Rheology of Complex Materials
Prof. Abhijit P Deshpande
Department of Chemical Engineering
Indian Institute of Technology, Madras

Elasticity in fluids: normal stress differences and stress growth
Lecture - 49
Normal Stresses – 2

So, in the previous segment of the lecture we saw that macromolecules under shear lead to the presence of non-stresses, then we saw that how stress tensor for simple shear looks like for viscous or linear viscoelastic materials and also argued that it would be normal stresses would arise in case we have a non-linear response, which implies at large deformation response of materials. And to account for such large deformation response of materials, we need the model which is also non-linear.

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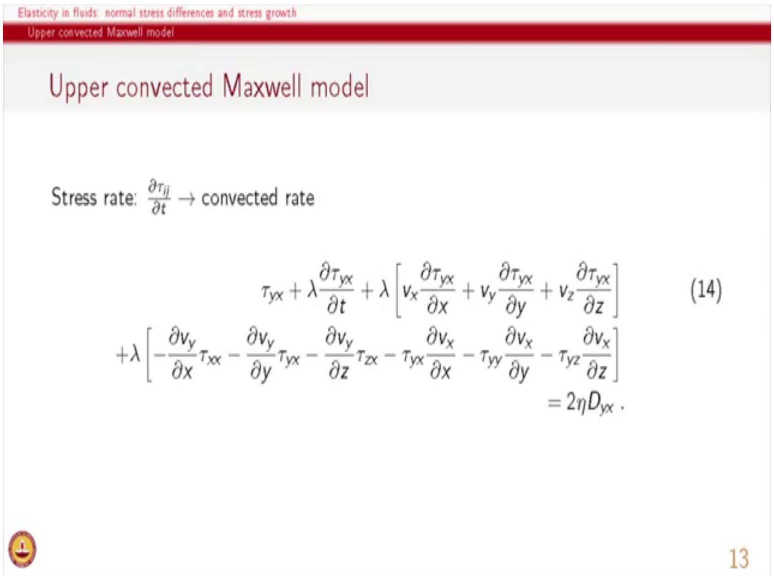

And the example that we will look at now is the upper convected Maxwell model. So, the upper convected Maxwell model when we use the convected rate the convected rate of the stress gives rise to additional terms. So, the expression here is given for the upper convected Maxwell model, but only the yx component. As you can see that its a fairly complicated equation with lots of terms there and therefore, for at least these segments of lectures where we are discussing behavior more qualitatively it is important for us to understand some of the idea behind these equations and then for the more advance learners we can have complete mathematical development of these equations.

So, the first term here which is basically same as what we had earlier, which is the partial rate of change of the stress itself. The next set of terms that you see are basically what are also there in Navier stokes equations or also there in any time we have the material derivative. So, therefore, these two together are basically the total derivative of the stress.

One which accounts for change in stress with respect to time only keeping the position fixed, the other one is when the overall change in stress due to change in position and of course, while change in position the material also changes in for example, vx is del x by del t. So, partial derivative of a x with respect to time is vx so. In fact, these terms are arise also naturally due to chain rule given that tau yx is the function of not only time and x also.

So, therefore, the material derivative is of course, measure of rate of change of tau; however, that is not a complete measure of stress rate in which is frame in variant and which can be used for arbitrary large deformation. There is an additional term which basically you can see where this velocity gradient is getting multiplied with the stress components.

So, this is the term where ones term where velocity gradient is multiplying with some components of stress and other case the some components of stress are being multiplied by with the velocity gradient. So, these are. In fact, the convected terms because for example, if there is velocity gradient if is 0 of course, naturally these terms will fall off and. So, these are the convected terms which lead to the overall rate of the convection in the material. In fact, what you can see is, even if we have a situation where the partial derivative of stress with respect to time is 0 we can still have the convected rate non zero. So, therefore, we have the convected rate includes terms.

