Rheology of Complex Materials

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Elasticity in fluids: normal stress differences and stress growth
Lecture - 48
Normal Stresses – 1

In the course on rheology, so far we have looked at the response of complex materials in terms of steady shear, in which case a constant strain rate is applied and then steady state when is reached we measure the stress and therefore, measure the viscosity.

So, we saw for example, that viscosity can be shear thinning shear thickening and the simple model that we used to understand some of the variation was the Carreau Yasuda model and then eh similarly we looked at the linear viscoelastic response in which case we looked at a stress relaxation we looked at oscillatory shear and creep. And again in each case we defined material function and we looked at the response of Maxwell model or a standard linear solid model and.

So, with that we reviewed the response of materials in two broad classes categories of response, one is the steady shear response and where only the shear stress is involved and the other was small deformation or the linear viscoelastic response.

Now, there are many situations in which in steady shear in shear a normal stresses are key indicators of elasticity in materials. So, therefore, the measurements of normal stresses during simple shear experiments is very useful in terms of understanding the viscoelasticity of complex materials. Similarly the time dependent nature of stress during simple shear or a study or an extensional flow is also of interest. So, therefore, in these few lectures, we will look at the response of materials to understand how do normal stress differences arise in viscoelastic materials and how stress growth is useful in terms of characterization the viscoelasticity. So, both of these are key features of elasticity in fluids.

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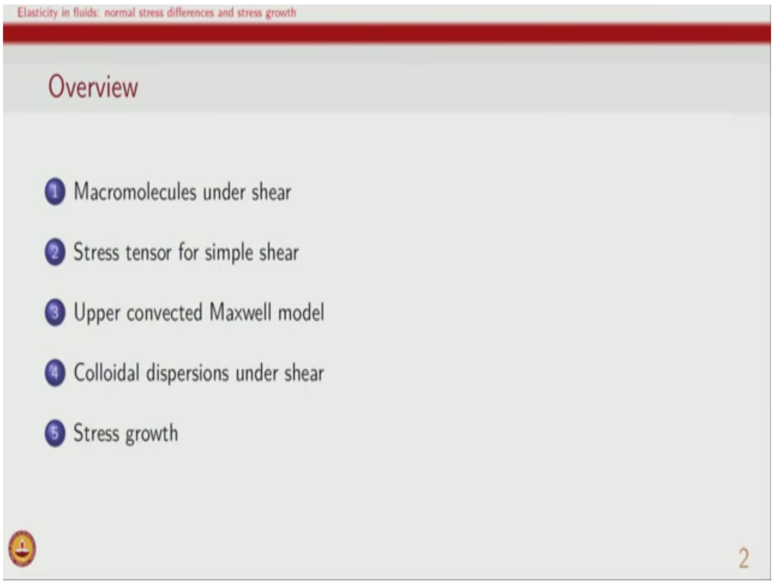

So, therefore, in this set of lectures we will first look at macro molecules under shear which we have seen in earlier and therefore, we will review it quickly, and then we will see what are the state of the stress given let us say a system such as macro molecule is undergoing simple shear and then we have we will look at the upper convected Maxwell model, which will be useful in terms of understanding the normal stress differences and how do they arise.

Finally we will also look at a colloidal dispersion under shear and then try to understand what is meant by stress growth during steady shear experiment.

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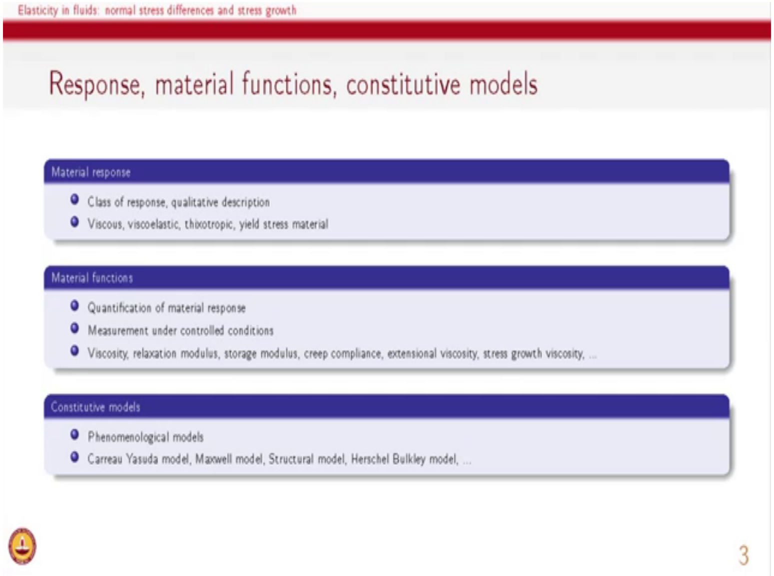

So, again to the overall frame work that we have been following that we can understand response qualitatively, but we define material functions to quantify the material response, and then we also look at some simplistic constitutive model so that we can understand how the material functions for realistic materials are. So, therefore, we have looked at viscous response, we are still continuing to look at viscoelastic response and in some of future lectures we will look at the thixotropic and the yield stress material. Similarly we have already started doing quantification of material response.

