Rheology of Complex Materials
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Lecture - 09
Velocity gradient and strain rate

In the previous few lectures, we have got some ideas about what are stresses and what are strains and strain rates in the material system. In this segment we will look at the velocity gradient and then the strain rate associated with a precise definition.

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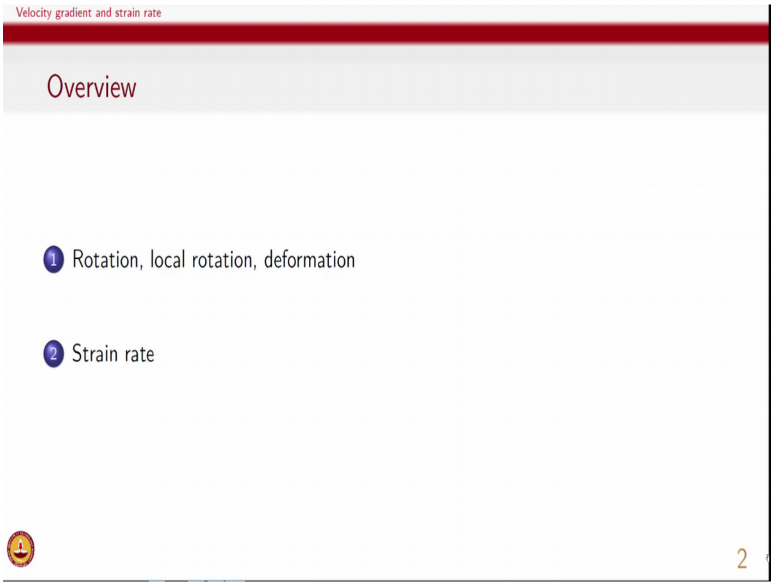

What we will do is we will also look at what is the difference; because velocity gradient contains information about rotation as well as deformation, while velocity at an individual point cannot tell us anything about either rotation or deformation.

So therefore, we will try to understand what is meant by rotation, what is meant by local rotation and what is deformation. And then we look at components of strain rate for couple of examples of flow.

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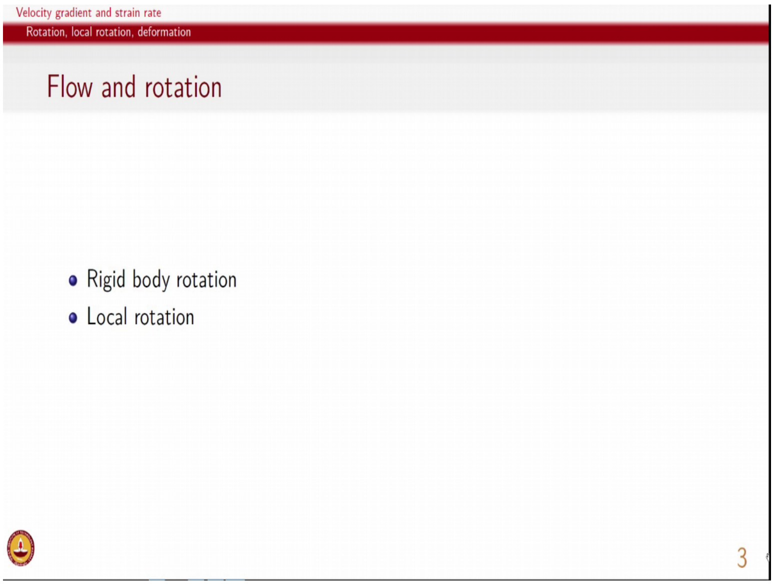

So, when you look at flow and rotation what we have is an individual material point; its velocity, its position as we have seen is defined as x tau.