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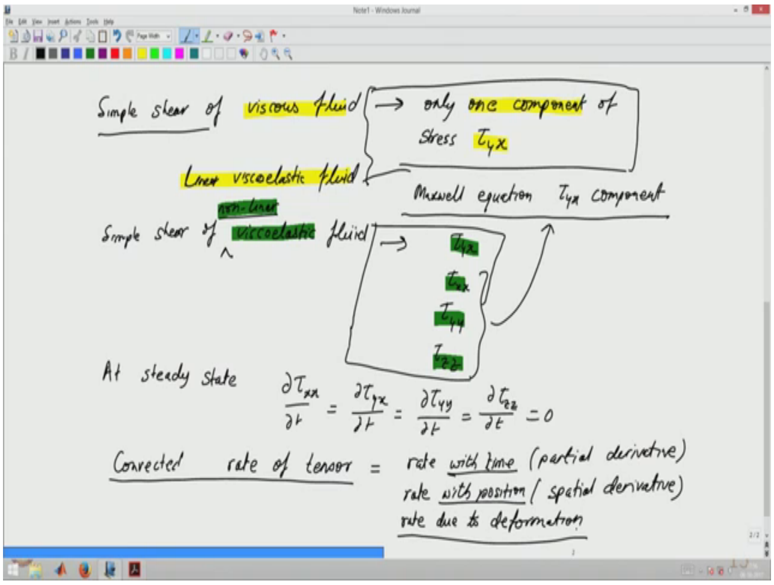

So, given that we are instead of choosing the partial derivative we are going to choose the convected rate. The convected rate of a tensor quantity includes the change or the rate with time which is the partial derivative, which is the partial derivative then what is rate with position which is basically the inertial terms form the navier stokes equation very similar to those and so, there are terms which indicate basically special derivatives and then we finally, have rate due to deformation.

And so, even if things are not changing with time, even if things are not changing with position the overall convected rate of tensor will in fact, have non zero values. So, that is the key characteristic of convected rate and that is what we will see in the example that we are looking at of simple shear. So, in case of simple shear we are going to look at a steady state situation, in which case we will see that this term will fall out we will also see that the special derivatives will not be there ; however, this term will contribute additional this overall convected rate which is due to deformation will contribute additional terms.

Now why is this model non-linear? If you see these terms which were there in the original Maxwell model itself are of course, linear. But here velocity is multiplying with there is multiplication involved in stress derivative with velocity. So, therefore, these set of terms are non-linear, similarly here also the velocity gradient is multiplying with stress. So, again these set of terms are again non-linear where two variables are in available to us in the form of a product the right hand side of course, still remains the same.

So, if you see due to this upper convected Maxwell model where we have replaced the partial stress rate with a convected rate gives us a lot more set of terms and each of these terms signify in terms of change with time, change with position and change due to deformation. So, now, let us look at what happens to upper convected Maxwell model in simple shear flow. So, we are again for this segment of the lecture we are not deriving the equations and we are not starting from the initial equations.

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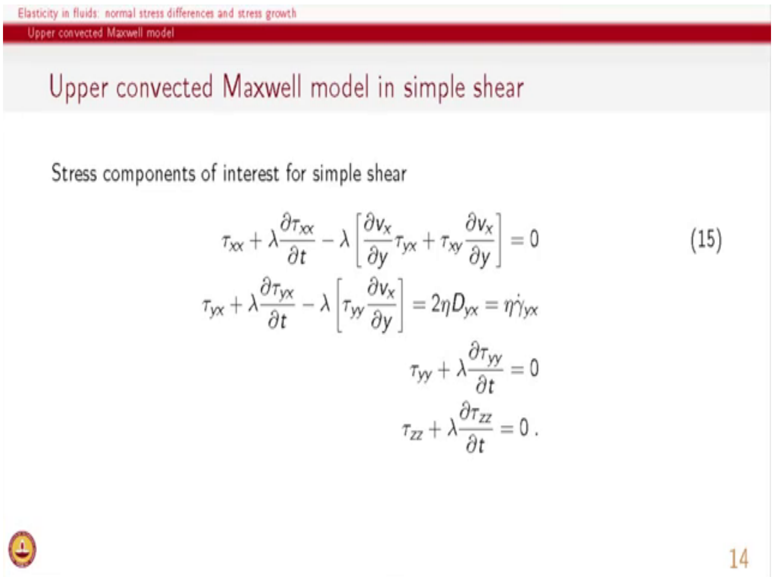

But the final equations which are governing equations for simple shear of upper convected Maxwell model again there will be four components because we have tau xx, tau yy, tau zz as the three normal stresses and then of course, we have the shear stresses. And so, what we can see is the tau yy and tau zz term are very similar to what they were for Maxwell model.

