.

So to understand this for example if I write Y of s as some numerator by denominator and I of s also as some numerator by denominator now if you think of this itself as a whole Y of s transfer function as long as there are no cancellations in terms of routes between N 1 s and D 2 s or N 2 s and D 1 s is there are no poles zero cancellations as they call it for this transfer function the roots of this transfer function will be a collection of roots of D 1 s and D 2 s ok. So just for the sake of illustration let us say D 1 has as two roots p 1 p 2 D 2 has as one root p 3 and as long as there is no cancellation if I write this Y of s as some N of s by D of s clearly the D of s polynomial will have all the roots p 1 p 2 p 3, so when you expand this in partial fraction you are going to get something like c 1 by s minus p 1 I am assuming each of these are distinct roots c 2 by s minus P 2 plus c 3 by s minus P 3 and when you do the Laplace inversion of this you are going to get c 1 e power p1 t plus c 2 e power p 2 t plus c 3 e power p 3 t right.

So what you need to understand this every root of the denominator polynomial in G of s will introduce one term and if it is repeated you will introduce as many terms as a repeat and every route in the I of s transfer function denominator also will introduce a term, so if you want Y of s to be stable basically we are looking at this whole sum of terms to be stable that means I of s also has to be stable ok.

So when we come to Y of s we use this notion of bounded input bounded output stability, so what we are saying is if u t is bounded then is y t bounded right, so given that u t is bounded when is y t bounded, so can I say for every bounded input y t will be bounded. Now in the last slide we saw for g of t to be bounded he said G of s should have all the poles in the left half plane.

So if the poles of I of s or U of s is in the left half plane then we call that as bounded u t or i t, now, now I am using this term of s here because this y need not be written in terms of just U of s that is the open loop transfer function later we will see that Y of s can be written in terms of some transfer function times a disturbance transfer function or some transfer function times a set point transfer function and so on.

So I have generalized that and then I am calling it as I of s, so as long as this input I of s has all the poles in left half plane then we will have a bounded input, so when we have such a bounded input then the output will also be bounded if all the poles of G of s are in the left half plane, so that is an important idea right. So a BIBO stability would mean that my poles in the G of s should be in the left half plane assuming that I have bounded input that is the reason why I call it bounded input bounded output stability.

So notice how when we talk about the stability of g of t itself which where we talked about the stability of G of s correspondingly the stability of Y of s just becomes the stability of G of s if I of s is bounded right, so if I of s is bounded we are only worried about G of s right. Now the reason why I left that line in the middle which I did not talk about we will come to later.

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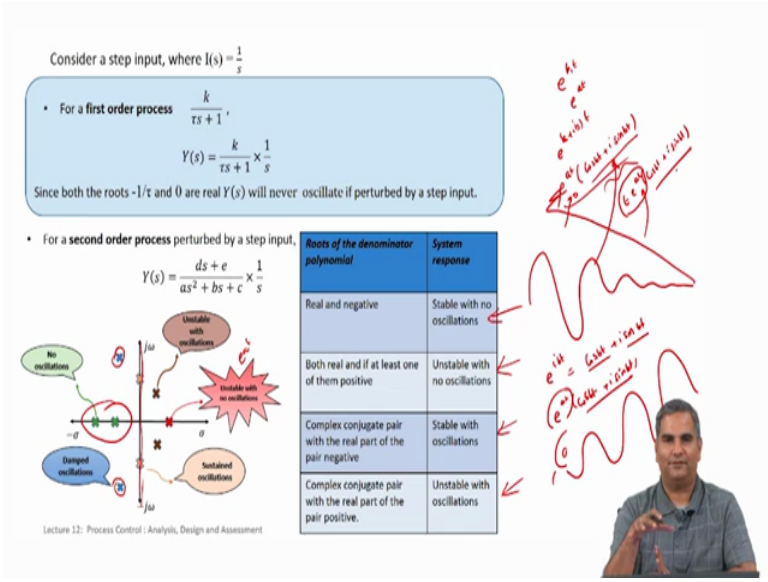

So I will introduce an interesting idea of how we think about that imaginary line and what happens if G of s has pole on the imaginary line and that is a very interesting idea and we will get back to this and understand that well as we go along but just remember that the BIBO stability really depends on G of s, if U of s is bounded and we call U of s bounded when we have this the poles of U of s or I of s in the left half plane again you could also ask the question what happens if one of the poles of I of s is on the imaginary axis that could still be a bounded input function I have left that now because I am going to come back to that it introduces some interesting ideas and we will see that presently.

