Rheology of Complex Materials
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Lecture - 07
Stress and Strain rate

In the previous lectures on rheology, what we saw our unusual flow phenomena and the complex materials that are involved in such flow phenomena. We also took a close look at some of the molecular and microscopic mechanisms which are responsible for the behavior of these material systems and while discussing the flow phenomena as well as microscopic mechanisms. We referred to the deformation that is being applied on the material we talked about the stresses that are generated in the material. And therefore, it becomes important for us to define these quantities quantitatively define them well so that when we start looking at characterization of rheological response we can use these quantities effectively.

So, in the next 2 lectures, we will define stress and strain rate we will also get a physical significance feeling in terms of; what do these quantities imply what do they represent and very importantly for rheological analysis stresses as well as strain rates will vary depending on the flow conditions that are there. Therefore, it is important to understand what is meant by stress and strain rate and the various stress and strain rate components.

So, the overall outline for the discussion will be the following we will quickly run through the concept of continuum again, because it is very central to what we will be discussing. We will just round it up with what is meant by kinetic and kinematic variables of which stress and strain rate are examples and then we will define the stress we talk about deformation and rates of deformation. And then finally, define the strain rate also.

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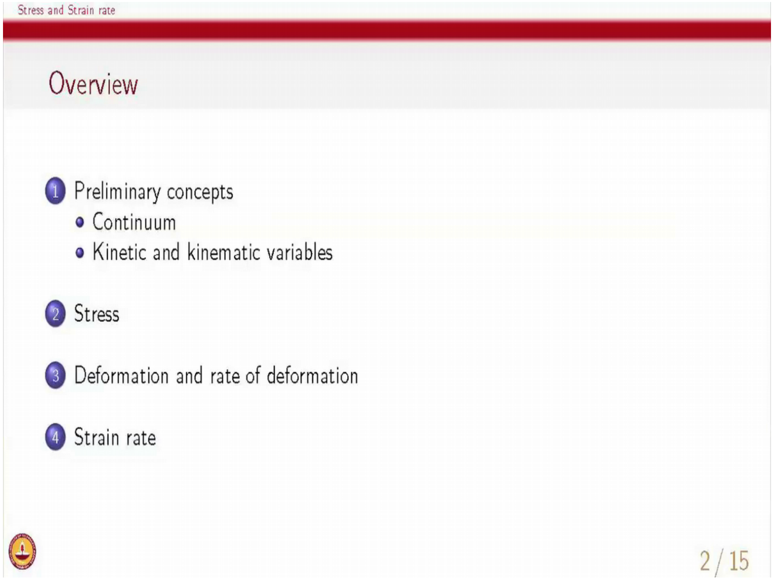

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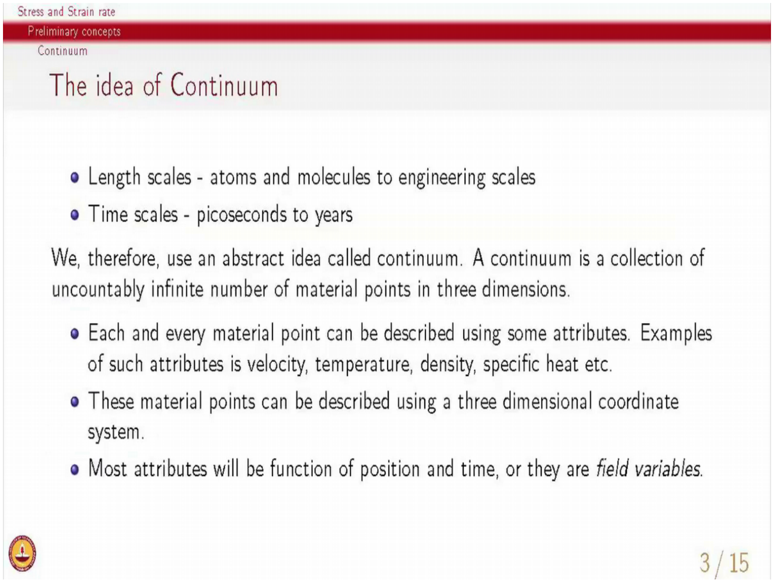

So, let us begin by looking at the idea of continuum again in a nutshell, we have the material being composed of at atoms and molecules scale and in case of multi phase system in terms of droplets and particles in case of macro molecular system individual macromolecules and so on. All the way till engineering scale which is could be of millimeter and meters length.