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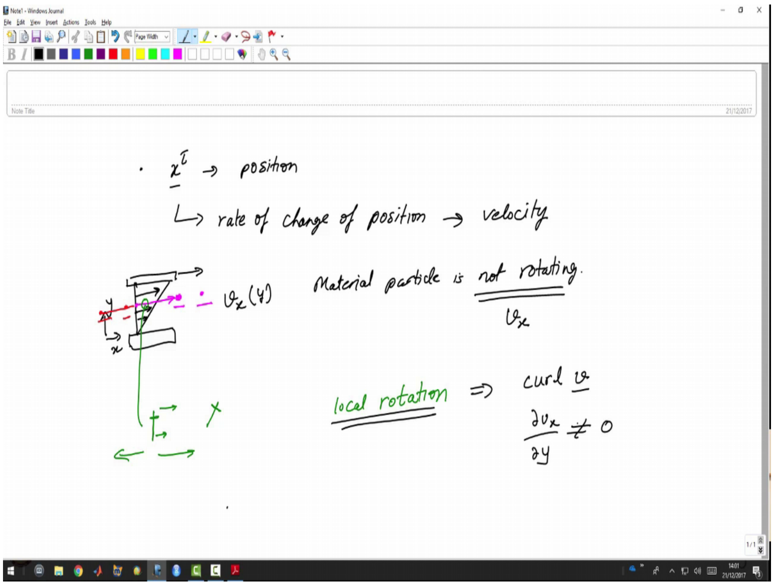

So, this is the position. And rate of change of this will give us the velocity; rate of change of this quantity position is velocity.

And so, the velocity tells us about what is the overall direction of flow. So, if we let us say have flow between two parallel plates which we have discussed several times and we will continue to discuss if the top plate is moving and bottom plate is stationary, then particles will be material particles will be moving in the x direction. And therefore, we will have velocity vx. And only the x component of the particle changes, and therefore if a material particle is here with time it will continue to move like this. And after some time it may occupy here and it may occupy here. And if you go sometime in the past then the same particle might have been here and it was in earlier. So, there is only x direction motion.

So, in this case we can clearly see that the overall motion itself is not rotational. So, the material particle itself is not rotating; so material particle is not rotating. However, locally what happens? So, if we let us say take a very small portion of this and then zoom out and the let us say I put a small object like this in the flow. So, what happens is the top end of the flow has slightly higher velocity than the bottom end. And of course, this overall object will move just away these material particles move. So, we saw that material particle was here and then it moved here or in the past it was in some other two locations. So, basically there is a motion; so therefore this green object will also move. So, with sometime it will move to towards the right and in the past sometime it was towards the left.

But you can see that with this small object the top end of it has a higher velocity. So therefore, this object will have a tendency to tumble or it will have there is a local rotation in this flow. So, overall we can see that the material particle is not moving in a rotating motion, so because there is only vx but locally every neighboring set of elements are experiencing local rotation. This local rotation is also related to curl of velocity. And if you remember curl of velocity will have terms like del vx by del y. And in this case we know vx is a function of y. So, del vx by del y is non-zero. So therefore, local rotation which is related to curl of velocity is also non-zero.

We can also look at another situation where let us say there is a rotation.

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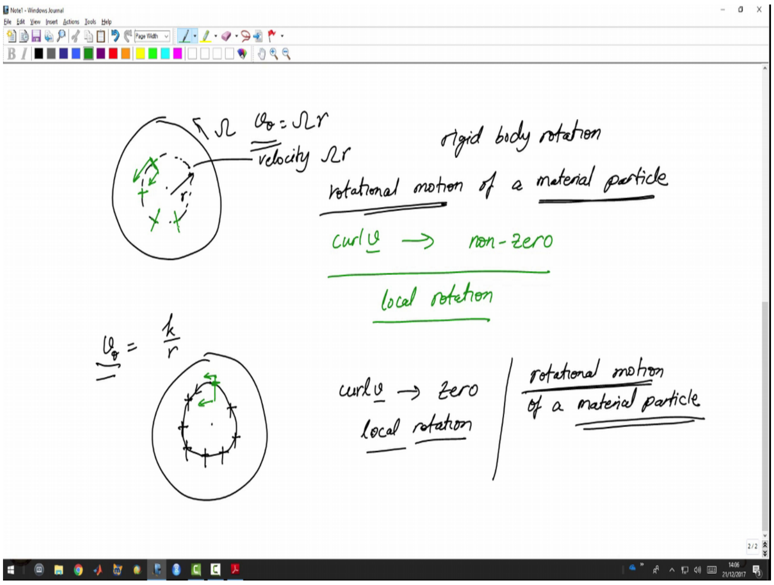

In this case we have let us say a disc and this disc itself is rotating. And let us say if the rotation rate is omega, then we know anywhere if the distance is r then the velocity here is omega r. So, this is called a rigid body rotation. So, in this case the material particle keeps on rotating, and so there is an overall rotational motion; rotational motion of a material particle. But if I let us say embed an object at any given location. So, let us like in the last time we embed a small green object here; what will happen is this object will rotate and it will really tumble in its place.

