Rheology of Complex Materials
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Strain measure and convected rates: conceptual framework
Lecture – 45
Strain and Convected Rate- 2

In the previous segment of the lecture, we saw that how relative displacement is there depending on which two points we pick and this is what I have shown graphically in the slides here.

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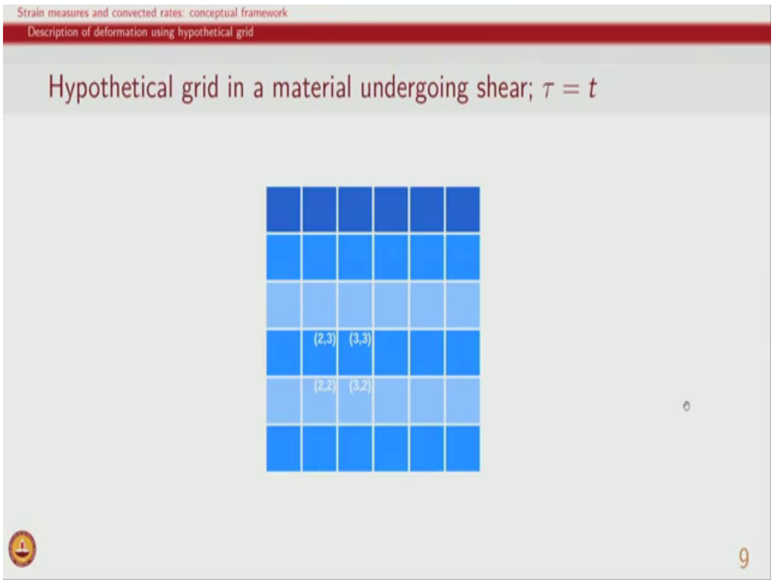

For example, if we put a grid of the coordinate system at the present time, and this is now embedded in the material system let us say and now when any time future what will happen is, these points are moving faster.

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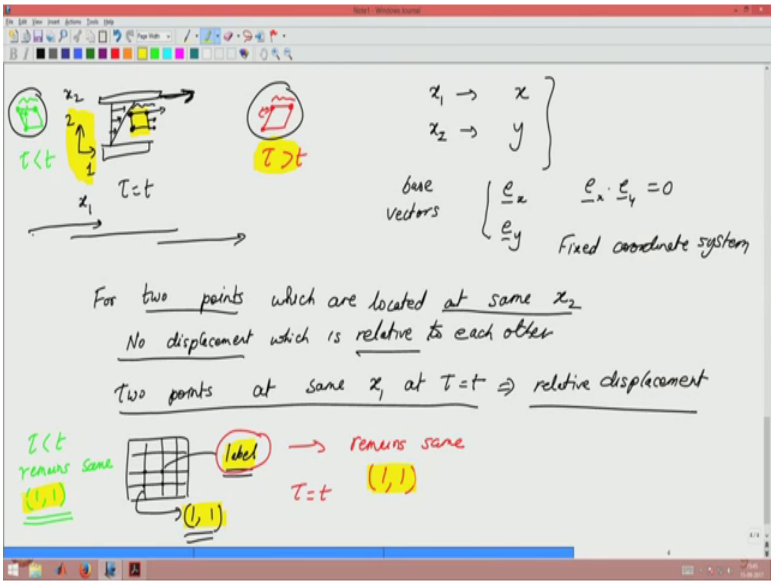

Remember that we have a simple shear flow and therefore, the points which are at the top are going to move faster and the points which are below are going to move little slower.

So, what we would expect is, the point which is at the top to move the fastest. So, this point here will move quite fast all these points in fact, then these points will move little slower and these points move little slower and so on. So, if you just focus on these 2 points because let us say this is the 0 0 point. So, this will be 1, 1 and this is 2, 2 and 3 3 and so on. So, I have just marked these four points with the coordinate numbers and now we can see what happens to it when if it was in the past.

