Rheology of Complex Materials
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Lecture - 44
Strain and Convected Rate- 1

So far, in the course, we have looked at the various strains and stresses and velocity gradients and using these quantities. We did some analysis related to viscous fluids or linear viscoelastic materials, in case, we have to look at the non-linear response of complex materials or in other words, we have to examine their behavior at large deformations. We need measures of deformations strain tensor which is appropriate for large deformation similarly since in rheology, we are interested in rate of change of quantities.

For example, rate of change of stress or rate of change of strain or even rate of change of strain rate in such for such rate quantities we need to use materially objective or freeman variant rates and convected rates are very used quite heavily in such considerations. So, therefore, if we have to look at the non-linear response of complex materials we have to make ourselves familiar with the finite strain measures as well as the convected rates convected rates are one example of rates which are used.

We also can use co-rotational rates or co-deformational rates and so, in the next few lectures we will give the mathematical background to how are these measures defined and how are these rates defined.

So, the overall framework which we will follow for today's lecture is we will quickly reiterate how the position of a material particle was defined, then we will formally define the displacement and displacement gradients using which we can define also deformation gradients and once these are defined these are basically the building quantities over which we can define the strain measures.

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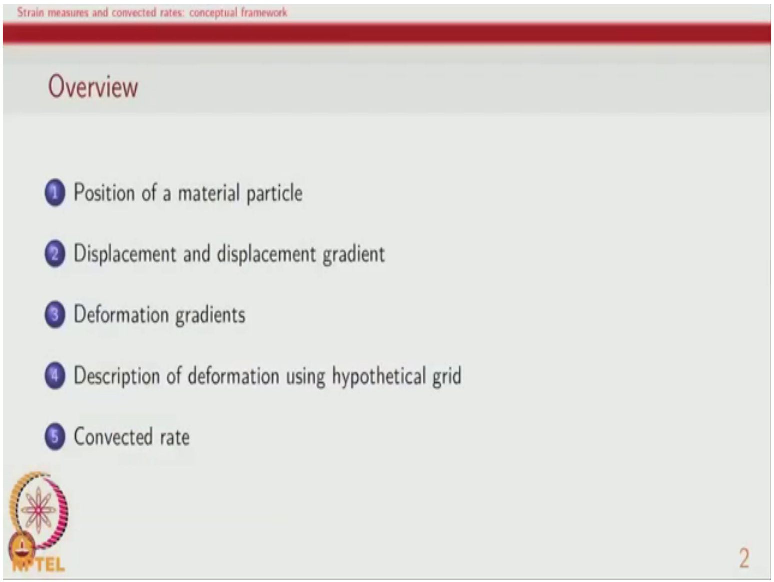

So, in todays and the next class, rather than define the overall strain measures as well as convected rates formally, first, we will look at the conceptual frame framework. So, therefore, we will look at the description of how the deformation happens, if we are able to put a hypothetical grid in the material and what happens to that grid.

So, using this example of shear deformation as well as extension deformation we will try to examine what happens to the coordinates what happens to the material points and how could this information be used in terms of defining either strain measures or also the convicted rates.

And so, just to end with we will then summarize few concepts which are related to convected rates. So, after this set of lectures then we will formally define both strain measures as well as convected rates later on.

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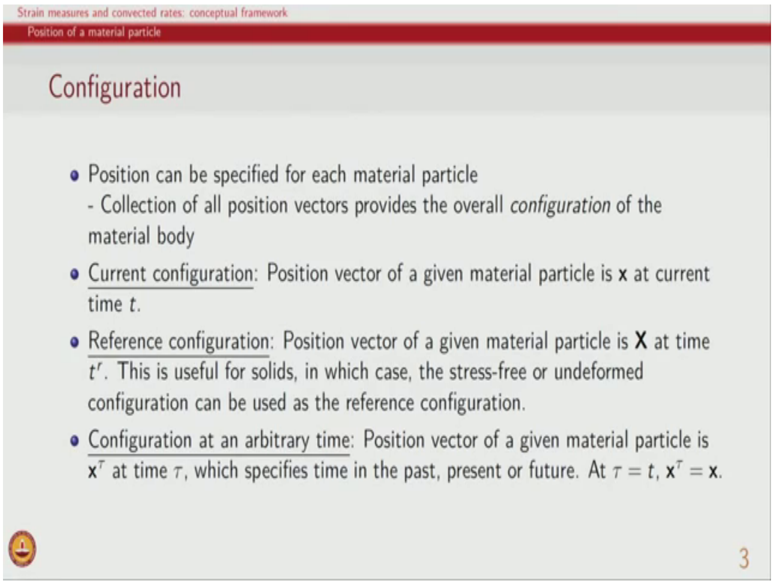