So let us take a quick look at how some of these come together, so when we look at the behaviour of let us say Y of s then in typical terms you could have let us say a system which responds which is stable with no oscillations it could be unstable with no oscillations it could be stable with oscillations it could be unstable with oscillations and so on, so these are possibilities when we talk about stability in oscillations and then if you want to understand from a transfer function viewpoint how does all of this come about it is a simple idea here.

So if you have let us say a transfer function where the poles are strictly in the left half plane and let us take the case where the poles are on the real axis in the left half plane then you know whenever I have poles like this the kind of terms I will get will be just e power p 1 t and since this pole is on the real line I am going to get something like e power at where a is negative, so I cannot get any oscillations if the poles are strictly on the real line on the left half plane however if I have poles like this right then what I am going to get is I am going to get terms of the form a plus ib times t this is going to be e power at Cos bt plus i sine bt.

Now as t tends to infinity because I am showing this poles on the left half plane a is negative so this is going to go to 0 but it will take a while before it goes to 0 however in the meanwhile because of the Cos and sine terms I will have oscillations so I will have oscillations which are damped which means that I start like this and then the oscillation will keep decreasing till it goes 0 so that is why I get damped oscillations, these oscillations come from this term and the damping out or dying out comes from the e power a theta if a is negative.

Now if I have poles roots of G of s only on the imaginary axis then I could have a 0 so I will have just e power ibt which will give me Cos bt plus i sine bt, now if you notice that this is oscillating this is oscillating because it is directly on the imaginary axis there is no e power at term to either let it die or let it increase, so basically this will be sustained oscillations whenever you have poles on the imaginary line and now the extensions are quite simple if you have a pole on the real line on the right half plane this will be unstable because I will have e power a theta where a is positive, so astT tends to infinity it will go to infinity and if I have on the right half plane I have imaginary part to the pole root also then I will get e power again at Cos bt plus I sine bt, so these terms will introduce oscillation this will keep increasing, so you will have oscillations which keep growing and then becoming unstable.

So these kinds of plots which you have seen before you can quite easily understand when you think about this simply in terms of partial fractions right you do not need any other mathematical machinery other than this partial fraction idea because all you are looking for is terms of the form t power r e pt that is it right and this here I explained without repeats and so on.

So the same idea is valid if you have roots that repeat for example if I had one imaginary root that repeats twice then I will have terms such as this and I will have another extra term t e power at Cos bt plus i sine bt for the repeat, now notice that this oscillations will still be there here and this term we said will die down to 0 if a is negative, so the same idea works so it will you will have oscillations which get damp as t tends to infinity this term will go to 0 and this oscillation will get damned out.

So look how beautifully we can understand all the behaviour with just partial fractions and I do not have to understand anything other than the fact that every term in this expansion is of the form t power r e pt.

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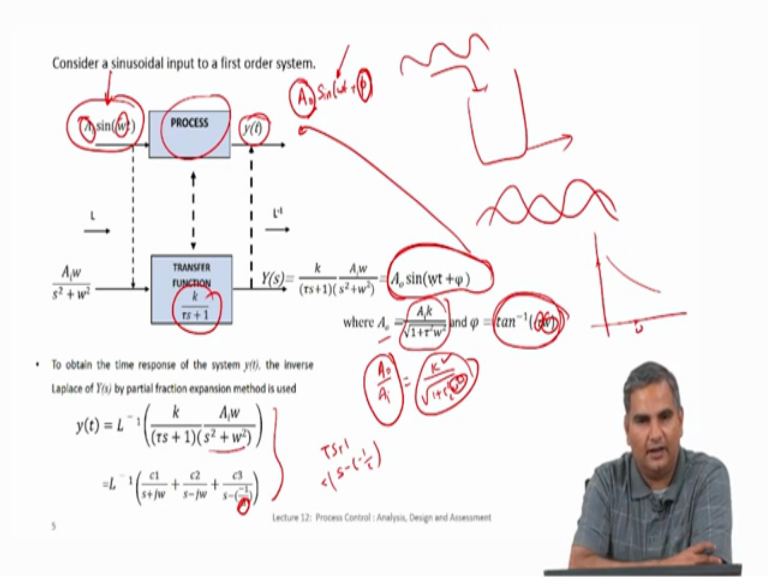

So there is a particular input function that is of importance in control systems this is called frequency response analysis of the system we will come back to this in more detail later. So typically if you have a physical process what you could do is you could give a sign input to the process and then see how the output looks so remember the tank example that we keep talking about there is an outlet there is an inlet now if you keep increasing and decreasing the inlet and see what happens to the outlet you expect it to increase and decrease.