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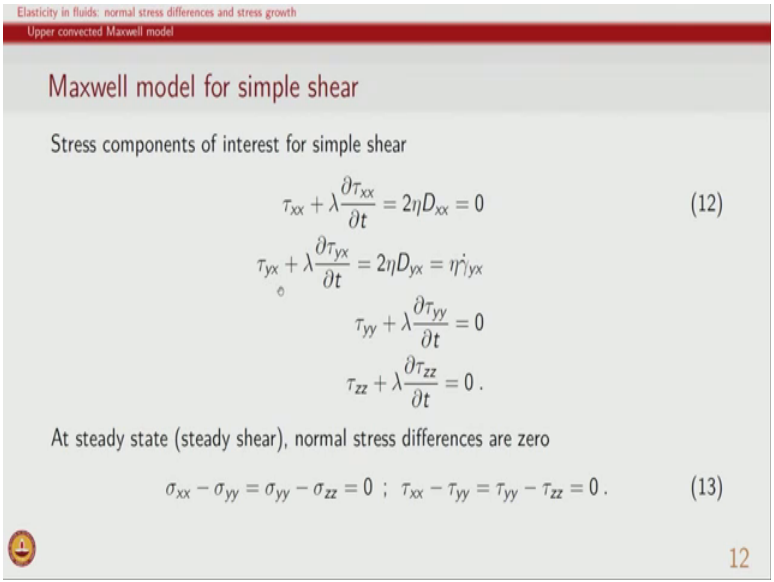

So, just to remind us again these are the governing equations for Maxwell model tau yy and tau zz. So, they remain entirely identical, but the terms for tau xx and tau yx get modified due to the non-linear terms which are present from this equation. Now the special derivatives will always be 0 in many of the situation of simple shear example V x is non-zero, but it does not depend on anything does not depend on x and z of course, we are dealing we only two dimensional situations.

So, z derivative is not involved and V y itself is also 0. So, therefore, these terms will rarely contribute to any of the rheometric flows equations, its only these terms which are deformation change due to deformation in the material which will contribute. So, for example, we are looking at V x as a function of y.

So, this term will certainly be present, while V x is not a function of z. So, therefore, this will go to 0 and since V y itself is 0 flow is only in x direction all these terms will drop out. So, therefore, there is a greater simplification and tau yx del tau yx by del t the partial rate as similar to Maxwell model itself, but there is only one additional term which is based on the non-linear terms. Similarly in the x component also we have similar terms. So, now, we can actually look at what happens to this in the steady state.

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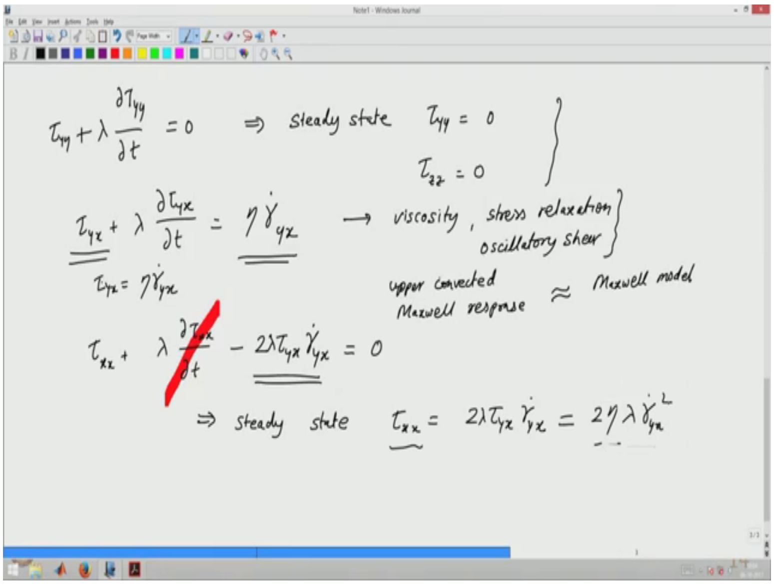

So, we will first look at the derivatives which are with the equations, which are with yy component. So, since we have tau yy plus is 0 and we are looking at steady state when the derivative with respect to time goes to 0 we have tau yy going to 0 similarly tau zz will also be 0; so given these two situations these two conditions.

Now we can look at the shear component which is the second equation that we have written here. So, we can see that that also involves tau yy, but at steady state tau yy itself will be 0. So, therefore, the overall equation will reduce to what it was earlier. So, in case of shear component the upper convected Maxwell model will again lead to an equation, which is similar to the Maxwell model itself.