So, let us just quickly reiterate that a position of a material particle is eh defined in each and every configuration we call the collection of all such material points a configuration and we could define three distinct configurations and as we saw earlier that for more solid like materials, it is the comparison between the current and the reference which is used and for fluid like material.

We said that configuration at any arbitrary time will be compared with respect to current configuration in case of fluid because there is no such thing as a unique stress free state or in other words there are multiple states in which fluid can be liquid can be stress free, we will define current configuration as the bases.

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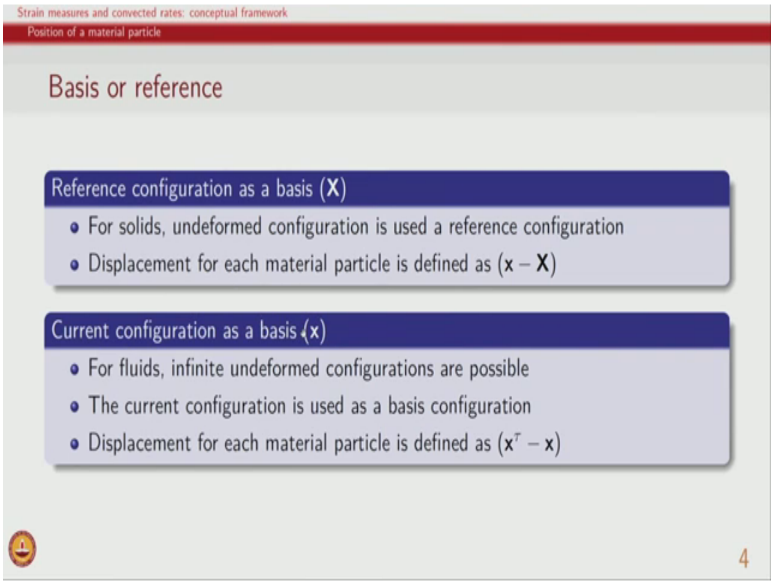

So, it is extending it this idea. So, we already said that reference configuration is the basis for solids while current configuration as a basis for fluids.

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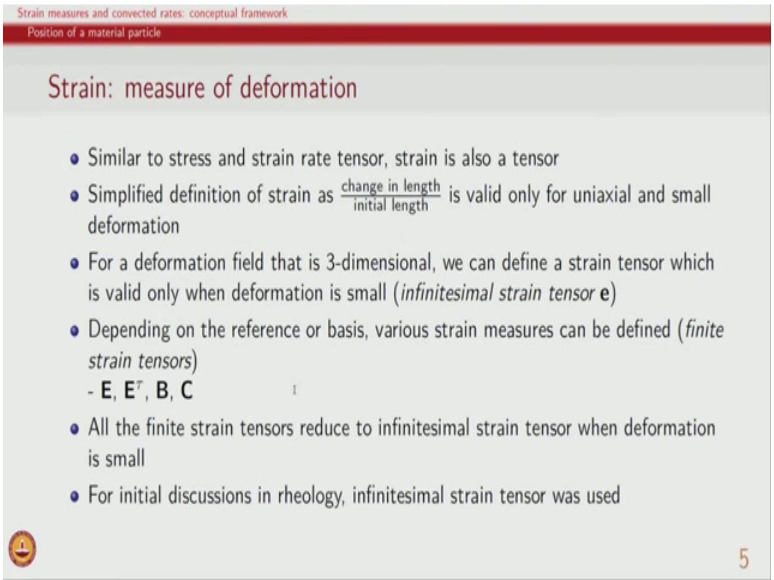

So, the strain measures also, we had said similar to stress and strain rate tensor strain will also be a tensor and simplified definition of strain of course, we know change in length versus initial length, but for a 3 dimensional quantity we will define these strain tensors which are E or E tau or B or C and we will see that all the finite strain rate tensors reduced to infinitesimal strain tensor when deformation is small.