Similarly, in terms of timescales we have the atomic molecular phenomena which happened in very short time. In fact, Pico seconds or even shorter time scales or nanoseconds and then of course from an engineering point of view we design products to have life of a year or any even number of years.

So, basically when we look at both length scales and time scales the interests that we have spans actually orders of magnitude several orders of magnitude given this broad length scales and time scales that are involved what we do is we use the idea of continuum to deal with matter in a far more systematic manner the fact that atoms and molecules are there. There is discrete nature of matter and the fact that there may be multiple mechanisms involved at the microscopic scale what our hope is that by using the idea of continuum. And the subsequent development that each of those phenomenology can be captured and effectively by ascribing certain attributes of this continuum we will be able to capture all the phenomena.

So, this continuum is basically a collection of material points and just the way we have a real number line where between any two numbers there are infinite numbers similarly continuum also is collection of infinite number of material points and between 2; any 2 material points that are again in finite material points. So, therefore, each and every material point can be described using some attributes. So, basically we define set of attributes to characterize each and every material point and this attribute can be temperature or density or specific heat or velocity.

So, basically we are saying that instead of the atoms and molecules which compose the overall material and the phenomena which may be happening at extremely different time scales we represent the phenomena using a collection of material points which have these attributes. And we describe these quantities and their changes over length and time scales which are of interest in specific engineering problem and quite often the length scale as I said would be of interest may be from micron meter all the way to meters and kilometers. And similarly time scales that may be of interest may be from some hundreds of microseconds all the way till years.

Therefore, the material point and continuum effectively capture many processes that happen at atom and molecular scale which are at small length scales as well as very short time scales. So, the advantage of such an approach is the material points, and then can be described using a 3 dimensional coordinate system. So, 3-dimensional coordinate system in which axes are real number lines we can use the same properties of real space and the material space and therefore, do a mapping between the material and the real space. So, each and every material point is one point in physical space.

So, therefore, the material points can be represented or described using the 3 dimensional coordinate system and due to this what happens is therefore, all these attributes that the mentioned in terms of specific heat temperature or velocity they all are functions of which material point and what position does it occupy and time, therefore their functions of position and time. And therefore, they are called field variables and using this idea of continuum and using this approach, we will write our overall governing equations in terms of these field variables.

The idea is to track these field variables and solve for them and the quantities that we are discussing in this and the next class are stress and strain rate and we will see that they are also examples of field variables depending on the deformation that a material is undergoing the stress may change. And similarly the strain rate may change the strain rate may be different in the same material at 2 different points. Similarly stress may be different, so these are going to be functions of space and time.

Clearly, you can also see that there are multiple ways of tracking and the changes in these quantities in the sense that we can keep the material point fixed and then track how the stress change at given material set of material points or we could also basically keep a fixed volume of interest and then look at the what happens to the changes. So, we will discuss both of these possibilities as in when we are developing the subject further.

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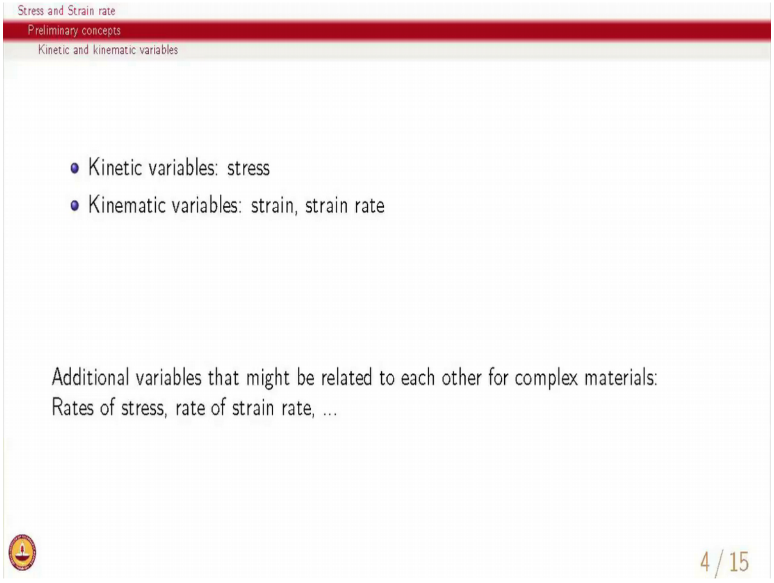