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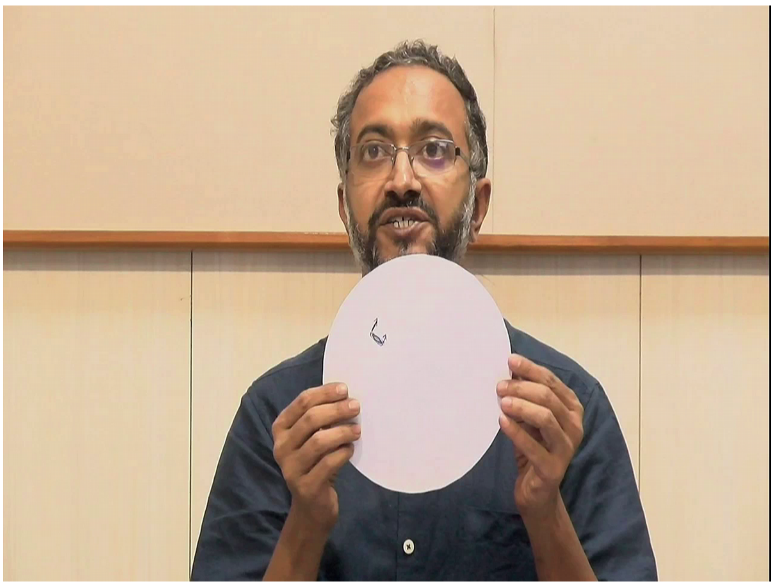

And this can be seen also where what I have done is basically let us assume that we have a small clip which can be embedded, so therefore it is embedded like this in this disc. And if we rotate it and this is a rigid body rotation because, the disc is rotating. And you can see that basically the clip itself is rotating and it is acquiring different orientations, but given that this velocity is higher than this velocity because this radius is larger. And so therefore, this end moves little more compared to this end and in the end the clip is rotating like a rigid body; there is local tumbling in this case.

But because of the rigid body rotation the overall curl of velocity in this case will also be nonzero. So, we do have local rotation in this case because the object itself is rotating we can also look at another type of flow. So, this flow is basically v theta is equal to omega r because there is only theta direction velocity we can also look at another type of flow which is v theta is equal to another let us say constant by r.

So, in this case what we can see is velocity is very small when you go to higher r and velocity is very high when you go to near the origin and so, in this case also, again velocity is only in the theta direction, but the velocity keeps on decreasing closer to the; so, if I again put a same kind of an object here. So, now, you can see that the velocity at this point will be larger and velocity at this point will be smaller. In this case, we had velocity at this point was larger and velocity at this point was smaller. So, in rigid body case the local rotation was present now in this case what happens is we will have curl of velocity to be 0 and therefore, local rotation will be absent.

So, what we see is the overall material particle is still. So, rotational motion of material particle is still the same material particles still exists. So, in this case also we have rotational motion of a material particle here also we have rotational motion of material particle because in both cases we have v theta. So, therefore, the material particle in general is moving in a rotational, but the local object itself is not really moving.

So, if I were to; let us say embed the same clip object and what will happen to this object is basically it will not have any local rotation at all. So, the object the way, it place; it is placed, it is going to rotate without any local rotation and so that is what can be drawn. Here, if we just look at this object, it is going to continue to remain like this. While in this case, it keeps on rotating or based on how the flow is.

So, what we have seen is there is a difference between the overall motion being rotational to there is rotational local rotation and clearly when we have a disc which is rotating such as what we talked about earlier in this case there are no stresses being generated in the disc because it is a rigid body rotation and there is no deformation involved, but in general in the course on rheology. We will be interested in flows where there is deformation and so, we will see that stresses which are generated are related to the deformation and they are not related to the local rotation.

So, the velocity gradient in general contains information about local rotation as well as deformation. This can also be seen based on how does the velocity of tow neighboring material particles change.