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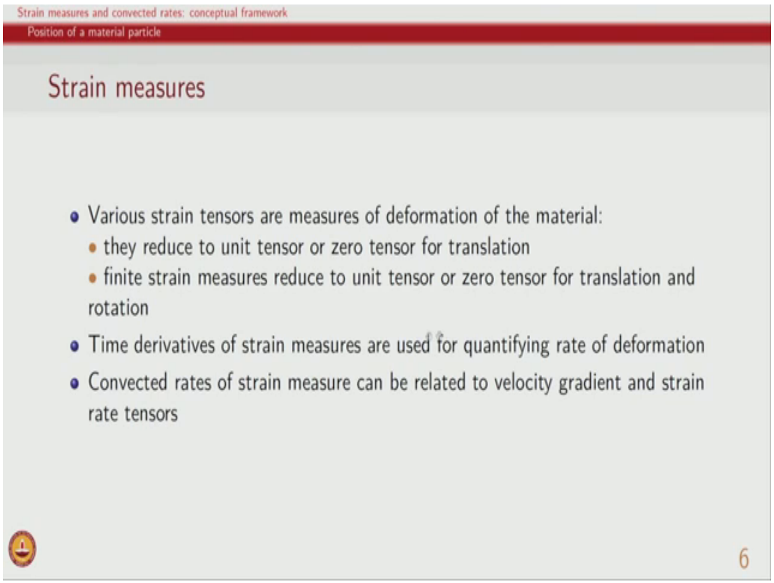

Now, the various strain measures which we will define they will reduce to unit tensor or 0 tensor for translation and they will also reduce to unit tensor and 0 tensor for rotation and of course, the time derivatives of these strain measures are will be used full and convected rates is what is required to get the derivatives when we discuss linear viscoelasticity.

We have already been saying that partial derivative of strain is equal to the symmetric part of velocity gradient or is related to strain is equal to strain rate tensor, but we will see that convected rate of the finite strain measure is what will be required to relate to strain rate tensor.

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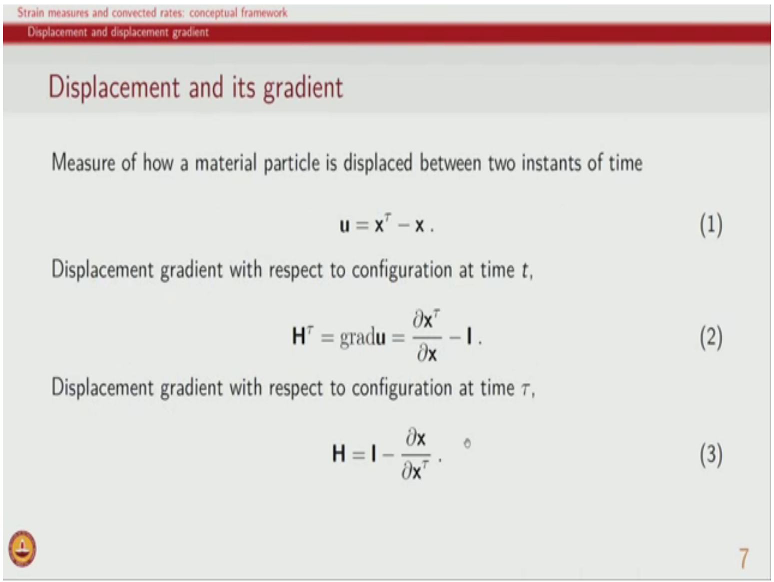

So, now let us look at the beginning the definition of a displacement.