So, what we can see clearly in this case is as far as viscosity or stress relaxation or oscillatory shear behavior are concerned, the upper convected Maxwell model response will be identical to the Maxwell model response, response is same as the Maxwell model. In fact, these two components being 0 also is similar to the Maxwell model response. However, if we look at the governing equation for tau xx, which is the first equation here we can see that there are two additional terms which are they coming from the stress rate which arises due to deformation in the material.

So, we have del V x by del y which is gamma dot yx and similarly del vx being multiplying tau yx and tau xy of course, we remind ourselves that this is the symmetric tensor. So, tau yx and tau xy will be similar. So, what we have therefore, is tau xx plus lambda del tau xx the partial derivative minus 2 times tau yx gamma dot yx is equal to 0. So, you can see that given that dxx is 0 the right hand side is 0, but these terms together actually give us two different terms which are related to they get added on to each other and therefore, we get an additional term.

So, at steady state steady state of course, we know that the time derivative will go to 0 and therefore, what we have is tau xx is equal to 2 tau yx into gamma dot yx. There is a factor lambda also which have omitted here remember this lambda, which has to be written and therefore, I will add that here. So, there is a lambda here and that lambda has to be also added here and we know that steady solution for shear itself is that tau yx is equal to eta times gamma dot yx. So, just recall our self that tau yx is equal to eta dot gamma dot yx.

So, that can be substituted here and therefore, we will get two lambda gamma dot yx squared. So, what we can see here is the, normal stress which arose in the material in this case the upper convected Maxwell fluid is related to the square of the strain rate in the material and of course, it is also proportional to the relaxation time.

Remember that relaxation time is an indicator of the elasticity in the material higher the relaxation time more is the elastic contribution and therefore, the normal stress which arises in an upper convectional Maxwell fluid is also more if we apply a higher strain rate or the relaxation time of the material itself is higher. Of course, whenever we apply higher strain rate we are giving material less response time and we know that as we give less and less response time we expect material to exhibit more and more elastic behavior.

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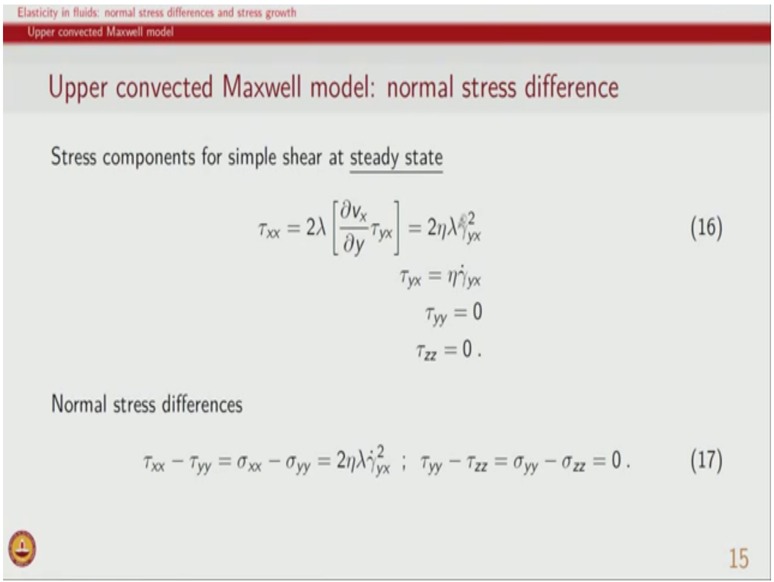

So, these features are very nicely captured in upper convected Maxwell model and. So, we have tau xx given by two eta lambda gamma dot squared tau yx of course, means very similar to what a Newtonian fluid behavior is, and we have seen earlier that this eta therefore, is interpreted as 0 shear viscosity, because we cannot get shear thinning or shear thickening or other responses from these kind of models.

So, we will also therefore, should not be surprised that they are far more complicated models which are available in either computational packages or in terms of deriving some of the behavior of either of colloidal systems or macromolecular systems, because even a more complicated model like upper convected Maxwell model shows that normal stress arises; however, shows that there is no shear thinning shear thickening. And similarly it also shows that the not all three normal stresses are non-zero. In fact, two of them are 0 and only one is non-zero. And so, if for the upper convected Maxwell model the normal stress difference is tau xx minus tau yy and of course, same is true for sigma xx minus sigma yy and it is two eta lambda gamma dot y square, while the other normal stress difference which is yy zz is 0.