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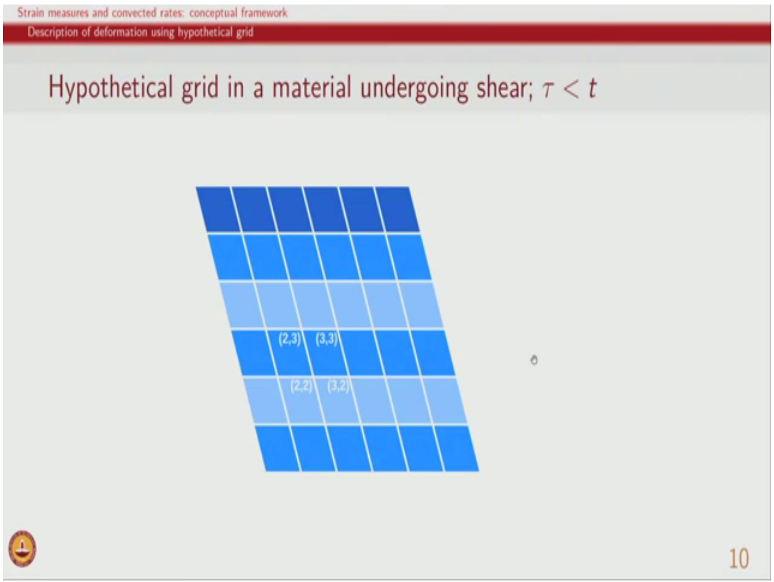

Since to come to this present state we had to, so the clearly since these particles are moving faster at summer time in the past, they must have been to the left. And similarly these points which are moving the whole set of material points will also move, but at the same time we will also have little more motion for these particles compared to this. So, therefore, these points must have been to the left compared to these points and this is what we see here. The bottom points of course, remain fixed because at the bottom plate there is no velocity. However, this 2, 2 point has moved with respect to this 02 point similarly these 2, 3 and 2, 2 point and 3, 3 and 3, 2 point.

So, these points have moved to were at the left position compared to this. Similarly if we take a look at the present position and then take a look at the future position.

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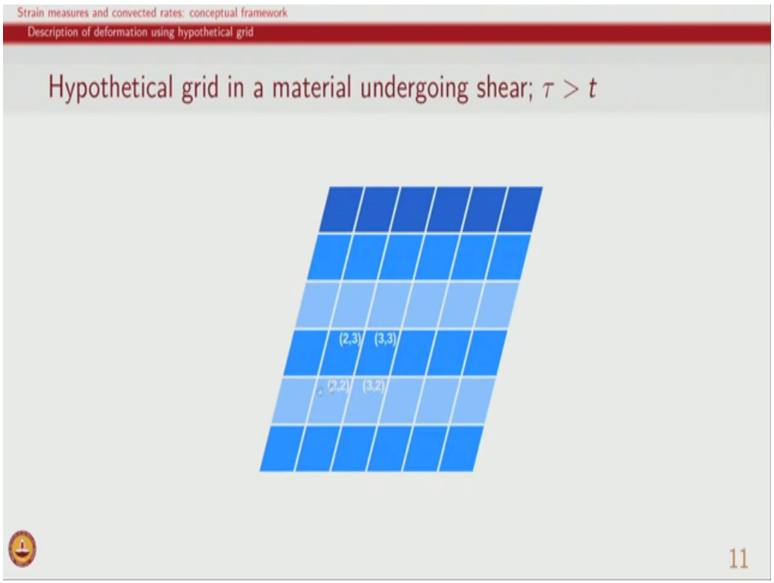

Then we will again see that the particles have actually moved to the right and again the particles which are closer to the top plate have moved to the right. So, what we can see here is the grid that, I have drawn is actually an indication of the deformation that is I being the fluid is undergoing. So, if at all I can keep track of these coordinates and grid points and how the grid points are changing then I can in fact, keep track of deformation itself.