Now understanding how the output behaves for a signal like this is what is called as frequency response analysis and we call this a frequency response analysis because the input is let us say a sine wave then this w represents the frequency with which we are oscillating this input, so we want to see how this frequency oscillation changes the output. So the way we usually do let us illustrate this for a very simple first order example let us say I have a process which is first order then the corresponding transfer function is k over tau s plus 1 and remember I said the way you do it is you do the Laplace transform of the input to the Laplace domain and you will see from your table quite easily this is A i w s square plus w square Y of s is simply a product of this and this, so I will have tau s plus 1 A i w by s square plus w square.

Now if you were to do the partial fraction expansion method here are details I would let you work this out but if you finally simplify all of this you will get g of t in a time form like this I encourage you to really work this out for yourself and we will also give you a homework assignment on this so that you really practice this and understand this because this is very important from a control viewpoint where we talk about frequency response analysis.

So notice that for an input so now I translate this to here so for an input A i sine omega t I am going to get an output a naught sine omega t plus phi ok. Now the interesting thing to notice is what this is if you are process is linear if you perturb the process at some frequency omega r w, the output will also be perturbed at the same frequency omega r w, however the input amplitude will get modified to an output amplitude A naught and there will be something called a phase lag or lead that is introduced here from the sign ok.

So if I have something like this as the input sustained sign so the output could be lagged but at the same frequency as the input so that is what this is, now in this case you will notice we can generalize this later in this case you will notice that you can compute this A naught if you do all of this computation here as A i k divided by root of 1 plus tau square omega square so this will come out of this computation and there is a significance to this and this phi will come out as tan inverse minus t omega.

So now if you notice this expression right so I can take this A naught by A i, A i to the other side I have k divided by root of 1 plus tau square w square, now if you notice this this is what I am going to call as a gained ratio why do we call it as a gain ratio? Because I sent in an amplitude of A i but they out the output amplitude is A naught so A naught by A i tells me the gain in the amplitude and if you notice this gain in the amplitude is some function which has the parameters related to the transfer function itself k and tau and also w which is the frequency at which I send in Inlet.

So what it basically says is if you send the sine wave at different frequencies I am going to get different gain ratios because the gain ratio also becomes a function of the input frequency similarly if you look at the phase the phase is also a function of the transfer function parameter tau and again it is a function of the frequency of the input signal. So the upshot of all of this is when I perturb the system using a sine function of a certain frequency I notice that the output will also be of the same frequency however the gain of the system will dictate what will be the output amplitude.

So if you define the output amplitude by input amplitude as the gain right then that gain is a function of not only the transfer function parameters but also the frequency at which you send your input signal similarly the output is as going to be lead or lag from the input and how much that is again depends on not only the parameters in a transfer function but also the frequency at which the input is given.

So basically we will get back to this in much more detail but I thought I will introduce this to you here because we can give you some assignments to understand this better you could conceivably now say at each omega I will get a particular gain right, so because A naught by A i this function, so at different omega as I might get some gain like this so I could plot the gain as a function of omega and similarly I can plot the phase as a function of omega.

So basically what this means is that I am understanding how the output gain is going to change how much it is going to be lag or lead from the input as a function of the system at different frequencies, so we can generate plots like this and this is what is called frequency response analysis we will come back to this later however the key point that I want to explain here is all of this comes out of nothing more than the partial fraction expansion that you have.

So here you have this so the same way we write there is c 1 by so this has two roots plus j omega and minus j omega, so I write c 1 by s plus J Omega plus c 2 by s minus j omega plus c 3 by the root of this, so we can write this basically as this tau s plus 1 we can write as s minus, minus 1 over tau right, so then you can multiply this by a tau ok so that will get absorbed in your constant c 3 then you can actually do the partial fraction expansion you can get c 1 c 2 c 3 using the techniques that I taught you in the last class and then basically invert this and after you do all the algebra which is well not trivial it is also not very complex it is slightly laborious but if you do this and you get to this result here then you have really understood how when I give sine input to process I get a sine output of a certain amplitude and phase.