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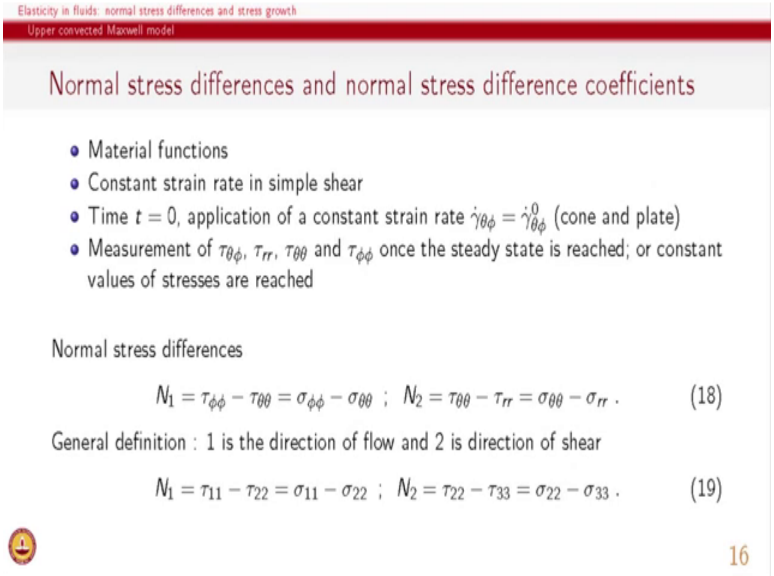

And for most general fluids we can therefore, try to characterize by doing an experiment or by doing a simulation or doing a characterization, and trying to see what is the normal stress differences in these fluids.

So, therefore, we can define our material function in terms of normal stress differences and the idea is to apply a constant strain rate in simple shear mode. At time t is equal 0 therefore, we apply a constant strain rate and if we use lets say device such as cone and plate the velocity is in phi direction and the velocity changes as a function of theta and therefore, we use gamma dot theta phi.

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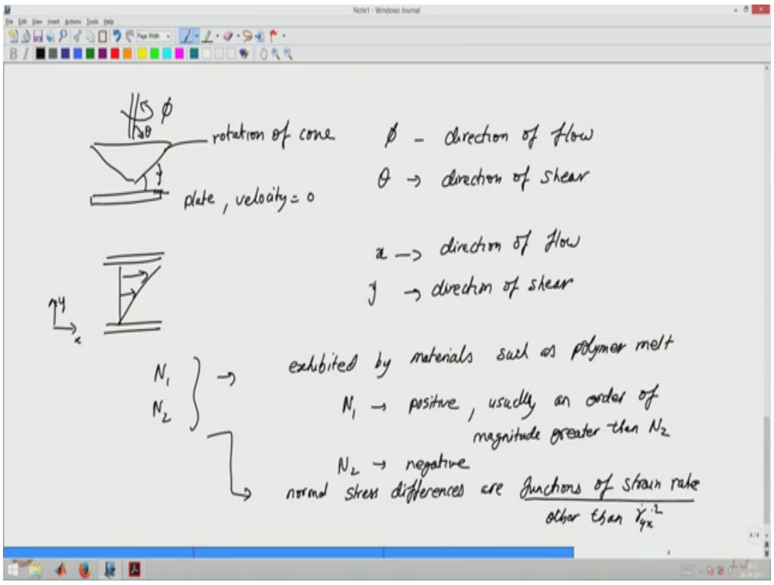

This is just so, that we become familiar with more and more what are the types of components involved in different types of geometries. So, if we have a cone and plate device the rotation leads to phi motion and the angle this angle which is measured is theta and therefore, the velocity here is because of rotation of cone and here velocity is 0 because plate is stationary. So, velocity is 0. So, therefore, in this direction there is change in velocity. So, phi is the direction of flow and theta is the direction of shear. This is very analogous to what we saw earlier in case of parallel plate in this case of course, we had y nd x. So, x was direction of flow and y was the direction of shear.