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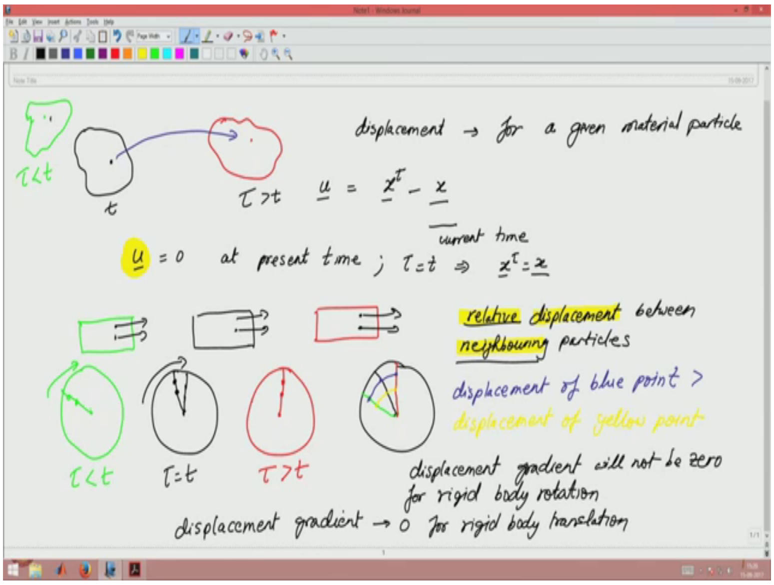

So, displacement is basically how a material particle is getting displaced due to deformation and flow. So, if we have a material and a point and let us say at the next instant of time this material has moved and deformed there is material has point has moved from one location to the other and therefore, we would like to know what is the displacement.

So, displacement is for an individual material particle for a given particle material particle and we define that as basically x tau which is the position of the material at time tau minus x because this is at current time and we denote this by this. So, basically u is 0 at present time and that is the case because at tau is equal to T which is the present time x tau is equal to x. So, therefore, since our the current configuration is our basic basis for comparison displacement with respect to the present configuration or the current configuration is always 0.

So, now what we will try to do is what happens to the displacement when you go back in the past or when you go in the future. So, for example, this particular material particle which is at time T was located somewhere else and at some time T tau which is in the future is located somewhere else and maybe was located at some other point at some time less than present. So, displacement material particle therefore, keeps track of what is the position of the material particle with respect to the current position.

And so, that is how we have defined it we say displacement is nothing, but the present position and position at any time tau and so, if we look at let us say a rigid body motion then a rigid body motion will also actually imply let us say this is some time in the past and since this block is moving it will move to the right let us say and even at some other future time, it will move to further to the right. So, this is a block which is in all the material points in this case are moving identically.

So, the all the material points; so, if you take any two neighboring material points they have moved exactly the same way and in the next instant also they will move the same way. So, therefore, even though there is displacement there is no deformation in this case. So, what is of interest to us to know is actually relative displacement relative relative displacement between neighboring material particles because only if there is relative displacement between neighboring material particles then only we the deformation can be there one caution here.

For example, if we have a rigid body rotation then let us say we have a disc which is rotating and there are two material points let us say we pick here and we can see that at some instants of time. So, at some instance let us say if we pick two different material points. So, at some instants of time they were in this position and at some time in the past actually their position will be towards the left because this overall body is let us say rotating to the right and so, since this disc is rotating to the right again in some time in future. So, tau greater than time which is present time this one is the present time tau is equal to T and what we had written earlier was tau less than time.

And so, given that this block is rotating to the right what we will have is this set of two points actually moving and now if you see the relative displacement of the two points is different because the further away from the square you are the more will be the point. So, let us say on the same graph. Now, if I draw the green curve and the black curve which is at the present time and the red curve which is at the. So, what we can see is actually the point and I will just choose another color to denote two different points.

So, you can see that this point has moved this much distance while another point which we had donate denoted earlier has moved. In fact, much less distance. So, we can see that displacement of blue point is greater than displacement of the yellow point. So, this gives us an indication that even if we are looking at this relative deformation gradient as a way of measuring. So, we are using relative and we are using neighboring particles to get this displacement gradient we can clearly see that this displacement gradient is going to be nonzero.