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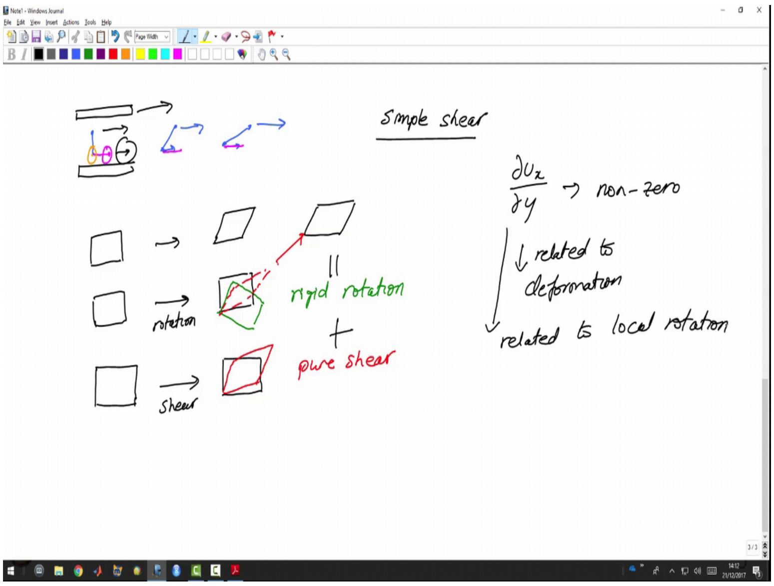

And we can look at this by basically placing. So, let us look at just the same flow that we looked earlier where we have two parallel plates and let us maybe pick up three points in this; so three points in this flow. And we have some hypothetical material fiber which is joining these points and what happens to these material fibers when there is flow and flow is initiated because this plate is moving the top plate is moving the bottom plate is stationary.

So, this kind of an object which is placed in the flow basically will have higher velocities at this location and lower velocities at the closer to the bottom plate because bottom plate itself is not moving. So, you can see that the blue wire will actually tend to rotate and become like this and this we will see also when we discuss the convicted coordinate. So, the distance between the 2 points will keep on changing because the top point is moving at a higher velocity while the bottom point is moving with a lower velocity. So, both of them will move to the right, but the top point will move in a different way.

Similarly, the other two points; the other material fiber will basically remain exactly the same because both of them are both the points the point here and the point here they are all moving at the same velocity which is given by this. So, therefore, this based on this the shape of this object we can see that there is local rotation as well as deformation involved this flow is called simple shear flow and if I; let us say start with an object of rectangular kind due to this deformation the flow is basically becomes a rhomboid. Now if we have a pure rotation, then this object after rotation would basically look like something like this. So, there they block itself is rotating. So, this is simple rotation rigid body rotation.

We could also have a shear in which case the same block if it is getting sheared then we have if it is the same block and it is getting sheared then what happens is with due to shear the same object will become deformed and shear deformation will cause the block to become like this. So, this is your shear. Now if I add these 2, then I get this. So, basically if rotation is there and on top of this rotation if I impose a shear then I will get simple shear. So, this plus this together will give me the simple shear that we see.

So, therefore, in general you can see that in all these cases velocity between 2 neighboring points is what we are looking at. So, in this case when del vx by del y is nonzero this means this is also deformation related to deformation and this is also related to local rotation. So, therefore, in general the velocity gradient and its different components include information about deformation in the material and for the course on rheology we would be interested in finding out the deformation and not local rotation.