So, therefore, we apply this gamma dot theta phi and we have phi a constant value and we measure the stresses and these stresses are of course, measured once the steady state is reached and therefore, we are measuring values which are independent of time, and once we measure these values we can define a set of normal stress differences basically two normal stress differences tau phi phi minus tau theta theta and of course, its same as sigma phi phi minus sigma theta theta or tau theta theta minus tau rr.

In general we use the first normal stress to be based on the direction of the flow and the second normal stress based on direction of shear. So, therefore, n 1 is defined as tau 11 minus tau 22 or sigma 11 minus sigma 22 and similarly the normal stress difference second one is defined as tau 22 minus tau 33 and sigma 22 minus sigma 33. So, given these definitions we can see that for the upper convected Maxwell model N 1 is shown to be non-zero; however, N 2 is known to be 0. Now as far as realistic materials are con are concerned it is generally known that normal stress differences both are present.

So, we have N 1 and N 2 are both exhibited by materials such as let us say polymer melt of course, same would be true for a colloidal dispersion or a macromolecular solution also and generally for many of these material N 1 is generally positive and it is also usually an order of magnitude greater than N 2 and N 2 many times is observed to be negative.

So, these are based on general observation of variety of materials. So, when we develop new models or when we try to understand the mechanistic features what is leading to its important for us to try to say why the normal stress difference is positive in one case and why it may happen to be negative in few materials or similarly what is the reason for normal stress difference, which is first normal stress difference N 1 is much larger than the second normal stress difference. So, given these behavior the other feature which is also commonly seen is the fact that the normal stress differences are functions of strain rate. So, therefore, the normal stress differences are not constant as is predicted by the upper convected Maxwell model.

So, like we saw earlier case where Maxwell model by itself was able to give us a good response in terms of an exponential decay for stress relaxation, in case of oscillatory shear it was able to show from terminal viscous response to a glassy elastic response; however, when it comes to realistic materials there are several other feature which have which are observed and therefore, Maxwell model is not really adequate though it does capture some elemental ideas of viscoelasticity.

So, now, in this case also upper convected Maxwell model shows in elemental behavior in terms of normal stress difference which are non zero; however, it shows the normal stress differences which are only related to strain rate in terms of square while what is observed in most cases is that they are functions of strain rate other than gamma dot yx squared. So, therefore, the behavior is not exactly what is shown by upper convected Maxwell model.

So in fact, just to just the way we use here the behavior that we can define material function which is this divi the stress divided by strain rate, in this case also we could define a material function which is this two eta lambda if it derived divide tau xx by gamma dot y x square. And in fact, that is what is precisely done in definition of material function.

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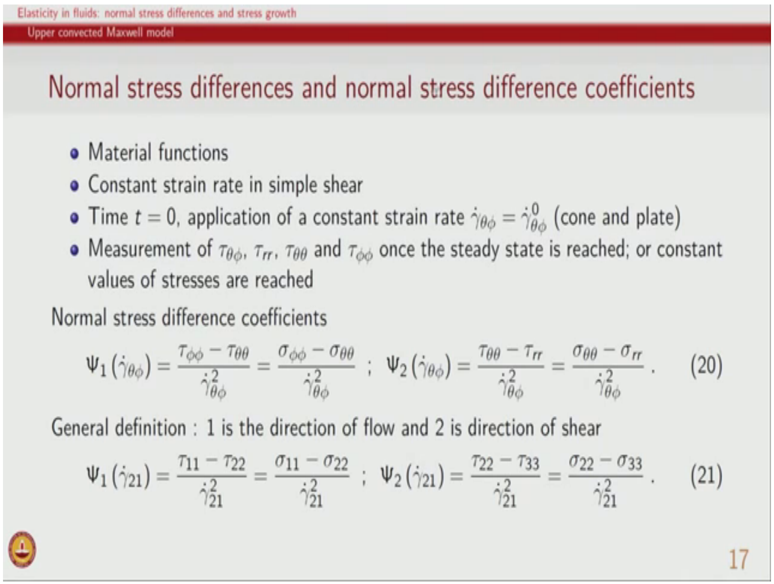

We define a normal stress difference coefficient and the normal stress difference coefficient are divide defined based on the basically the ratio of the normal stress difference to gamma dot theta phi squared.