Through viscosity relaxation modulus storage modulus and creep compliance and this set of lecture for example, we will look at stress growth viscosity and normal stress differences as additional set of measurements, which are again done under control conditions. And then of course, we continue our journey also by parallely looking at some phenomenological models, which are also simple model, which can eh explain to us it can show us the basic mechanisms which are prevalent in terms of generating the behavior that we are studying using these material functions. So, for example, we already looked at Carreau Yasuda and Maxwell model and in these set of lectures we will look at for example, the upper convected Maxwell model.

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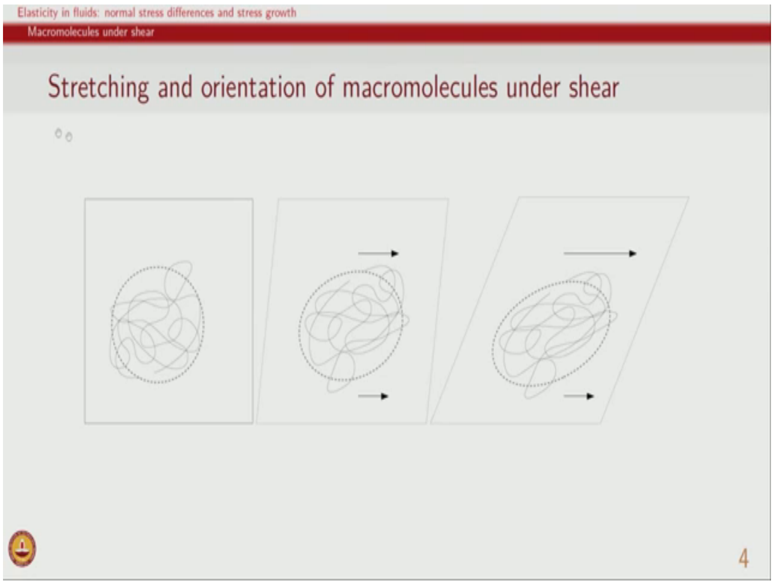

So, memorize again our discussion related to macromolecules under shear, macromolecules in that equilibrium can be considered to be spherical blob because the malo macromolecule is coil like, and when the shear because these some segments of polymer are dragged faster because of their overall shear flow we have stretching and orientation of macromolecules.

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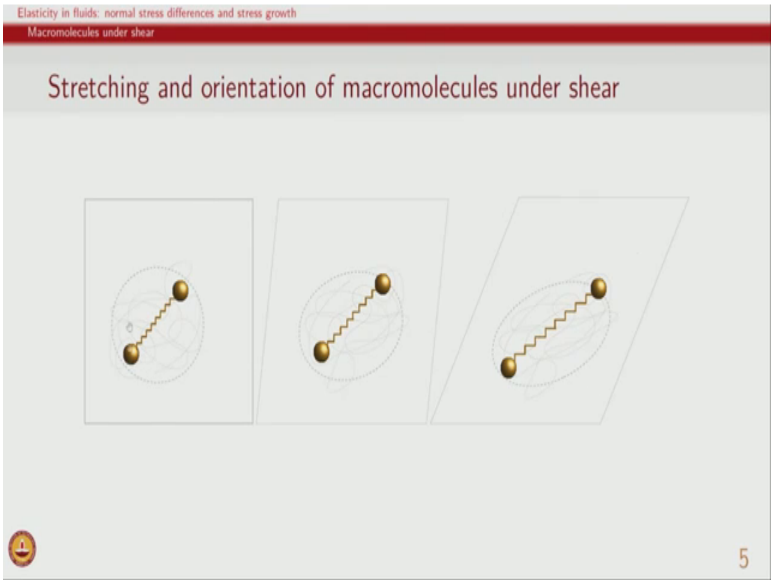

And we have also seen this picture in terms of dumbbell, where two beads are connected using a spring and we understand that when shear is applied the spring gets extended or therefore, there is stretching involved and similarly the spring on average may also orient in the direction of the shear and therefore, there is a change in orientation of the object which originally to begin with spherical and even in all these different pictures we should always remember that it is a basic at the macro molecular scale its a fluctuating picture.

So, therefore, the molecule will always not have the shape, but its fluctuating in an average it may be oriented in the direction of the shear. Just a way here there is no differential orientation once the shear starts there is a preferential orientation. So, therefore, both stretching and orientation takes place when we have this macromolecule under shear and because of this what are the consequences.