So, going on to look at we have the forces that are described and stress is basically a representation of force per unit area and that is an example of kinetic variable. On the other hand related to position of material particles related to the motion of material particles related to the relative deformation of the material particles and related to the relative rates of deformation of material particles are basically kinematic variables. So, they describe the position of material particles and using the position of material particles we can define quantities like strain and strain rate and various other quantities.

So, in general; in a continuum in a material we would have the expectation that kinetic variables are related to kinematic variables. So, before we seek what those relationships are before we try to understand how is stress dependent on strain or stress dependent on strain rate or various combinations of their relationships we must define them well. And so, we actually will first look at what is the definition of strain and then strain rate. At this point we should remember that there may be additional variables that might be related to each other for complex materials and this is the feature of complex materials that we will come across very soon. In the sense that instead of stress being involved alone we may also have rate of change of stress.

So, rather than stress being important alone we also have its derivative important similarly strain is involved derivative of strain in terms of strain rate is involved we also could have derivative of strain rate which is rate of strain rate. Therefore, any combinations of how these properties are and how do they vary with time could also be involved in trying to describe the behavior of the complex materials.

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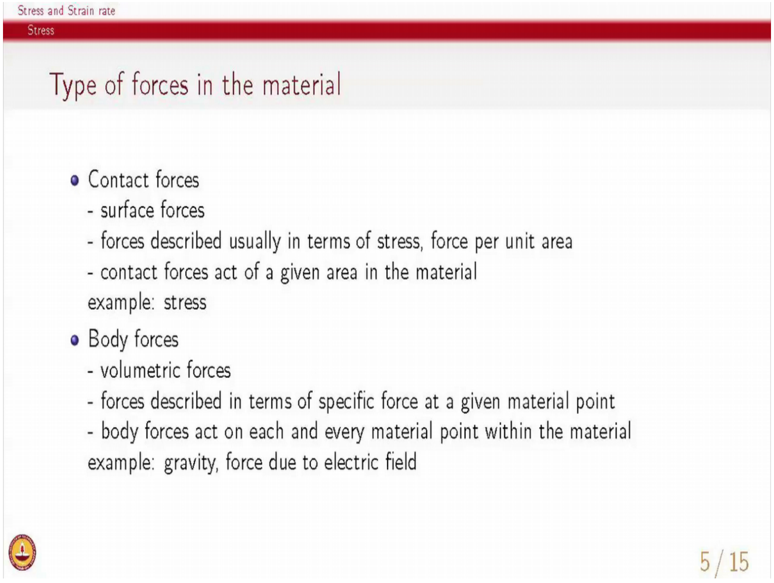

So, let us look at to define stress what are the different type of forces which we can most generally define basically what we have are contact forces in the material and body forces. So, the contact forces are essentially forces which act on surfaces. So, that is why we can refer to them as surface forces they are forces described only in when we define a surface within a material.

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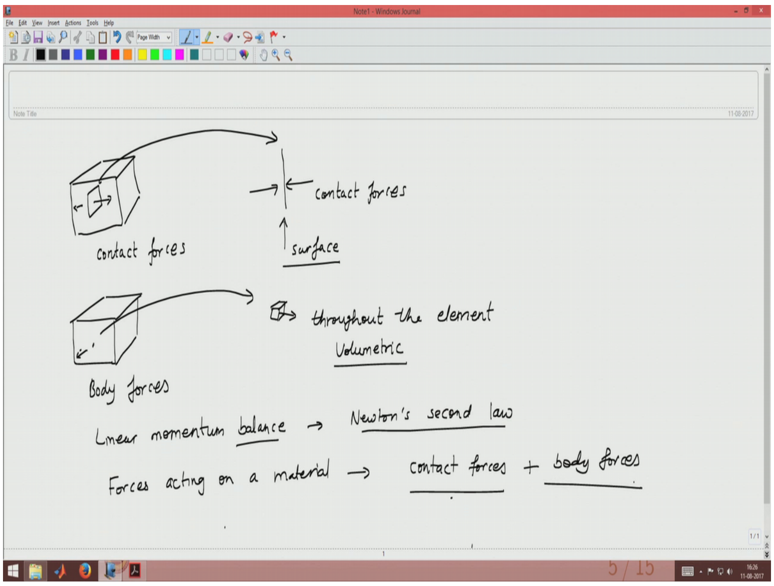