So, the displacement gradient displacement gradient will not be 0 for rigid body rotation; however, we saw that displacement gradient is 0 for rigid body translation. So, displacement gradient was 0 for rigid body translation. So, clearly relative displacement or displacement gradient is useful in terms of tracking the deformation in the material, but if digit body rotation is involved then it will give us a picture that there is nonzero displacement gradient; however, we know that material is; In fact, not deforming.

So, just to continue the definition of the displacement gradient therefore, the displacement gradient then we define as we can define it in terms of with respect to the current time or we can define it with respect to time at any arbitrary time tau and so, given that in this course we may be using this configuration more time as the basis we can define the H tau which is the relative deformation gradient which is nothing, but gradient of u which is the displacement vector and since we are finding gradient of a vector quantity the displacement gradient is actually a tensor and it is denoted by this in since we are now familiar with the index notation also.

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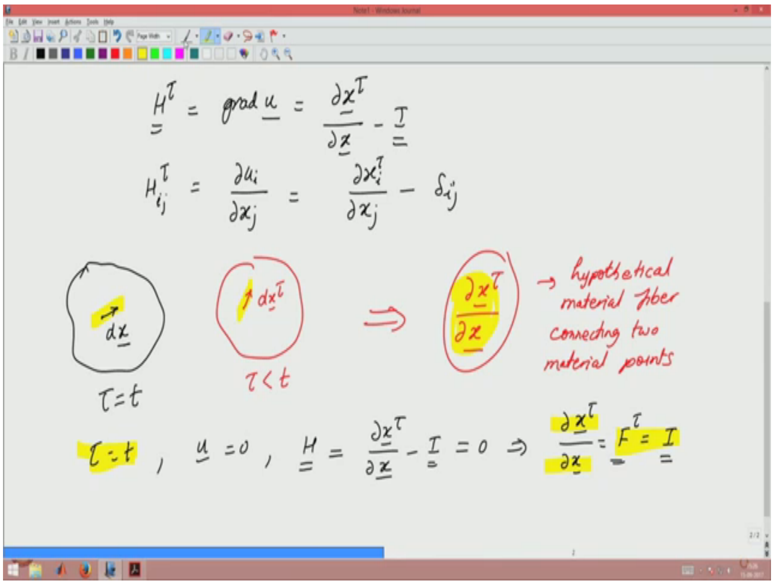

So, we could write the same expression which we have written there which is in terms of gradient of u is equal to del x tau by del x minus identity. So, the same equation can be written in index notation as H i j tau is equal to. So, del ui by del x j is equal to del x i tau by del x j minus delta ij. So, this is called the relative displacement gradient because it with respect to the present configuration in solids as we said quite often this x tau will be a specific reference configuration and in that case this displacement gradient is defined as a unity minus the deformation the displacement the change rate in change of position x with respect to change in position at any arbitrary time tau.

Now, these quantities which are involved in defining this gradient themselves can also be used as a measure of deformation because this is telling us how the current configuration is related to the configuration at any time tau if we look at what we are trying to say here is if we have a material with and two neighboring points at any time tau, let us say this tau is present time.

So, therefore, this is nothing, but the material fiber which connects the two neighboring points and so, in anytime in the future the same let us say two material points are now of course, connected through another material fiber which we will call tau where tau is greater than time and so, how is the relative location as well as orientation of these two material fibers is what is indicated if we evaluate a quantity like this. So, therefore, this is comparing the hypothetical material particle hypothetical material fiber material fiber connecting two material points again just to emphasize one material fiber was at present time somewhere the same material fiber at some other time tau has become here and therefore, this is the comparison between the two material fibers and so, we define this quantity as a deformation gradient.