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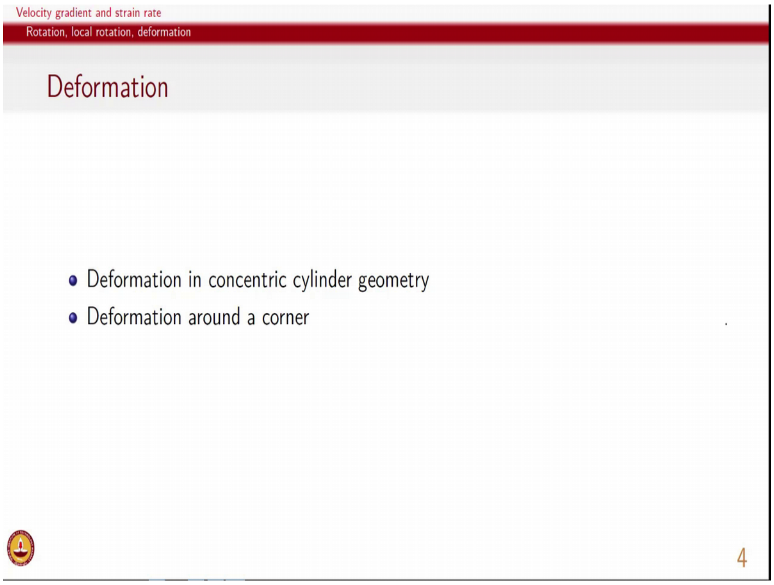

So, let us just look at an example geometry which is concentric cylinder and what happens in the case of concentric cylinder.

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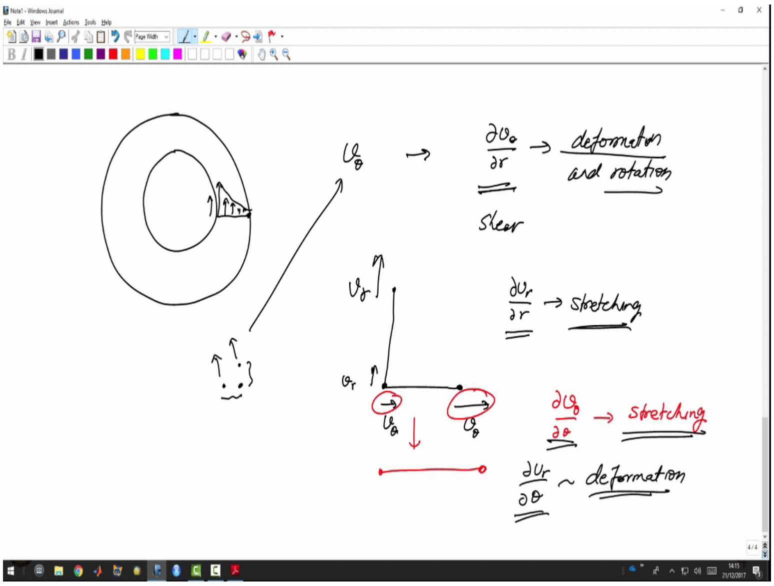

We have an inner cylinder which rotates with certain velocity and then the outer cylinder which is kept fixed and because of this the flow profile is again where the inner cylinder keeps on moving and therefore, the velocity would be something like this. So, the velocity decreases and goes to 0 at the top edge. So, again if we keep some, so in this case, we only have v theta and again del v theta by del r will indicate about deformation and rotation and to understand how we can obtain pure deformation and pure rotation information what we can do is we can again look at 3 points which are there this. So, let us say if we look at 3 points which are there in the neighborhood and all of these 3 points are moving in theta direction because there is a v theta.

What we can see in such cases is the fact that relative velocity between these 2 points relative velocity between these 2 points will tell us whether there is a rigid body rotation whether there is deformation whether there is any flow at all in terms of the 2 points having different flow and. So, just to look at this thing in a little more detail we can look at let us say 3 different points and so, this one let us say is v theta and little bit later let us say the velocity changes then this material fiber will keep on getting stretched. So, this after some time this material particle would become stretched because this v theta is small and this v that is large. So, the distance between them would keep on changing.

So, in this case since this is the theta direction del v theta by a del theta will tell us about stretching, we have already seen that del v theta by del r will tell us about shear and rotational component will also be informed by del v theta by del r we if let us say in the same case we have vr as the velocity and here again vr is velocity which is slightly higher. So, again del vr by del r will tell us about stretching and similarly del vr by del theta will tell us about deformation.