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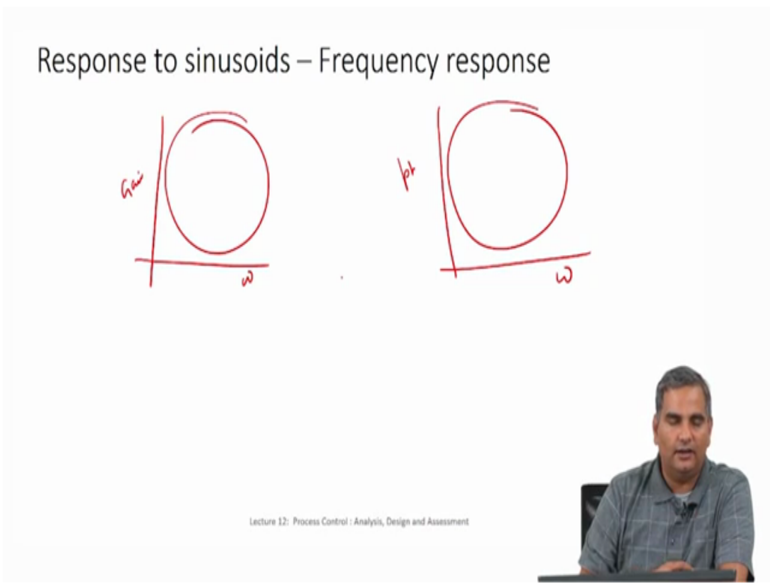

So this is what I called as frequency response analysis so you could basically plot the gain as a function of omega and the phase as a function of omega and the way this plot and this plot looks actually describes the underlying system, so you can understand what the underlying system is by looking at these two plots carefully and when you do that this is call the frequency response analysis of the system and now since I focus on the notion of partial fractions and doing this you do not need to know much more to analyse this lot more carefully because you could take a second order system and then see how the gain varies as a function of omega, how the phase varies as a function of omega and so on.

And there is some standardization in terms of how do you get the gain and the phase as a function of the transfer function which I am going to give that as a homework assignment for you to do I am not going to explain this here but if you do that then you will get a much better understanding of this on your own instead of me showing you exact the main result. So in fact you would be able to guess how the result will look based on what you got for the first order transfer function in the last slide.

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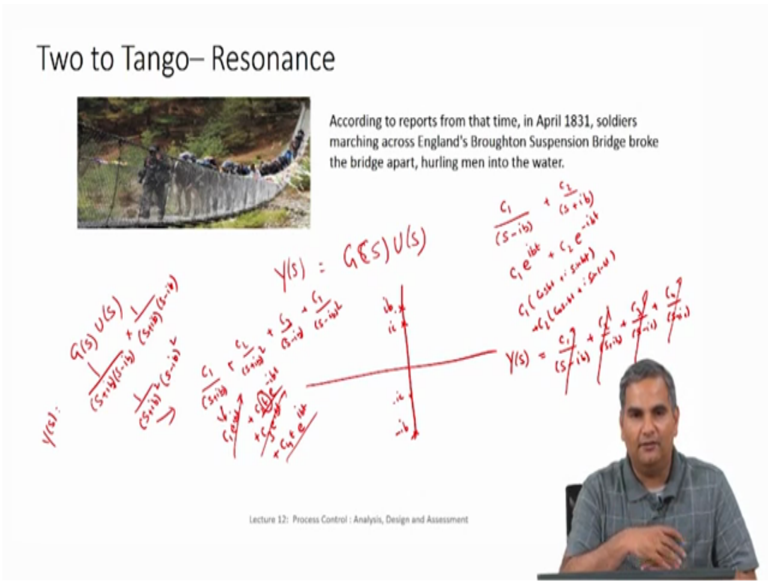

This also brings about another very interesting idea of resonance you might have heard that when soldiers march past and when they come to a bridge they are instructed to break their marching pattern and then walk normally and this is simply because the idea of resonance where if the natural frequency of the bridge and the frequency at which the soldiers are marching match each other than the bridge can break, so this is not something that is just theoretical supposed to happen in 1831 when soldiers were marching across England is Broughton suspension bridge this thing broke because of the frequency match.

So if you were to understand this phenomenon based on what we have seen so think about what does it mean to say the systems natural frequency so whenever you have an output Y of s so that is G of s times some U of s or I of s and let us say your system is having let us say two poles here on the imaginary axis right, so let us say if these are the two poles then this is ib this is minus ib and for each one of this when you do this expansion to get g of t you will get terms likes c 1 divided by s minus ib plus c2 divided by s plus ib, so this will give you c1 e power ibt and this will give you c to e power minus ibt and we have already done this before.