So, let us look at what do we mean by this, if we have any material which has a certain shape if we take a surface hypothetical surface in this material basically the 2 sides of the material will be exchanging what are called contact forces. So, let us I just look at the one dimensional picture of it. So, this plane that I have drawn here now I will represent it as a line. So, we will have the contact forces which are being exchanged.

So, unless we define a surface, we will not be able to specify; what is the nature of the forces. And so in general what we will do is define surfaces in the material and then on them define contact forces contact forces again act on a given area in the given material the other example is that of body forces. So, body forces are volumetric forces in the sense that they act throughout the body.

So, now if we again look at the same material and what we can see is body force will act on a given point. So, if I take any small material volume the body force will be acting throughout the element and therefore, it is called volumetric. So, we basically have surface forces and volumetric forces. So, these are the 2 broad types of forces which act in the material and body forces. Therefore, are described at specific force at a given material point body force act on each and every material point within the material. And therefore, they are called volumetric forces and of course, common examples of body forces gravity which is one of the main forces that we might encounter during flow of materials depending on the materials of interest if there are charges in the material then based on electric field there will be forces that material will experience similarly magnetic field and so on.

So, there are all of these again are felt throughout the material wherever there are charges or wherever there is mass given that continuum is composed of material points in each and every material point as a density. Therefore, each and every material point will have gravity. Similarly in the continuum picture whenever we have a charge the charge will be distributed on each and every material point. And therefore, each and every material point will feel the force and we know from physics that the force will be the charge times the electric field.

So, therefore, the body force can be felt at each and every material point. And so the question is now how do we specify them more and why; what is the need for talking about these forces. So, given that in the end what we will have to do is; we will have to do a linear momentum balance or we will also this is also Newton's second law which states that the mass into acceleration is equal to the forces which are acting on a material.

So, therefore, we will need to know what are the forces acting on the material and clearly therefore, to define those forces acting on the material we will need to say that what are the type of contact forces which act on the material and then what are the body forces which are acting on the material and so when we do the balance or when we write Newton's second law we will have to specify contact forces we will have to specify the body forces.

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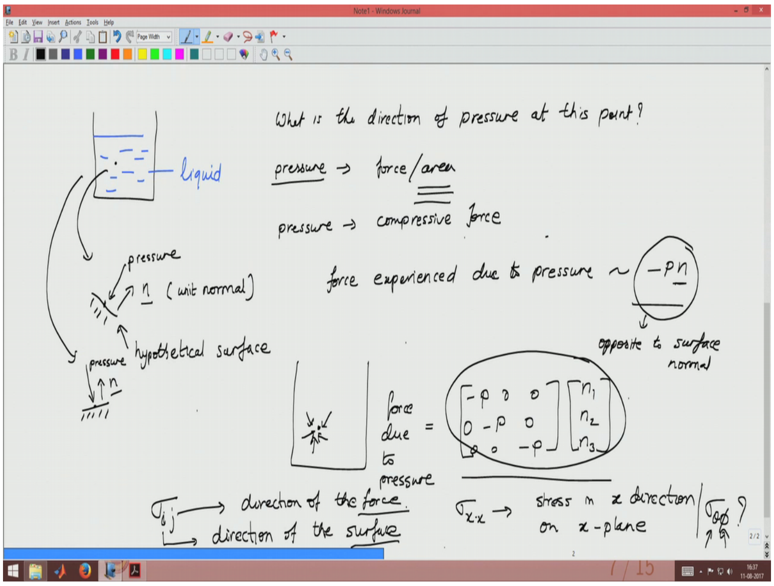

So, now let us go and look at one example of contact force which we know from school times which is pressure and in general what do we mean by pressure. So, if you look at a stagnant pool of liquid. So, we have a liquid and in this if we have specific point and we asked the question about what is the direction of pressure of pressure at this point and the answer of course, we know from again school definitions that pressure is uniform in all directions or it is the same in all directions, but central to defining pressure because pressure is nothing but force per unit area. So, an implicit assumption is that you are defining an area.