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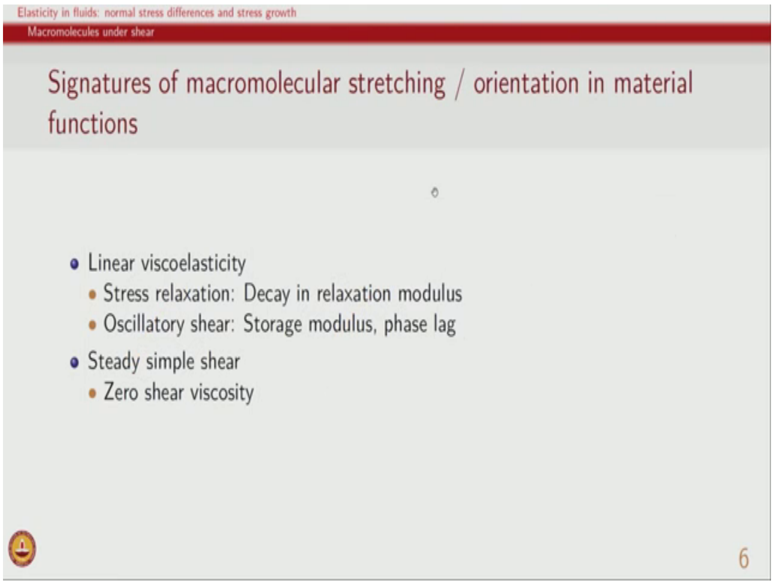

And so, we have summarized this consequences in the two sets of results that we have seen in material functions, we saw for example, that stress relaxation there is a decay in relaxation models.

So, what happens is, when we apply a shear molecule gets stretched and oriented, but if the deformation is kept constant because of the thermal energy available to macro molecule the segment starts relaxing the segments starts moving and in the end it reverts back to this and therefore, there is a decay in relaxation modulus. Similarly when we apply oscillatory shear there is a storage modulus which is the elastic contribution due to the stretching and orientation of molecules and similarly of course, there is a phase lag also because not the contribution is viscous is as well as elastic therefore, there is a phase lag. In terms of steady shear property we saw that the consequence of the stretching and orientation was that we get higher viscosity. So, because of the macromolecules also present and the fact that is exchanges a friction with solvent, and it stretches in orients will lead to a 0 shear viscosity, which is different compared to the solvent itself. And so, these are the consequences or the signatures of macromolecules stretching and orientation in material functions which are related to the polymer solutions.

So, today we will look at one more such a feature, which arises because of the basic mechanism which are stretching and orientation.

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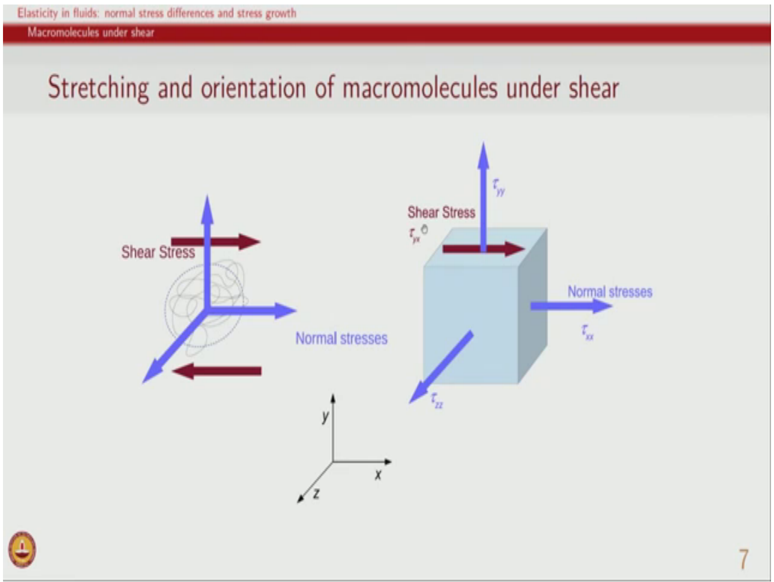

For example we during introduction introductory lecture we also saw and it was expressed that when we apply a shear stress generally for viscous material, we would expect only shear stresses to be there, but in case of polymeric material such as polymer solution we will also get normal stresses.