So this one will be c 1 cos bt plus i sine bt plus c 2 cos minus bt plus i sine minus bt, so you will notice that since all of these terms are bounded Cos sine functions, so there will be sustained oscillation so this is what we call as a natural oscillation of the system. Now imagine you have U of s which has a pole let us say at ic and minus ic then when you write this Y of s as G of s times U of s and I told you all the polls will be collected for Y of s so you will have something like c 1 by s minus ib plus c 2 by s plus ib plus c 3 by s minus ic plus c 4 by s minus s plus ic.

Now you notice that this will give you Cos and sine terms, this will give you Cos and sine terms this will give you Cos and sine terms and so on but all of them will add together and there will be oscillation but you will not have any term that goes to infinity right, so for any input where the frequency does not match the natural frequency you will have some of these terms and it will be such that they will all oscillate but they will never blow up to infinity but now imagine that the c is made into b right so basically what I have is I have G of s times U of s ok.

Now G of s has two roots s plus ib and s minus ib and when I write U of s let us assume that normally if it does not exactly match the system frequency it would be s plus ic and s minus ic but when I make c equal to b that means resonance right I have the same frequency input as the system frequency then I am going to have s plus ic times s minus ic, now something that strange or crazy happens.

So now if I make this b then the Y of s now becomes s plus ib square right s minus ib square now you know when you expand this in partial fraction you will have 1 by s plus ib right c 2 by s plus ib square and corresponding to this you will have c3 by s minus ib c 4 by s minus ib square now this term will not create any problems because this will be c 1 e power ibt but when you look at this term now and if you expand this, this will give you c 2 t e power minus ok here minus ibt minus ibt and you will get c 3 e power ibt plua c 4 t e power ibt ok.

Now this term and this term will not give you any problems but look at this term as t tends to infinity now this goes to infinity and this is bounded still right however because of this going to infinity these two terms will go to infinity so once it goes to infinity that means you have become unstable, so for every other frequency march past which does not coincide with the natural frequency I will have a stable system but when they exactly coincide then I will get instability and that is the reason why when they march pass and that frequency matches exactly are very close to the system frequency then you see the resonance and you do not have to go to infinity in a bridge like this if it oscillates quite a bit and the materials properties are such that at certain oscillations this thing can break.

So it is a very interesting and very nice way of understanding this notion of resonance from simple partial fraction expansion and see how just when these two poles exactly become the same you get this square term which introduces the t term in the time domain which shows that you can have behaviour that goes to instability.

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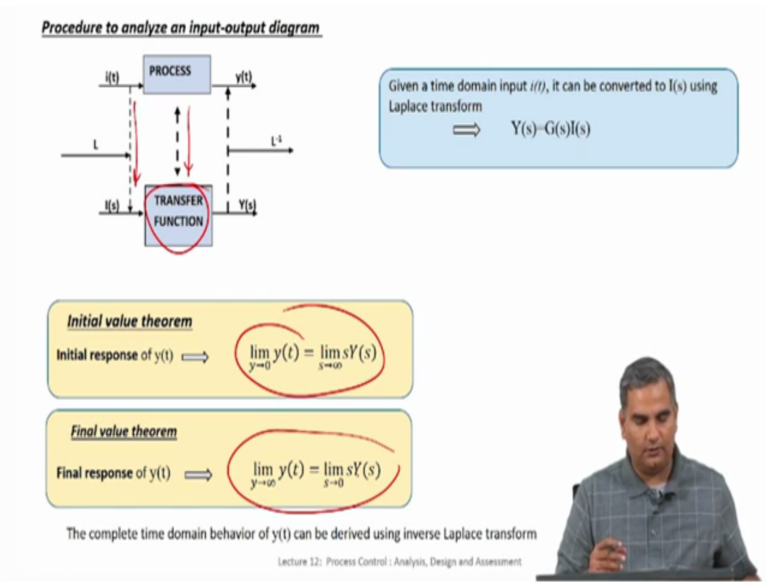

So if you put this all together and then we talk about a procedure to analyse an input output system so the way you do it is there is some input i of t to process and y of t is what you are interested in. So what you do is you get a Laplace transform transfer function your Laplace transform i of t to get I of s and Y of s is simply a multiplication of I of s times transfer function.

Now if you want to get the initial value of y of t you can use the initial value theorem if you want to get the final value you can do the final value theorem and so on. So this is how a time domain problem is converted to a frequency domain problem in process control. So with this I will end my lecture 12 and I will see you again for the next lecture, thank you.