This is an idea behind a convected coordinate system. So, what we can do is at present time, we fix a grid onto the material point. So, we have already said that we are using a coordinate system to describe the overall parallel plate shear problem and so, what we are saying now is the grid which was fixed and I explained it using couple of points, now what we will do is the present coordinate system is basically being. So, this present coordinate system, which I embedded into this material, what I will do is I will pick each and every material point here and actually I will label the material point.

So, therefore, each and every material point it gets the label and this label will make sure that I can track this material particle. So, in any time in future, the label of the material particle will remain the same because it is a label of that particular material particle. Similarly the same material particle at some time in the past when tau is equal to t is again remains the same. So, in this case what we are doing is not really keeping track of material through a fixed coordinate system like what we did here, but we are keeping track of the material particles using a set of labels and so, these labels can be called the coordinates themselves. So, for example, this point which is 1, 1 in current time, I will say that this point will remain 1, 1 even in the future. So, its coordinate will again remain 1, 1 and similarly at any time in the past also its coordinate will remain 1 one. So, what we have now is an example of a convected coordinate because the coordinate itself is getting moved along so.

So, the same point is actually 1, 1 the same point is same point is 1, 1 at any time in the past it is 1, 1 at some time in the present as well as at some time in future. And that is what was depicted using these grid points also. So, this is a grid which is embedded in the material when actually deformation happens if we used a fixed coordinate system the coordinate system will remain fixed and therefore, we can track the how the material particles are moving.

Now in the alternate way to describe the overall flow is to say that I will label the material particles and I will embed them. Embed the overall grid into the material and this hypothetical grid itself keeps on deforming because the grid is actually attached to the material particle. So, this square which I had drawn here con connecting these four points, because these four points are moving at and they have relative displacement with respect to each other, at some time in the past it had become a parallelogram like this and somewhere in the future it also is again another parallelogram.

So, in both cases the shapes are completely dependent on how much is the deformation and what is the nature of flow that the material is undergoing. But the 2, 2 point which at some point in future is at this location based on let us say the fixed reference point which is the corner let us say of this particular screen, then that 2, 2 point has moved to the right because the same location in fact, was somewhere to the left and even the grid is moving to the right.

So, therefore, and similarly in the past the 2, 2 point was actually to the left and. So, you can see that basically you have this 2, 2 point itself somewhere in the left at some time in the past and somewhere to the right, but what we are going to do is continue to keep the label of 2, 2 on that and so, therefore, these numbers that I have draw written here are in fact, the convected coordinates.

Because the material particle is now labeled using these coordinates and these coordinates are labels of a material particle the material particle can change its position, but its label remains fixed and since our coordinate is same as the label, the convected coordinates do not change. So, this is something which is a unique way of following the material deformation and this is somewhat counterintuitive to the cylindrical or rectangular or spherical coordinate systems that.

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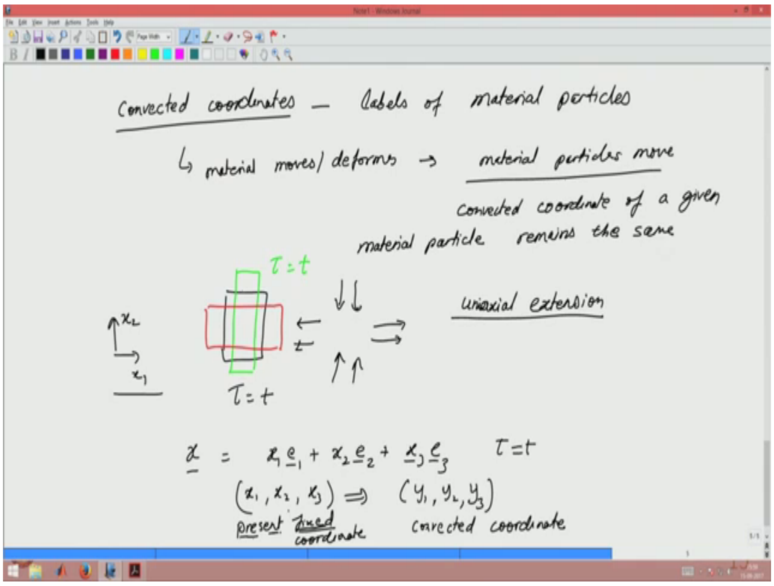

We usually use to solve engineering problems, what we are saying now is the convected coordinates are nothing, but labels of material particles.