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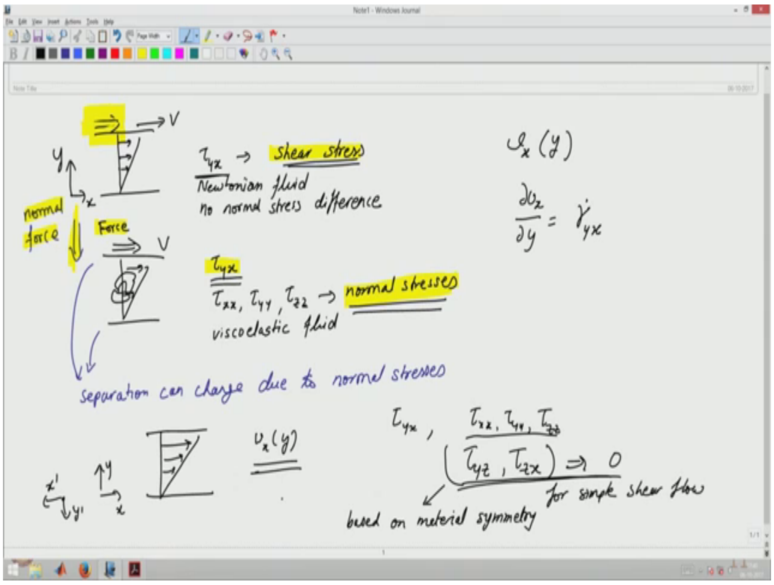

So, for example, if we have let us say fluid, which is let us say water and we apply constant velocity on the top plate and then all these molecule start moving and of course, we get the linear velocity profile. And so, in these kinds of cases what we will have in the material is only the shear stress. But because this macro molecule is there and since the macromolecule is getting stretched and the stretching leads to the normal stresses being generated in the material because basically there is a streche stretching happens and there is a tendency of the material molecule to actually revert back to its original spherical shape. So, that is why we are denoting it using a spring. So, since spring is there the spring develops a tension and therefore, this tension is consequent consequently leads to the normal stresses in the material.

So, if we use the rectangular coordinate system to describe this simple shear flow, we have tau y x which we have been talking about is the shear stress which is there in the material, but we will also have tau xx and tau zz and tau yy which are the normal stresses. So, therefore, in viscoelastic fluid, when we take it between the two plates we will have. So, let us say it is a polymer molecule and therefore, we are again applying steady shear experiment we will of course, have tau yx, which is the shear stress, but we will also have tau xx tau yy and tau zz as the normal stresses and this is in case of viscoelastic materials such as polymer solution. And so, given this eh situation where the normal stresses arise, even though the only force which is being applied is a leading to shear stress. So, the force which is applying for the motion is shear and therefore, there are shear stresses in the material.

So, what happens is in case of viscoelastic fluid because of these normal stresses which are generated in the fluid, in addition to the force that we have applied we will also need to apply another normal force. Otherwise due to the normal stresses there is the tendency of the two plates to separate. So, the distance between two plates the separation can change due to normal stresses. So, in case of a Newtonian fluid, we apply a force which is leads to shear and therefore, we get shear stress. In case of a viscoelastic fluid we apply a shear stress we naturally get the shear stress, but we also get normal stresses and because of the normal stress we get a normal force also. So, therefore, if we measure this normal force, then we can actually get an idea about what these normal stresses are. And that is the principle behind measurement of normal stress measure the normal stresses in the material. So, let us look at how do these normal stresses arise in different type of fluids that we already know.

So, what we have seen what we are saying is we are imposing a full shear flow as was drawn here and where we the coordinate system is basically we have the flow is in x direction therefore, we have only velocity vx and it varies only as a function of y. And naturally the derivative of velocity with respect to y is what we defined as gamma dot yx.

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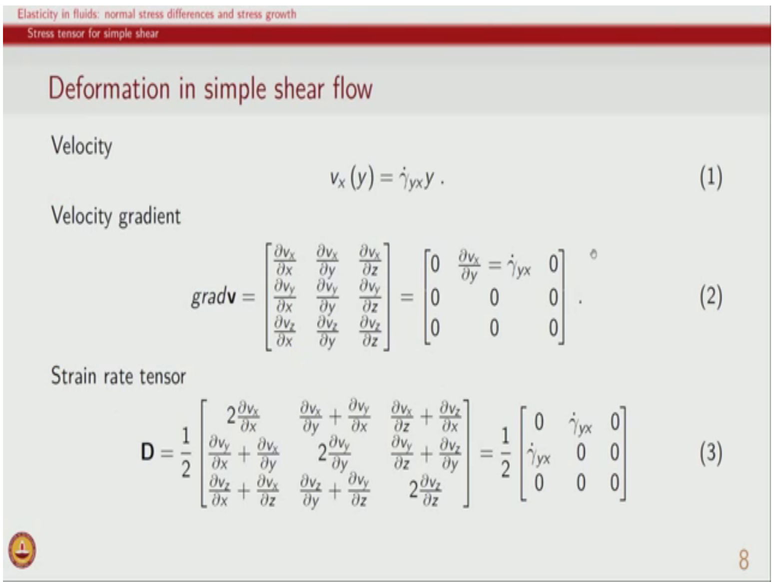

And so, in this kind of a flow what we have is linear velocity profile and that linear velocity profile will continue to change depending on how is strain rate. If strain rate is constant then it is a linear velocity profile even if strain rate changes as a function of time with respect to y the velocity profile remains linear; however, it can change as a function of time. Because of this velocity profile only one component of velocity of gradient is non zero and which is del V x by del Y because V y as well as V z are 0 and of course, there is no dependence on x and z.