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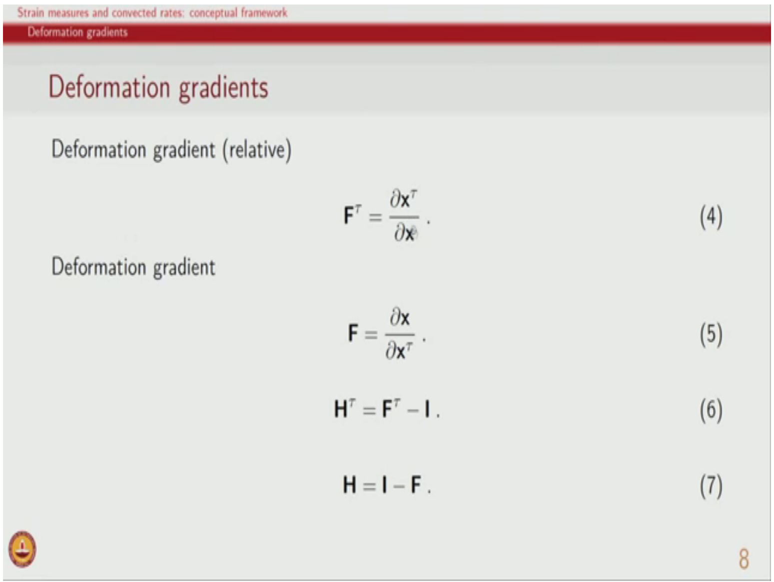

And so, which is what is defined here and deformation gradient again can be defined with respect to the present configuration or the current configuration in which case we will call it relative deformation gradient and it is denoted as F at given that it is at any time tau which can be present past or future now looking at this quantity we can clearly see that at the present time the deformation gradient will become unity as we saw earlier that at the present time.

So, when tau when tau is equal to T what we saw is the when tau is equal to T, we saw that u is equal to 0 and therefore, H will which is nothing, but del x tau by del x minus unity. So, this will also go to 0 this is also because del x tau by del x which is the deformation gradient that we define just now is actually going to be unity because we are using the same we are using basically the present configuration and this will also be present configuration because tau is equal to T. So, therefore, the deformation gradients are unity always at the present time.

So, what we can just emphasize here is displacement gradient becomes 0 at present time because x tau is equal to x and this also becomes 0. So, of course, there is no deformation with respect to present in the present itself and therefore, the other deformation measure also which is the deformation gradient becomes unity at the present time and deformation gradient when it is defined with respect to any arbitrary time tau is denoted and called deformation gradient for our course purposes most books which deal with solid materials actually will define x tau as a reference configuration and therefore, this will be called the deformation gradient or basis with base as reference configuration.

So, now having looked at the definitions of deformation gradients and displacement gradients these are the quantities which are involved in defining the finite strain measures and we have already seen some of the characteristics of these displacement gradient as well as deformation gradient we already saw that displacement gradient was 0 for a rigid body translation, but it was not 0 for a rigid body rotation similarly the deformation gradient will be unity for rigid body translation or also for no deformation itself, but we will again we can show that the deformation gradient will be non-unity when there is a rigid body rotation.

So, clearly we need a strain measure which for rigid body rotation also should go to 0. So, effectively what we need is really not just comparison of the material fiber and its orientation, but also the magnitude of the material fiber.

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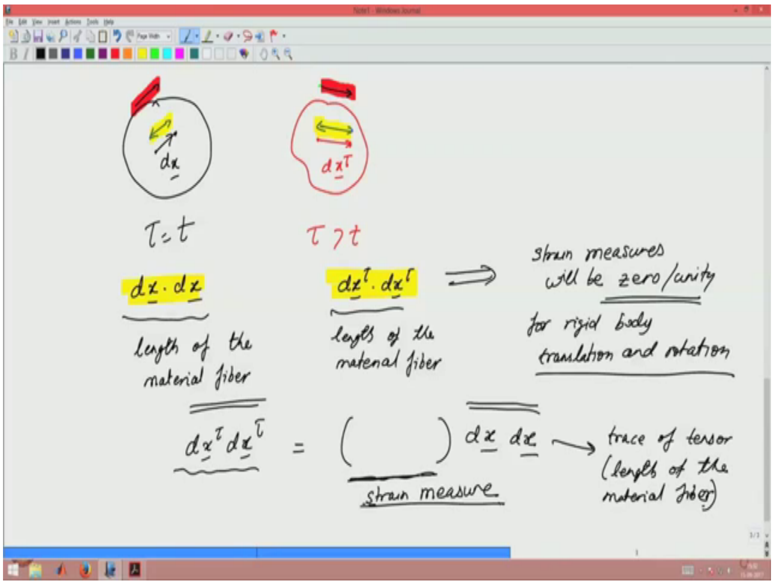

So, the for example this material fiber that we are saying is we are comparing and let us say this is at present time because tau is equal to T and then some other arbitrary time where tau is either greater or even less than and then if we have this material fiber which is dx tau we are not only interested in what is the orientation.