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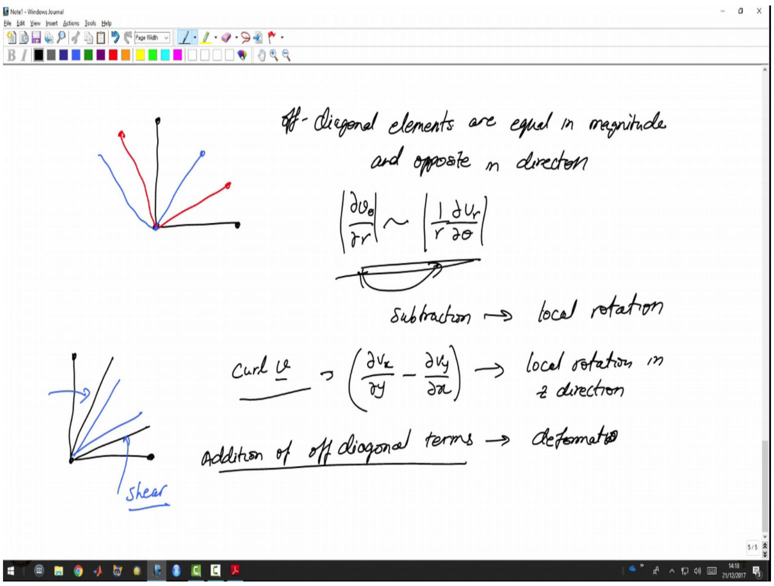

So, what we can see is rr theta theta r theta theta r. So, the diagonal components will tell us about stretching and the off diagonal components will tell us about deformation and if the same wire if we if it were to let us say if we have a wire like this in the fluid and with time let us say this wire keeps on doing this kind of a motion what we can clearly see is that this is a rigid body rotation. So, in this case the diagonal elements are equivalent diagonal elements off diagonal elements are equal in magnitude and opposite in direction. Therefore, del v theta by del r and 1 over r del vr by del theta these quantities will be similar, but opposite in direction.

So, therefore, subtraction of these points can give us an idea about. So, subtraction of them will give us an idea about local rotation; this is not surprising, if you just go back and think of curl of velocity and the components that we looked at earlier del vx by del y and del v y by del x subtraction of this is the local rotation in z direction . So, therefore, terms like this will indicate rotation and when you add the off diagonal terms addition of off diagonal terms.

So, in that case what happens is if we have a fiber like this it will get basically. So, this is nothing, but pure shear deformation. So, this is nothing, but. So, this material fiber and both of the material fibers are moving closer and. So, this is nothing, but shear deformation. So, therefore, addition in this case if they are equal and opposite then addition of off diagonal terms will give us deformation and so, that is what we can see when we look at the velocity gradient the type of deformation that can be there in the material of course, it can move as a rigid body translation itself.

In which case basically this is what is happening and in this case of course, there is no each of the velocity at each and every material point is identical and therefore, velocity gradient itself is 0, then we have rigid body rotation in which case the velocity gradient is not 0 because one point is moving at a faster rate compared to the other point and so, but only thing we have is local rotation there is no deformation, but in this course what we will be interested in is when the bodies are not just translating or rigid body rotation, but when the bodies are deforming and so, to get the deformation we will look at the off diagonal terms and add them from the velocity gradient.

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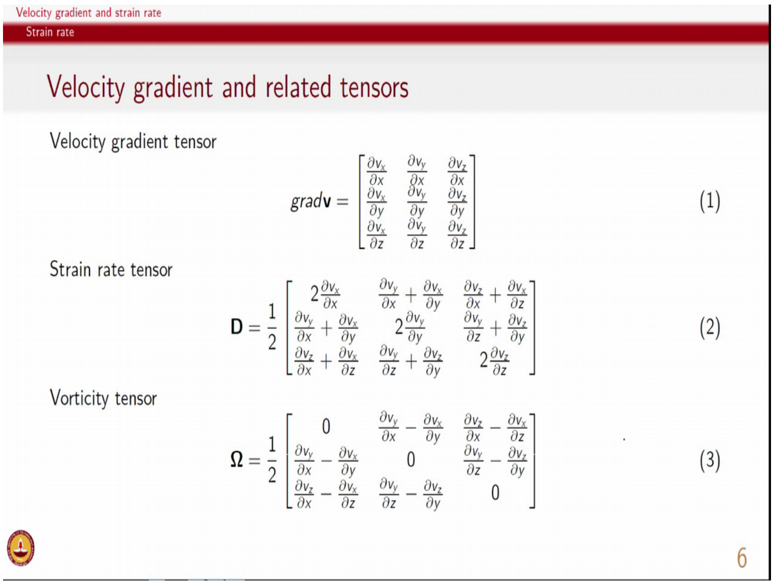