So, therefore, all these terms go to 0 and del V x by del y is the only non zero component of velocity gradient. And because of this the strain rate tensor which is nothing, but gradient of velocity plus its transpose and half. So, what we have is two components of the strain rate tensor being non zero and.

So, given situation like this where we have only two components of the strain rate tensor non zero, what happens is even the stress tensor. Therefore, only two components which are non zero.

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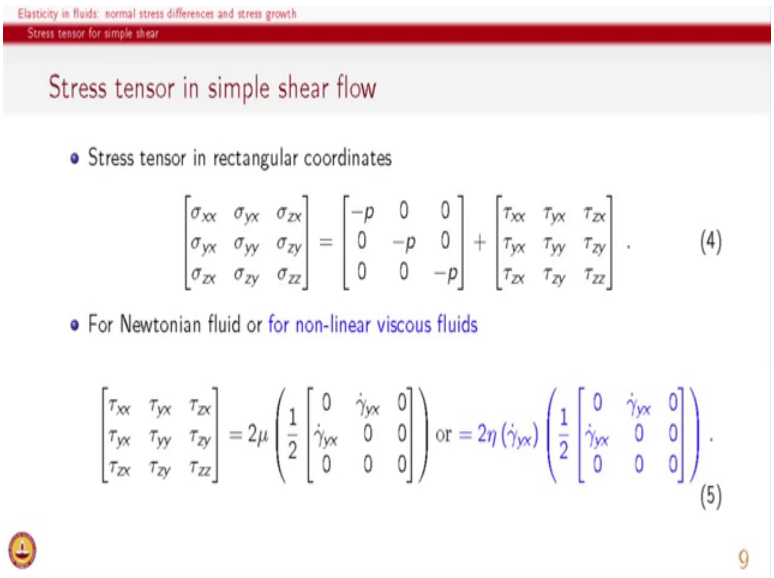

So, let us see how is, that the case. So, in general stress tensor of course, will have all nine components, but since we are dealing with symmetric stress tensor, we have three normal stresses and three shear stresses. Of course, we can split this into the isotropic or the deviatoric part.

The isotropic part is basically pressure and for incompressible fluid we know that pressure is an undetermined factor because for various values of pressure density does not change. So, if there therefore, there is no influence on material behavior; however, gradients in pressure are important, but the value of pressure itself there can be different values of pressure for which the gradient is the same and therefore, behavior will be the same. So, absolute value of pressure is not always not always determined in case of incompressible fluid then we have the deviatoric stress and again that six dependent components are there.

Now once we have simple shear flow where we mentioned that there are only two components of the strain rate tensor, for a Newtonian fluid where the stress is directly proportional to the strain rate tensor, we again have two new times basically d.

So, you can see that because these are the only two component which are non zero tau yx will only be the component which will be non-zero. Tau xx will be 0 because d xx is 0 similarly tau zy will be z0 because dzy is also 0 and by the way same is the case for a non-linear viscous fluid also.

In case of non-linear viscous fluid for example, the Carreau Yasuda model or the power law of fluids that we saw, the viscosity can be a function of strain rate that is being applied and the overall dependence on of stress on strain rate tensor again similar to what is there in Newtonian fluid case. So, the difference in between Newtonian fluid and the general non-linear viscos fluid is the fact that, the viscosity can depends on strain rate. And that is why if you recall we had called this type of non-linear viscous fluids as generalized Newtonian fluids. The behavior is generalized by rather than assuming mu to be a constant material constant, it is a material function which depends on gamma dot yx. But in case of non-linear viscous fluids also since dyy is 0 that multiplied by 2 eta will again be 0 and tau yy will be 0.

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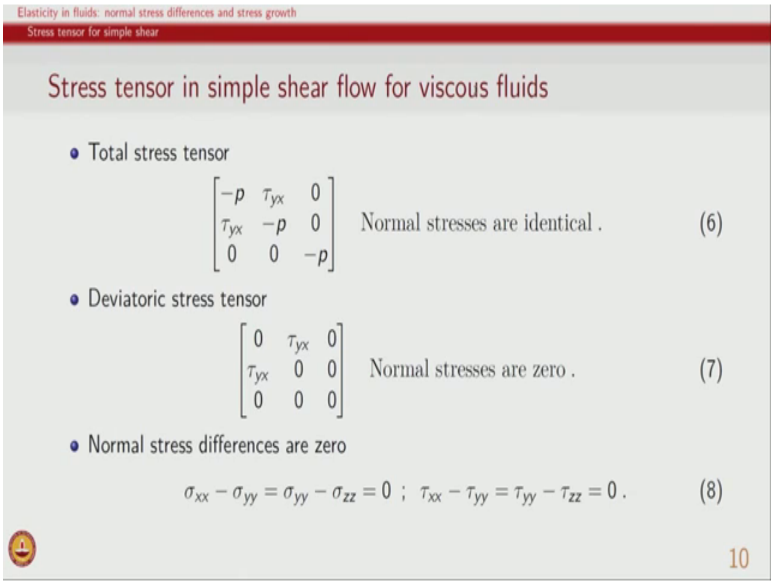