So, only when you define the area it is possible for us to define a pressure. So, generally what we do is we mean whenever we want to define pressure at this point let me just take this point and redraw it again if I draw a hypothetical surface. So, let us say this is a hypothetical surface and of course, each and every surface can be defined using its orientation which we usually define in terms of unit normal vector which we will tend to denote as n. So, this is nothing but the unit normal.

So, we have a surface now at that point and which is given by n. So, by definition we know that what is the pressure on this is basically in the opposite direction its pressure we know is a compressive we by convention we say that it is a compressive force. And so, what we know is that pressure will be in the opposite direction to the surface that I have drawn. Now same point if I take and at the same point if I draw another hypothetical surface whose unit normal is of course, different again the pressure on that point will again be opposite to the unit normal vector. So, therefore, this is also pressure.

So, basically what we know is that pressure is opposite to whichever surface we are writing. And so the force which is experienced due to pressure is basically given by minus p times n this says that the direction of pressure is opposite to surface normal. So, therefore, we are able to think of this as a knowledge that we know and the we draw some of this information in a school textbook by saying that; pressure is same in all directions what we do not do in each of these cases draw the surface over which pressure is defined, because without the surface we cannot define the area. And since we cannot define the area therefore, we will not be able to define the pressure.

So, effectively what we have said here is that pressure if I want to know the force due to pressure I need to know the surface normal and this can also be written in a vectorial form in terms of matrices like this that if I know pressure at each and every point and if I know the surface at which I would like to know then this is my force due to pressure. So, we can see that this information is the same thing that was written. So, what is written here in terms of p n is the same thing that is written here.

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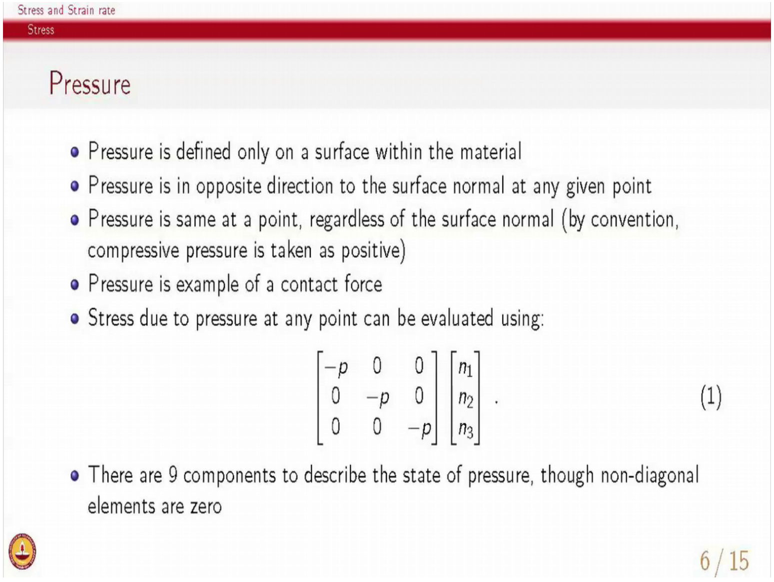

So, now let us go and look at pressure and summarize what we have learnt that pressure is defined only when there is a surface within the material pressure is in the opposite direction to the surface normal at any given point pressure is the same at a point regardless of whichever surface is being talked about. And therefore, we know that pressure only varies if let us say the height of the point that I am talking about varies, because then we say that that is the static head, but otherwise pressure at a given point will be same regardless of whatever is the surface we are talking about and also by convention we say that pressure compressive pressure is positive.