And so, when the material moves and deforms material moves deforms the material particles move, but the convected coordinates convected coordinate of a given material particle remains the same. So, what is the advantage of doing such an exercise you may ask and the advantage is the fact that, the if I do a convected coordinate like this and if I keep track of what is happening to the grid, I can immediately keep track of whether the material is deforming and which way it is deforming and this is also called convected, because I am getting convicted along with the material, the material coordinates themselves are getting convicted along with the material and I am looking at the overall material deformation only from that viewpoint.

Therefore, what is clearly also happening is the unit vectors which are describing which can be used to describe. So, since in the present time we have used a coordinate system, which is fixed in space which using x 1, x 2 or x y coordinate system which we have already described here.

So, given that this is fixed in space at any time in the future or any time in the past also the coordinate system will remain fixed. But now what we can do is since the coordinate system itself is deforming, there is no longer useful to use the base vectors which we had for a fixed coordinate system to describe. Therefore, what we need to do is to actually define the base vectors also in this convected frame.

So, therefore, using these I deformations we can actually define a convected frame and in which the coordinates will be given based on the material particles and the base vectors will also be dependent on how these material coordinates are connected to each other. For example, we define the coordinates in this case which is fixed basically the coordinates are defined in terms of how the 2 points are located. So, for example, as I said any two these two points here were at the same x 2. Similarly the points which are located here and then this are at same x 1, because that is because this x 1 is fixed. Now we are looking at a material coordinate, which keeps on shifting with respect to the fixed coordinate system, but we need to describe the relationship between material coordinate in the at any instant of time.

So, if you look at it here the 2, 2 and 2, 3 points are connected using this material fiber while at the present time the same material fiber was actually coinciding with x 2 similarly this 2, 2 and 3, 2 you can see that the material fiber is horizontal or its in the x 1 direction and in anytime in the future or in the past it still remains. Even if there is a relative displacement between these two material particles set of material particles it appears that the coordinates which are 2, 2 3, 2 they are not really changing with respect to it each other.

This is of course, the case because we are looking at a specific deformation example of shear and that is why we see that this deformation of the grid itself gives us an complete idea of how the material points are changing now let us carry on this exercise with let us say an extension deformation. So, again at time present we have the same set of four points, which because of extension will get extended in x direction and contracted in y direction.

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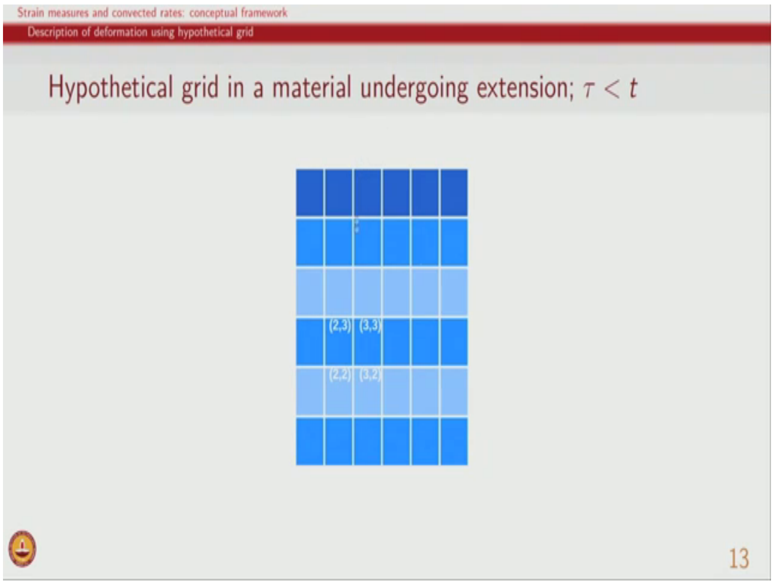

So, at some time in the past the in the y direction the length was much longer while in the x direction the length was less.