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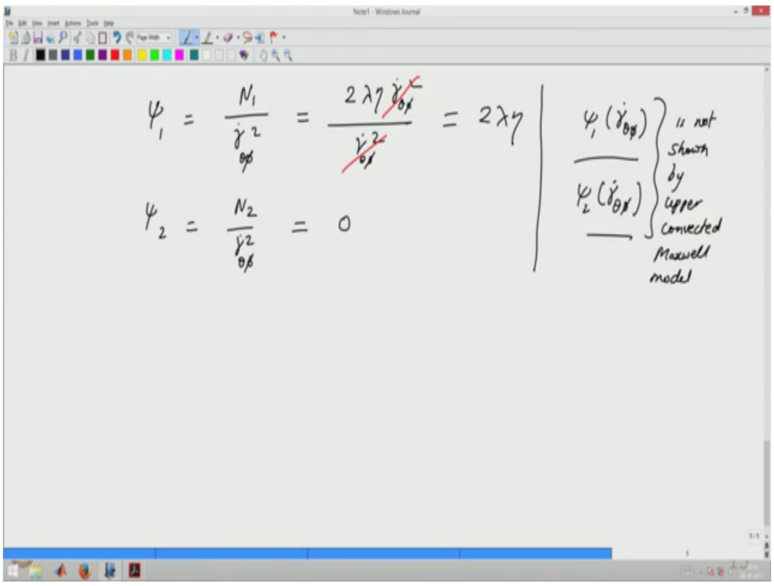

So, in case of upper convected Maxwell model, we can clearly see that psi one which is N 1 divided by gamma dot yx square or gamma dot theta y squared as we been writing this will end up the being in terms of 2 lambda eta gamma dot theta phi squared divided by gamma dot theta phi squared and therefore, we have both of these terms cancelling each other and in the end we get psi 1 which is a constant value similarly psi 2 is 0 because N 2 itself is 0.

So, since this itself is 0 we have psi 2 0. So, in for realistic materials psi one itself is a function of gamma dot theta phi and psi 2 will also be a function of gamma dot theta phi. So, this behavior is not shown by the upper convected Maxwell model and so, clearly the basic mechanism that we have discussed in terms of stretching and orientation, and if we do a microscopic theory of macro molecular solutions using a model in which stretching and orientation are included we can show that model is very similar to upper convected Maxwell model will arise in. And since therefore, that can explain only the presence of normal stress difference which is a constant psi 1.

So, naturally when we have more mechanisms. So, clearly stretching and orientation are not sufficient mechanism, to explain shear thinning or the fact that normal stress difference coefficients are functions of strain rate in case of a real complex material such as polymer melt or a solution so, clearly more mechanism have to be incorporated for us to explain the more complicated behavior of these systems.

So, in summary we have defined the material function in terms of normal stress difference there are two because we have three normal stresses that arise and we can define them in terms of either normal stresses or normal stress difference coefficients. And so, with this we can now conclude our discussion related to normal stress difference coefficients and now move on to the next the aspect of elasticity of fluid behavior which is related to the stress growth.

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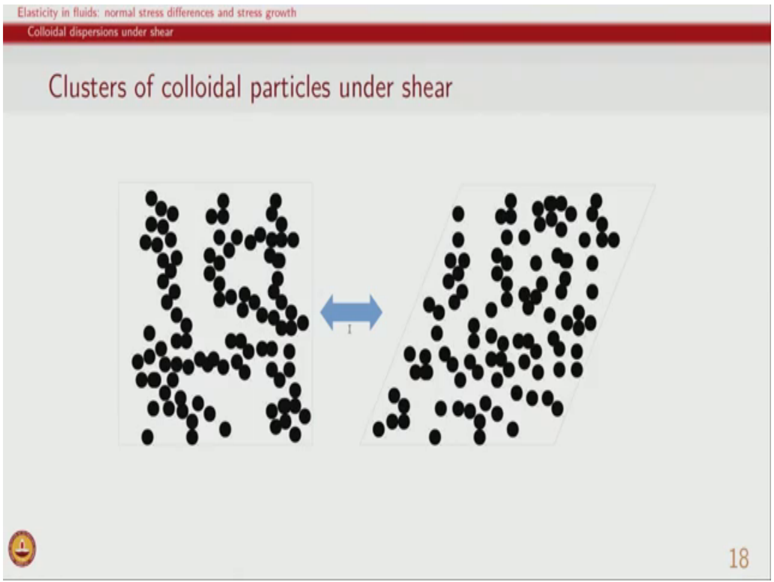

And to do that let us again consider the colloidal particles under shear and this qualitative picture we have seen earlier also, the fact if colloidal particle system is stationary it will form the this cluster of particles and they are all networked with each other and basically we have a percolated structure and therefore, in general the viscosity of such systems we said will be higher.