So, that can be seen here for cylindrical this is the rectangular coordinate and the strain rate tensor is obtained from velocity gradient tensor by just adding the velocity gradient and its transpose. Therefore, you have del v by del x added to del vx by del y. So, the gradient of velocity and gradient of velocity transpose and half gives us the strain rate tensor. So, all the components in this tensor tell us about deformation the diagonal components tell about stretching or extension and or compression and the diagonal off diagonal terms tell us about shear when we subtract the gradient of velocity from its transpose the diagonal terms all go to 0 and what we have are terms very similar to curl of velocity. So, therefore, this vorticity or spin tensor tells us about the local rotation.

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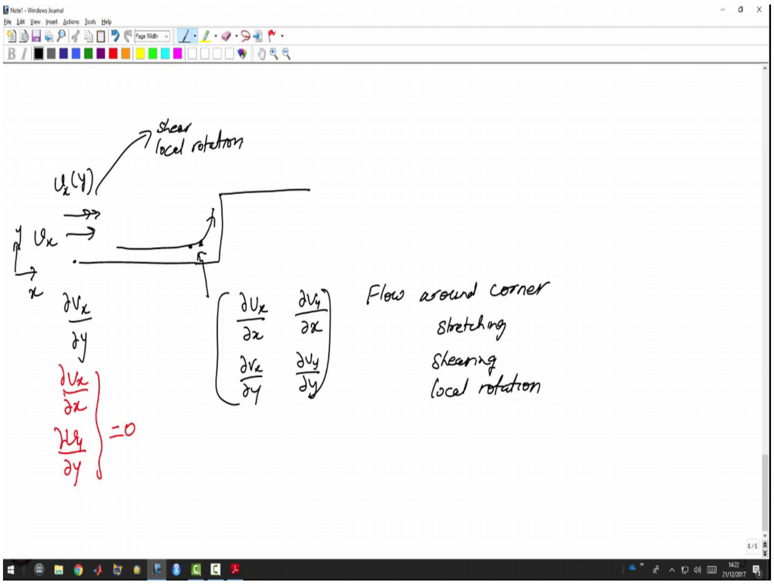

In the course on rheology, we will be predominantly interested in how is d for a given flow if we look at let us say flow around the corner, we can see this that somewhere very far away from the corner if let us say we have some surface like this and fluid is moving and then we have suddenly there is a corner. So, fluid will have to come and go like this. So, somewhere here velocity will be only vx because this is the y direction this is the x direction and the velocity will be a function of y only. So, here vx is only function of y and the only terms which will be nonzero are del v x by del y terms like del vx by del x or del v y by del y will all be 0.

But now once the material points a material point comes here now you can see that each and every point there is also going to be vy and vx and not only that the material is its velocity will change as a function of position both vx and vy will change. So, in this case we will have del vx by del x del v x by del y. So, as you can see here the 2 since we are discussing a 2 dimensional case we will only have these four terms and. So, we have del vy by del x and then del vx by del y and del vy by del y.

So, all these will be nonzero here and. So, flow around the corner we will have stretching and shearing and local rotation. So, all are present while in this case which was simple shear flow we have only shear and local rotation. So, in this course, our interest will be to impose some flows which are purely shear and some other flows which are purely extension and of course, in practice quite often in polymer processing or in applications of personal care products the flows that these products will experience will be a combination of shear and extension flow.