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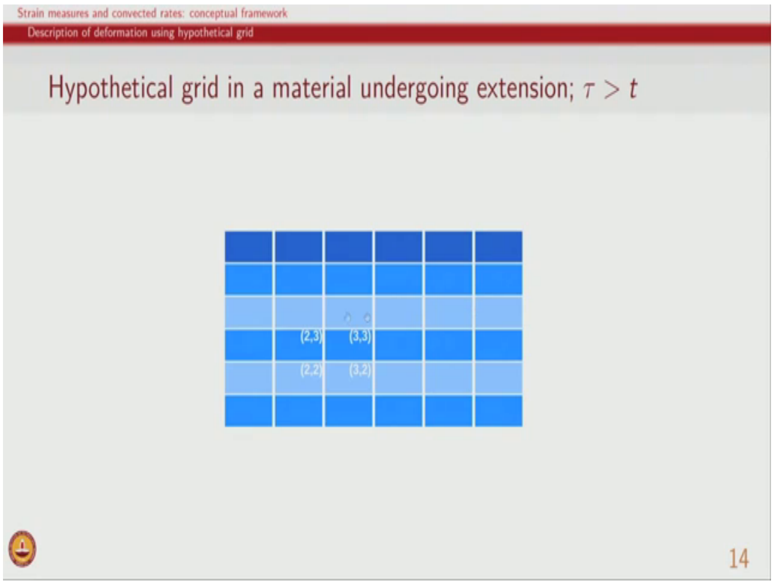

Or at some time in the future the x direction length is going to be much larger and the y direction or the x 2 direction length is going to be much smaller.

So, again we can look at this square which was present and since we have taken these four points which are each changing coordinate by 1 this is the square of unit 1 size as soon as we go to a next configuration, we see that now the x 2 distance between them has increased because this is sometime in the past and the material is getting extended. So, just to remind you the kind of deformation that we are looking at is the following that at present time, we have the material as a square grid and then sometime in the past it actually was at this and then sometime in the future it is actually this.

So, basically what we have is material is flowing inverts in the y direction or the 2 direction x 2 direction and the material is flowing outward in the. So, this is an example of uniaxial extension that we have already seen. And so, now, in this case we can clearly see that the position of 2, 2 and 3, 2. So, this material fiber which connects 2, 2 and 3; 2 was shorter in some time in the past though the 2, 3 2, 2 points the length was longer compared to what it is at the present time. Similarly if we compare sometime in the present and future then the points connecting 2, 2 3, 2 are further apart while the points connecting 2, 3 2, 2 are closer together and again if we look at the grid and how it is the grid itself overall is changing.

We can clearly see that this is an example where there is an extension and contraction involved. Extension in one direction while contraction in the other direction. But keeping these coordinates the same what we have again done is embedded a coordinate system along with the material and as the material deforms the coordinate system also deforms. So, again if we are able to keep track of these coordinates and keep track of how the base vectors themselves are changing, then in that case in we can then keep track of the overall deformation and that will be called the convected frame.

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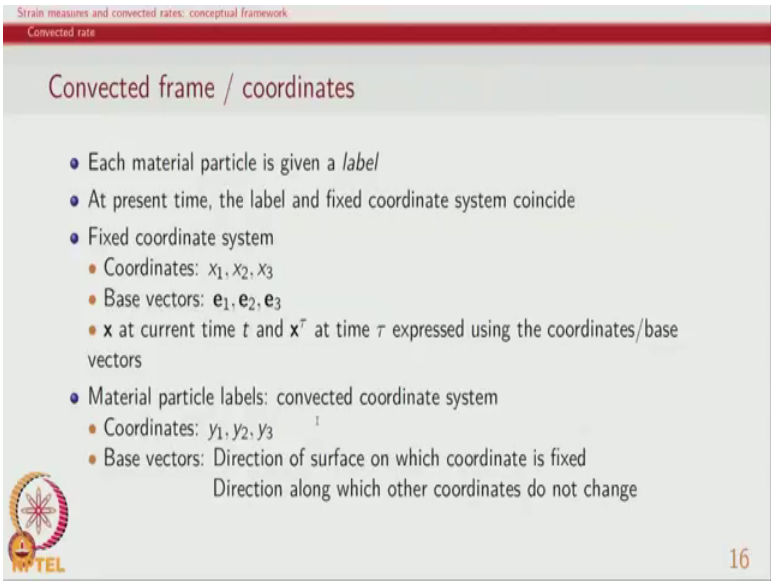