So, in summary what we have is in case of simple shear flow for viscous fluids, it does not matter whether it is a Newtonian fluid or a non-Newtonian fluid such as the shear thinning fluid as long as we assume viscous behavior the total stress tensor only has this form.

So, there is a shear stress which is non zero and of course, the isotropic part of stress itself is also non zero; however, the other components are 0. So, what we can observe is that the normal stresses are identical in the sense that all the three stresses are minus p. Now if you look at the deviatoric stress tensor it has only shear stress component tau xx tau yy tau zz is 0 and therefore, we can say that normal stresses themselves are also 0. Since the pressure is an undetermined factor as I mentioned earlier, what we do is rather than looking at the value of these normal stresses themselves we can look at the normal stress differences.

So, whether we look at the normal stress differences of sigma xx minus sigma yy or tau xx minus tau yy we see that they are both z0. So, sigma xx minus sigma yy or tau xx minus tau yy are both 0. Similarly tau yy minus tau zz or sigma yy minus sigma zz both of them are 0. Since there are three normal stresses we could construct two normal stress differences which are independent and therefore, what is listed here is both of these normal stress differences are zero.

So, in a summary what we are saying is that if we apply a simple shear flow on viscous fluids, which is steady shear flow and at steady state we look at the stray state of a stress tensor, the state of the stress tensor would lead to normal stress differences being 0. And that is what is indicated in this plot here when we said that the force which is being applied with only be shear and in this case no normal stress or difference will arise. So, there is no normal stresses, stress difference will be observed in fluids and for viscoelastic fluid normal stress difference will arise. And therefore, then it can be used as an indicator of the mechanisms as well as elasticity that is involved in the material.

So, now, let us look at the stress tensor in simple shear for a general fluid. In case of a general fluid what we have is again the same flow where we have taken the fluid in between two different plates and since this is the viscoelastic fluid and we are imposing velocity on it we can see that yx and of course, z is perpendicular to the screen and. So, with this velocity gradient we can try to argue and ask the question as to what will be the most general state that is possible.

So, we know for sure that tau yx must be there, because even for Newtonian fluid case tau yx was there and we expect that to be there also. And additionally of course, we have tau xx tau yy and tau zz which are the normal stresses, and we also have the other two tau yz and tau zx as the two other shear stress. So, using material symmetry arguments one can show that both of this will be 0 for simple shear flow. Basically if you look at this a flow for example, if I use another coordinate system which is let us say rotated by 180 degrees then what I can do is that I can use the coordinate system where y prime and x prime are basically rotated.

But in if you using these two coordinate system, you can actually show that the velocity field will not change even if we use two different coordinate system and since velocity field does not change the forces on any plane within the material will not change and using these kind of arguments it can be shown that the normal stresses will be the only stresses present, while the shear stresses associated with tau yz and tau zx will be 0. So, based on material symmetry this can be shown.

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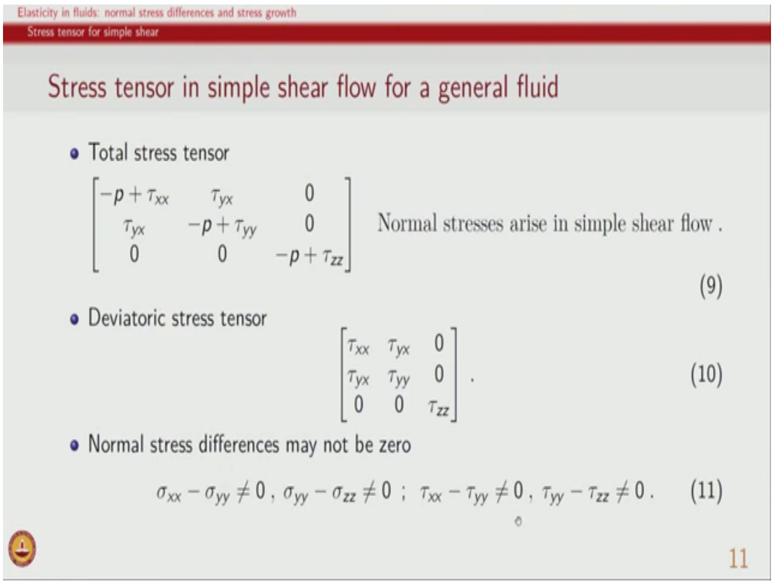

So, we will assume this to be the case and therefore, the most general state of stress tensor for a fluid which has viscoelastic effect may be the following. So, this components which are associated with tau zy and tau xz and tau. So, they will be 0.