And. So, clearly now pressure is an example of a contact force because we have to specify the surface area and this is again a hypothetical surface area which is being defined in the material. And so stress which is due to the pressure at any given point can be evaluated using a matrix notation we wrote, but this is basically can be written in vectorial or tensor notation also is minus p times n. And so the n unit normal vector can be written as a column matrix where n 1, n 2, n 3 are the 3 components and here 1, 2, 3, refer to the 3 coordinate axis. And so what we can see here is there are 9 components to describe the overall state of pressure 3 of them are non-zero and all of them are uniform. So, we all the diagonal components are minus p while all the non-diagonal components are 0.

So, generally to specify the state of contact force we will see that all 9 components will be needed because pressure is a specific type of contact force which is uniform regardless of the surface normal given that surface normal does not determine what is the amount of stress we have this specific form where the diagonal terms are all identical to each other and by convention we write it as minus p while all the off diagonal terms are 0; what we will see is in a stress when we want to specify the general state of contact forces in the material all the 9 components will might be non-zero.

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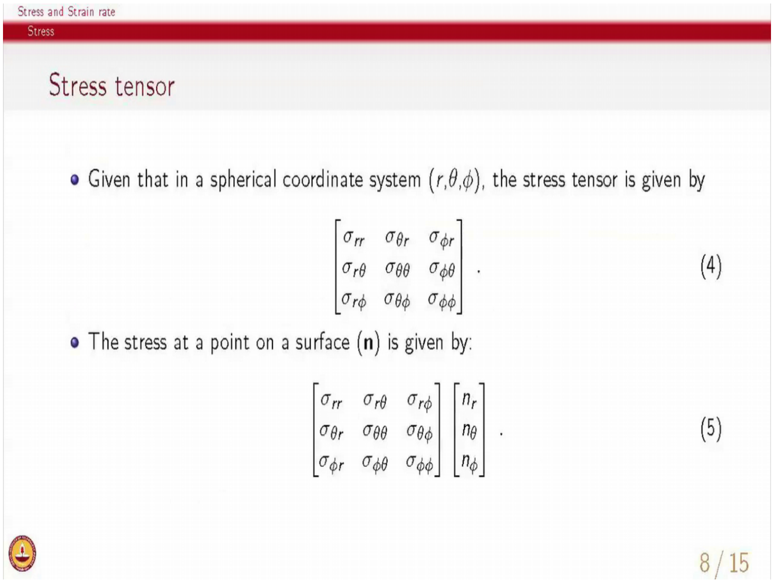

So, let us proceed further and then discuss what is called the stress tensor. So, as I said contact force can be specified using similar matrix notation we will use 3 dimensional space to describe most of the problems. And so we have choices of 3 directions for example, if we take rectangular coordinate its x, y and z if we take spherical coordinates its r theta phi.

So, since we are describing a force we will need 3 directions to specify the direction of force as we know force is a vector and we will need 3 unit vectors x, y, z or r theta phi to describe the overall direction of the force, but this contact force is acting on a surface area. So, surface area also needs to be specified as we saw in the previous discussion that its n denoted by unit normal. So, again unit normal can be specified using 3 planes basically.

So, if you have an rectangular coordinate system then we have an explain where y and z are constant we have y plane where z and x are constant and so on. So, therefore, we have 3 ways to specify the direction of the force we have 3 ways to describe the surface on which this force is acting. So, that gives us 3 into 3; 9 choices.