So, the idea of convected frame and coordinate in a summary is that each material particle is given a label and the labels that we have been using for example, are 2, 2 3, 2 and this particular material particle keeps the same label for all times. At present time the label and the fixed coordinate system coincide. So, that is why we have you chosen this square grid to describe. So, this 2, 2 could also be if origin is here then basically this is 1, 1 2, 2 point and therefore, it will be 2 direction and one direction.

So, a fixed coordinate system which is rectangular co-ordinate system and these convected coordinates completely coincide at the present time. This is just for us to have a reference to the fixed coordinate system because quite often we are more tuned to understanding motion and mechanical quantities in terms of a fixed coordinate system. So, the fixed coordinate system of course, we describe using x 1, x 2, x 3 as 3 coordinates since we were looking at 2 dimensional description we only looked at x 1, x 2 and the base vectors were of course, e 1 e 2, but more generally it will be even e 2, e 3 and of course, the position will be described at x at current time t and x tau at any time t and we can use this x and x tau to present at the.

So, we can for example, say that x at present time is x 1, e 1 plus x 2, e 2 plus x 3, e 3 and similarly at any other time tau also. So now, when we have these material particle labels being designated as convected coordinate system we will denote y 1, y 2, y three. So, therefore, what we are saying is given that it is at the present time same.

So, we are saying that x 1, x 2, x 3 the points are our convected coordinates y 1 y 2 y 3. So, it so, happens that the convected coordinate is identical to the present fixed coordinate. But at any other instance of time which is not present the coordinates of the material particle will be entirely different it is only the present state of the material is coinciding with the convected coordinate, but of course, once we have described the label or the convected coordinate y 1, y 2, y 3 the same material particles no matter what its deformation on motion is it will remain same.

So, now, the question is what kind of base vectors can be used to describe. In this case we are familiar with the orthogonal unit vectors which are used as base vectors and along with these coordinate we complete the description of the fixed coordinate system. Now we are looking at a convected coordinate system where coordinates have been identified along as the material labels, which are coinciding with the fixed coordinate system and now how will the base vectors be described.

So, in the next couple of lectures we will look at the definition of these base vectors and what we will see is using these base vectors, we will be able to capture the deformation. You will just need to recall that actually the shapes of these grids as they are changing with deformation, they contain information about the base vectors. So, in terms of base vectors it is useful to think that how is the coordinate changing in any given direction or how what is the unit normal to the surface where the coordinate is constant.

So, we can think of it in 2 way how is the coordinate itself changing or how when a coordinate is kept fixed what is that surface. So, we will see that therefore, there will be 2 sets of base vectors that we will be able to define and these 2 directions and the base vectors will be based on the question direction of surface along which a given coordinate is fixed or we could also ask the question as the direction along which the other coordinates do not change. We do not really encounter 2 sets of base vectors in rectangular or cylindrical coordinate systems or in a spherical coordinate system, because we will also show that using these 2 definitions we will get the identical set of base vectors.

So, since the set is identical we usually do not hear of 2 independent base vectors. But in case of simple shear we will be able to show that how there will be 2 sets of base vectors and these 2 sets of base vectors can describe the deformation and the convected coordinate system.

So, with this we will close this lecture and look at the convected frame coordinates and the overall strain measures in detail in the next couple of lectures.