But if this system is sheared for example, the top surface is being moving at higher velocity compared to let us say the bottom plate which is stationary then in that case these some of these particles are being forced to move at a different velocity compare to the others, and that will eventually lead to may be cluster breakage and therefore, much smaller sizes of cluster and also there is no percolation and clearly in this case therefore, viscosity will be much lower. And so, we discussed said this qualitative picture while we were looking at the shear thinning nature of many of the colloidal dispersion.

Now what we can also think about is what happens as a function of time when I take a material which is at rest, and then apply a strain rate and then look at how does this breakage of cluster happen as a function of time. And is there any elasticity that arises in the material during this period of breakage of bonds between different particles.

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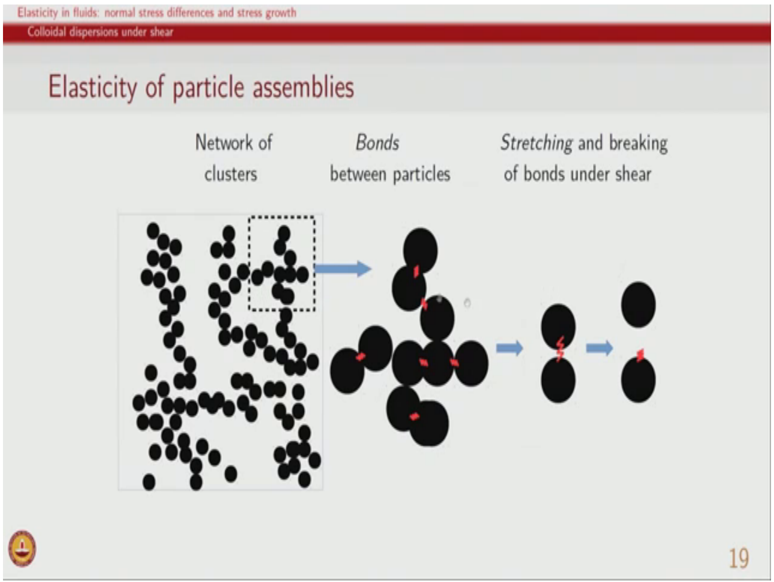

So, that can be graphically pictured and it is shown here that lets say if we have a network of cluster and stationary state the particles are of course, formed a network which is of percolated cluster and let us consider one of these clusters basically all these particles are joined, because they have an attractive force and that attractive force can be thought of as a spring. As so, there is an energy required to pull this particle apart we actually need to apply some force so, that these particles are pulled apart.

So, whenever we have shearing being applied on these materials what happens is these particles are pulled apart and therefore, the bonds between these which is in the form of an attractive force between these particles gets pulled and because of this spring is pulled there is an elasticity associated with it. So, if we let us say deformed for only some extent and do not apply large deformation then there is a chance this material can again the particles will come closer and therefore, again the network of clusters that was observed will be back there. So, therefore, there is again elasticity and recovery possible with such bonds between particles. However, if the strain rate which is applied is large enough and the deformation therefore, applied is large enough the particles will actually separate. So, in this case what happens is, we will again see the overall viscous behavior and the overall viscosity or the stress required to move may appear to be smaller.

So, in general what we expect is when we start shearing this material as a function of time, as soon as we apply the strain rate and as soon as we apply the deformation the all these bonds will lead will resist the deformation and therefore, the stress within the material will grow rapidly. And beyond a certain point when these springs actually break or the bonds between the two particle break, then we have these stress either decreasing or becoming constant.

And so, therefore, we generally have in these viscoelastic material what is called as growth and. So, stress growth is usually observed during either a simple shear flow or an extensional flow, again the idea is to apply constant strain rate and then look at stress as a function of time and by looking at stress growth and we can get an idea about what is the elas what are the elastic contributions within the material and therefore, we can again make a hypothesis regarding, which are the mechanisms which are important in determining the viscoelastic response of these materials.

So, in the next segment of the class we will define stress growth and look at how again Maxwell model response is there for stress growth and for a non-linear deformation, which means a large deformation what kind of stress growth is observed for a system such as colloidal particle system or for a macromolecular system.