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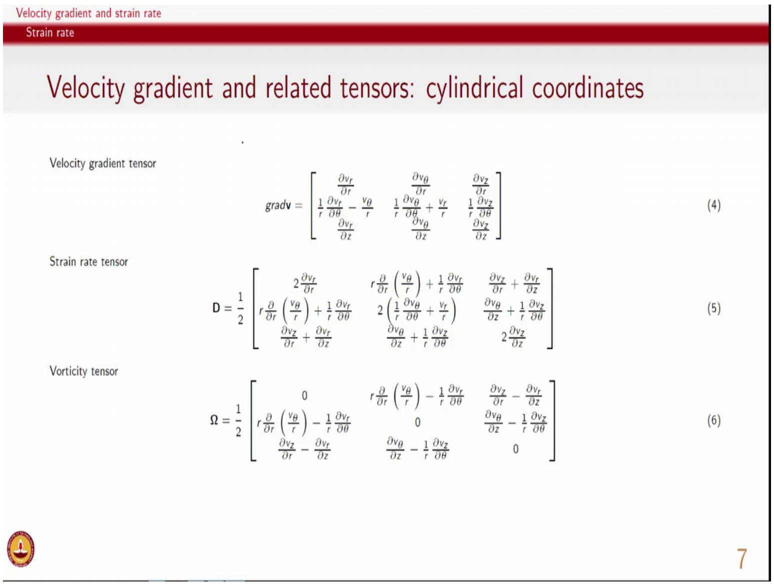

And so, the velocity gradient and its several components can also be seen for other coordinate systems when we use the other coordinate systems we have to be careful in terms of the additional terms which arise due to curvilinear nature of the coordinate system and so, for example, we have here vr by r or v theta by r and of course, there is 1 over r because with the dimensional consistency in this case the element is rd theta and not d theta itself the distance that is traveled in theta direction is rd theta. Therefore, the conceptually the cylindrical and spherical coordinate systems have identical components there will be nine components the off diagonal components indicate shear as well as rotation the diagonal components include information only about stretching.

And so, when we look at the deformation tensor or the rate of deformation or strain rate tensor, we have the normal components informing us about stretching and compression while the off diagonal elements informing us about shear deformation and the vorticity or spin tensor the diagonal terms are of course, 0 and the off diagonal terms tell us about whether there is local rotation.

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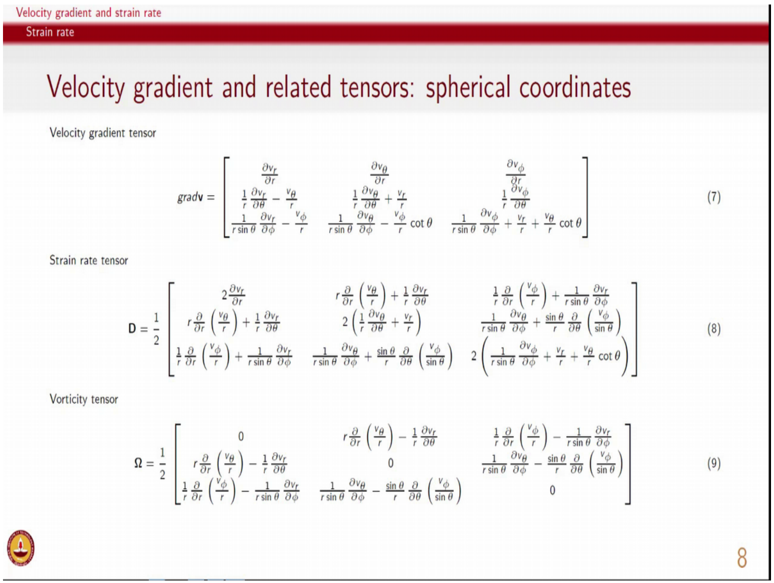

And the same set of terms can also be seen for a spherical coordinate system and again because of r theta phi system, there are additional terms; however, conceptually the velocity gradient again the terms in them indicate exactly the same thing.

So, in r course we will be interested a lot more in strain rate tensor vorticity tensor tells us about the local rotation that the material is experiencing and this is important for flows where local rotation can change the microstructure and during introduction we discussed for example, a glass fiber reinforced nylon and in those cases, if there is a glass fiber glass fiber can also tumble. So, the orientation of the glass fiber can change depending on what is the nature of the vorticity that is available near the glass fiber. So, if we are interested in knowing the orientation of glass fibers in such case we should also get information about vorticity, but as far as stresses are concerned, stresses would be related to the rate of deformation which is given by the strain rate tensor.

So, with this we have reviewed the overall basic quantities both kinetic and kinematic quantities which are required for us to do rheological analysis. In the next set of lectures we will start looking at some specific examples of flows; and how those flows are described by in terms of position vector in terms of strain rate and in terms of other quantities of interest.