So, therefore, normal stresses arise in simple shear flow and of course, just to remind us this we are saying again based on this picture the fact that this molecule is getting stretched and can develop a tension it will lead to the the normal stresses arising in the material. So, given that normal stresses arise in simple shear flow, the overall deviatoric stress tensor also then therefore, will have five components two of which are same tau yx. Therefore, the four independent components of stress which are to be solved; so unlike the earlier case where when we had a simple shear flow of materials.

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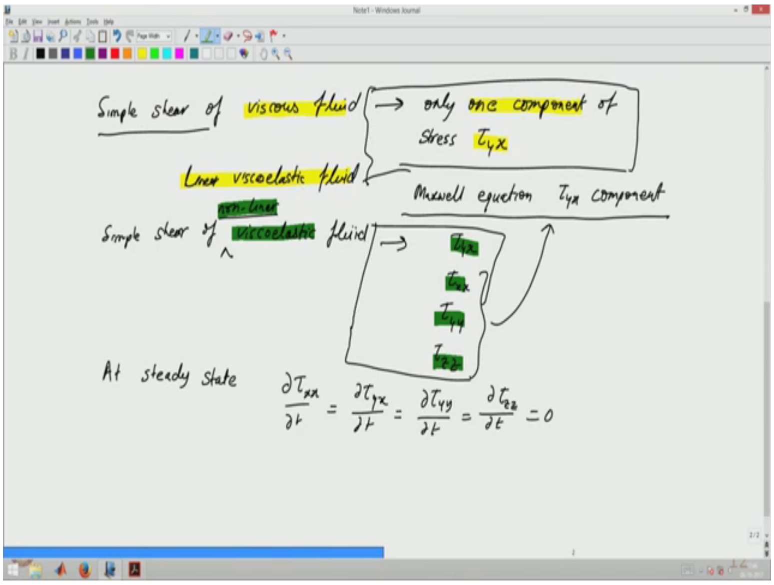

So, simple shear of viscous fluids we had only one component of stress and which is tau yx. So in fact, all our earlier development we did with this only one components. So, even when we rad wrote down Maxwell equation for instance we actually wrote it for tau yx components, because that was the relevant stress as far as simple shear flow is concerned. In terms of now what we are saying is that if there is a simple shear of viscoelastic fluid, then we will also have tau yx tau xx and tau yy and tau zz.

So, why is it that we did not consider for Maxwell equation, actually an example of viscoelastic fluid why was tau xx tau yy tau zz not considered? So, what we can show is since Maxwell model is a linear model it actually fails to show any normal stress difference. So, for a general fluid we expect that normal stress differences may not be 0, now let us see what does a Maxwell model behave when it is applied in simple shear.

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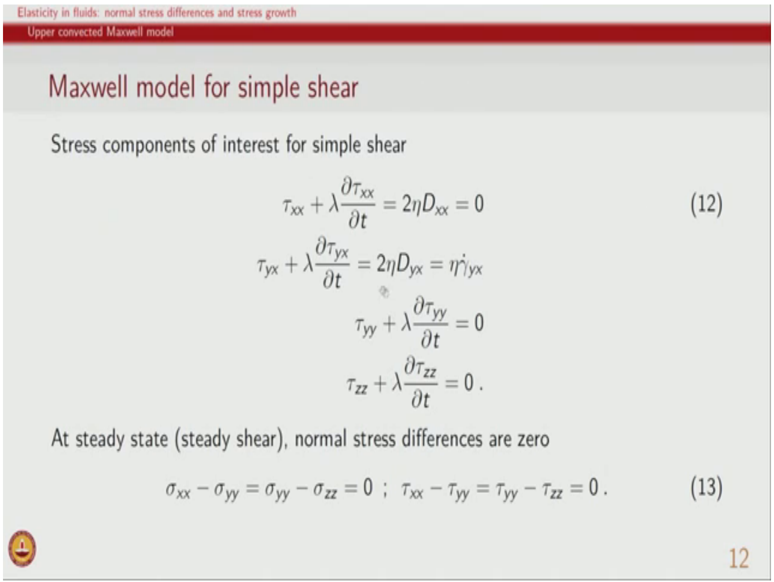

So, when we look at Maxwell model the stress components of stress, earlier we had only looked at tau yx and this is the standard equation which we have used earlier for analysis assuming that the shear stresses, shear modulus, shear properties were the only important things. Now given that we know that for viscoelastic materials, normal stresses can also arise the other three components of the overall constitutive relation of Maxwell model are also relevant.