So, orientation clearly in one case is here in this case, it is here so, orientation is different, but if you look at the length of it itself. So, in this case the length is given by this while in this case the length is much larger. So, clearly between the two cases, the length has increased at tau time and so, we are interested in quantities which are like the measure of the; so, therefore, we are interested in quantities which are of this kind. So, this is nothing, but the length of the material fiber and similarly here also the dx tau dot dx tau will give us the length of the material fiber.

So, what we will find is that strain measures are defined using quantities which are of this kind and so, in today's class, we will not really define these quantities, but for now the idea for us to understand is the fact that both orientation as well as magnitude of these material fibers have to be compared with respect to each other before we can actually define the strain measure and. So, if it is a rigid body more rotation, then what will happen we will find that this length will remain the same; however, the orientation may change.

So, in that case we can again since the length is remaining the same we will find that the strain measures that we use will go to 0. So, in case we define use these quantities then we will see that the strain measures such defined will be 0 or in some case unity depending on which strain measures we define for rigid body translation as well as rotation and so, actually what we are what we would like to know is therefore, the how is the material fiber oriented as well as so, the dyadic product that we have of this kind which is actually the orientational tensor for the material fiber x tau; how is this related to dx.

And this is the quantity then that can become a strain measure. So, we need a tensor quantity which can map these two orientation tensors and these two orientation tensors incorporate not only information about orientation, but also the magnitude of the fiber itself as the diagonal term because a trace of this trace of this tensor is the length of length of the material fiber and the different components of this will tell us what is the orientation and so, therefore, these two quantities have to be related using the strain measure tensor and so, this is what we will do define formally in a latter class.

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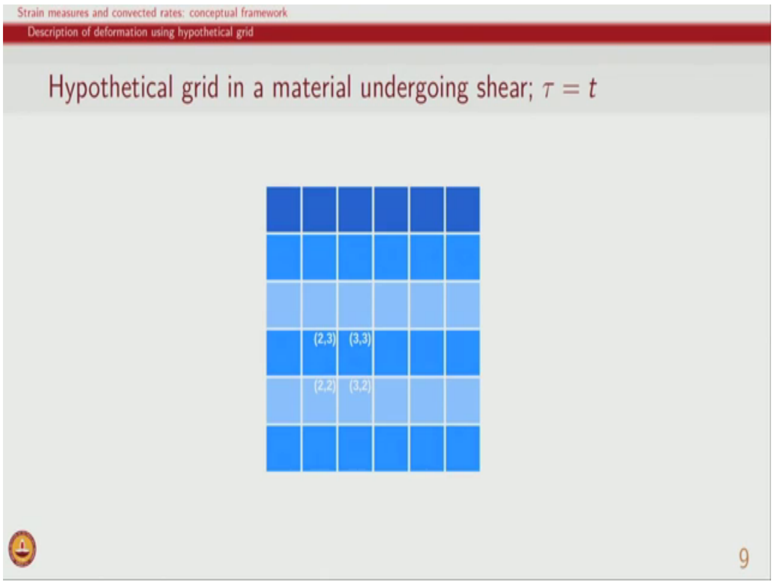

Moving on to looking at what happens to these displacement and deformation gradients we will look at a some graphical example and to understand as to how material particles are getting displaced in two hypothetical situations; one where material is undergoing shear and another one where material is undergoing extension.