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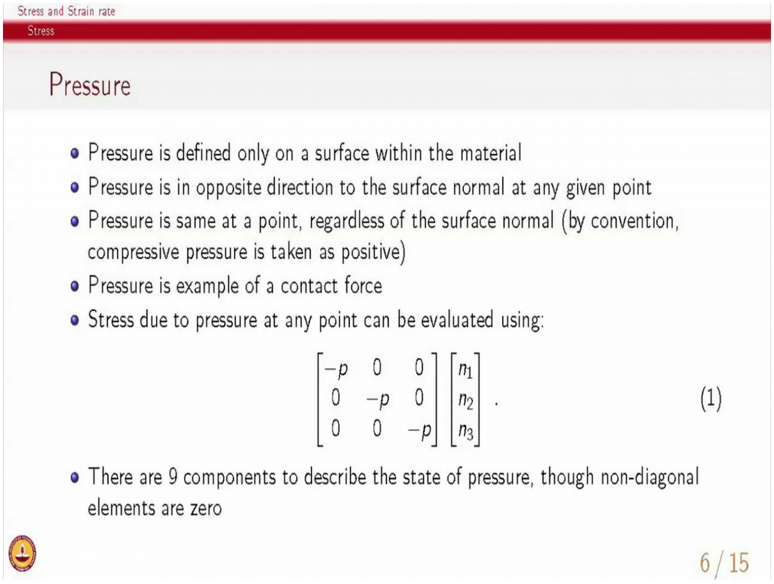

Therefore, the number of components in stress tensor are 9 and which is what we saw also earlier that the pressure also can be specified using all the 9 components, but because pressure is isotropic or pressure does not change depending on the surface normal at a given point we only have diagonal elements and no non diagonal elements.

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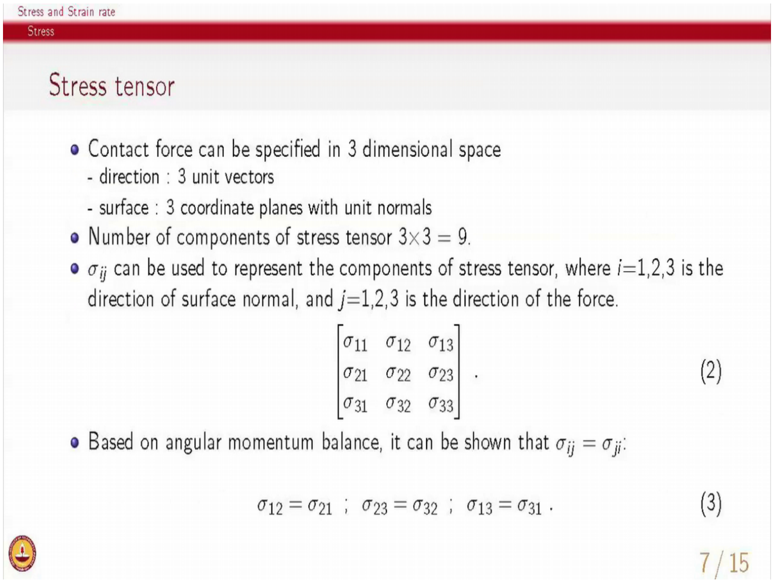

But in general, we will refer to the contact forces in the material using a stress tensor and the stress tensor will have 9 components in short we will denote it as sigma ij and where I stands for 1, 2, 3 which is the 3 dimensions it could be r theta phi it could be r theta z in cylindrical coordinates and so on and j specifies the direction of the force. So, just to make a note of this again when we say sigma ij I refers to the direction of the.

So, there are different conventions possible and the convention that we will follow in our course the direction of the surface will be referred to by I and the direction of the force will be referred to by the j. Therefore, sigma xx would imply stress in x direction on explained. So, similarly it may be helpful for you to think; what does sigma theta phi in spherical coordinate imply. So, as we are saying one of them theta describes the direction of the surface and phi describes the direction of the force.

So, now let us come back to and summarize. So, we have 9 components of the stress each component represents one specific direction of the stress as well as direction of the surface and we will not do this derivation, but it is can be shown that based on angular momentum balance that the stress tensor is symmetric which means sigma 1 2 is equal to sigma 2 1 and sigma 1 3 is equal to sigma 3 1. So, basically we only have six components we have sigma 1 1 sigma 2 2 and sigma 3 3 as the diagonal or what are called the normal stresses. And then we have sigma one 2 sigma 1 3 sigma 2 3 as the off diagonal elements which are called the shear stresses and of course, as I said sigma 2 3 will be sigma 3 2 and so on. So, we have the off diagonal elements equal to each other.

So, with this introduction to stress tensor now what we can look at is how are these stress tensor varies for a specific different geometry. And we will take a specific look at example of a cone and plate viscometer in which we will look at the state of this stress.