So, it would read very similar to how it is for the tau yx compound, basically we will have tau xx which is similar to tau yx term here then the rate of change of tau xx with time, which is similar to tau yx rate of change with time multiplied by of course, the relaxation time lambda and the right hand side is two eta times the strain rate tensor component.

Since we are looking at xx component, we also have dxx which is the strain rate component xx. And of course, we have seen earlier that how D yx which is the shear rate component is related just gamma dot yx. So, if we write down the governing equations we have this four governing equations which need to be solved for us to look at what is response of Maxwell model in simple shear, and since we are looking at steady shear at steady state basically all these time derivatives will go away. So, therefore, at steady state we can set all the time derivatives partial time derivatives to be 0. So, we have del tau xx by del t or del tau yx by del t or del tau yy by del t or del tau zz by del t all 0.

So, given that this is the steady state and we can clearly see that if these two terms fall out then tau yy and tau zz itself is 0, and since tau yy and tau zz is also 0 since dxx is also 0 tau xx this term is also 0 therefore, tau xx is also 0. So, the only non-zero stress in case of Maxwell model is really tau yx and so, at steady state normal stress differences is for a Maxwell fluid are 0. So, even though Maxwell model is a viscoelastic fluid in fact, it accounts for no normal stress differences.

So, we can add here when simple shear of viscoelastic fluid, which is linear we will observe that there is only one component of stress. However, if we have simple shear of non-linear viscoelastic fluid, then we have all the four component present. So, let us just to highlight again if we have a viscous fluid or if we have a linear viscoelastic fluid we will only have one component which is the shear. However, if we have a non-linear viscoelastic material, then we have all four components of stresses present in the material and therefore, we can characterize the non-linear response.

Therefore now, one needs to look at as to what is meant by a non-linear viscoelastic response, we have while defining linear viscoelasticity we have said that whenever we confine our attention to small deformations that is linear viscoelasticity therefore, Maxwell model can be used only for small deformations. When we do a steady shear experiment we are actually taking the material and applying very large deformations to it and therefore, for normal stresses which are measured cannot be predicted using the Maxwell model.

So, the non-linear contour part of Maxwell model is arises when we use the correct form of stress rate. The statement of Maxwell model is the fact that there is stress and stress rate are related to the strain rate in the material. So, we have the stress and the stress rate related to the strain rate and for stress rate we are using the partial derivative.

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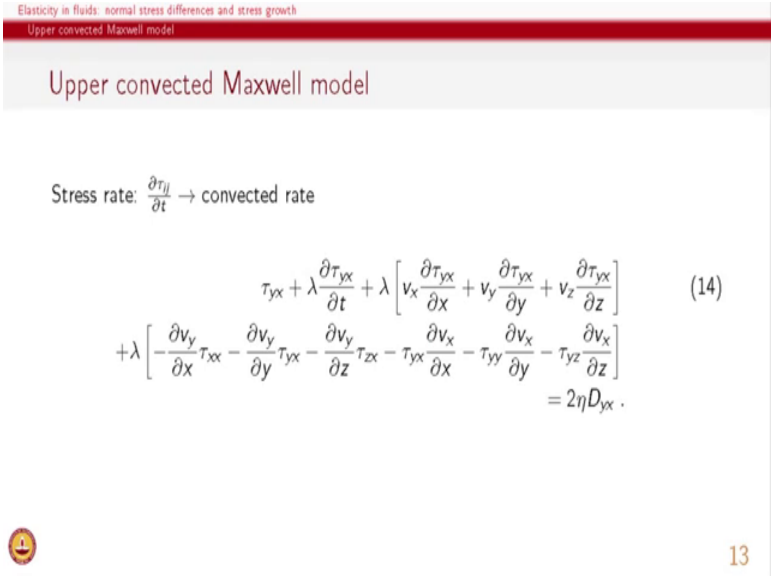

However from a material objectivity point of view only when we use convected rate for the stress rate, then we get a model which can be used for arbitrarily large deformation and this model is called upper convected Maxwell model and again. So, we can go through the overall development of how does the upper convected Maxwell model get derived, what we can do is do a phenomenological development to try to based on continu mechanics principle, what are the types of terms which can be involved in a constitutive relation and on the base of that we can going to collect terms together and propose convected models.

The other alternative is of course, we can also have kinetic theory and statistical mechanical theories of polymer behavior, and using such microscopic theories we can try to understand as to how convected rate arises in material response whenever we are considering large deformations.

So, in the next part of the lecture we will look at how is this upper convected Maxwell model useful in terms of understanding the normal stress difference in the materials.