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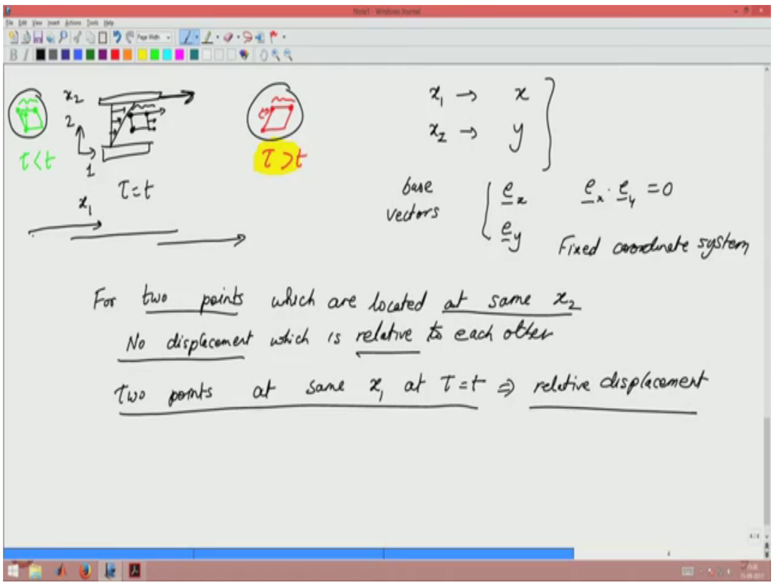

So, let us say that we have a fluid which is moving and we again in a case of extended shear we just take a simple Example where again, it is a simple shear flow. So, we have two parallel plates and the top plate moving and because of this there will be shear deformation in the material and if I take a small block in this material and let us say this is at time T is equal to tau because this particular point will be moving little faster than this particular point and this point what will happen is in some time in future this material block that I drew would have deformed like this, similarly, at some time in the past this material particle would the block would be actually oriented like this. So, this is at some tau which is less than T and then this is some tau which is greater than T.

So, now just to visualize this clearly what we can do is we can put a grid this is usually what we can say coordinate system. So, what we are doing is let us say at this instant of time I put a coordinate system and let us say this is our one direction and this is two direction. So, therefore, we could say that this is x 1 coordinate and this is x 2 coordinate or they of course, generally we know that x 1 coordinate is x and y 2 and x 2 coordinate is actually y.

So, therefore, we generally use an xy or rectangular coordinate system for this problem. So, just to keep the description generic let us say we have x 1 and x 2 coordinate and along with these x 1 and x 2 coordinates we have of course, the base vectors which are E and E y. So, these are the base vectors and since we are using a orthogonal coordinate system rectangular coordinate system we also know that a dot x ey is 0 and also of course, it is a fixed coordinate system.

So, with respect to this fixed coordinate system what we will clearly see is at any time in the future when we are looking at the material anytime in the future we have material which has moved to the right and the material has moved to the right as well as the shape of the material element that I drew rectangular element has changed and similarly it the same material point any time in the past had actually a different shape and of course, it was to the left and it took some time to actually come to the same move to the present position T.

So, therefore, we can see that each and every material particle is getting displaced. So, this particular point was here and in some time in the future has reached here clearly what we can see is if I compare the material points which are at the same x 2. So, for two points which are located at same x 2 what I can see is they since this is only the motion is only in the one direction this is basically because of simple shear flow that motion is only in one direction.

So, these two points would move identically and therefore, there would be no relative displacement between these two material points and so, this distance as well as this distance as well as this distance is completely identical. So, therefore, we know displacement which is relative to each other for these two points right the two points the two points which are located at same x 2 they in fact, have no relative displacement and so, clearly if we look at the displacement gradient here some components of displacement gradient will be 0.

However, similar if we take two points which are let us say in different x 1 or a different same x 1. So, two points at same x 1 at present time actually will not even be at same x 1.

So, let us say the two points actually one is displaced by some amount similarly when it was in the past the two points actually have been displaced while in the present time they were at the same x 1. So, clearly now there is a relative displacement. So, there is a relative displacement. So, we can see that the displacement tensor some of the components are 0 and some other components are non-zero.

So, in the next part of the lecture we will examine further using an extensional example and also, then we will get an idea what if we embed a coordinate system which is in the material itself and it moves along with the material system.