Rheology of Complex Materials

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Lecture –29
Governing equations for rheometry

So, in this segment of the course we are looking at governing equations for rheometry in the previous lecture, we saw how mass balance can be used for a couple of specific examples of rheometric flows. In this lecture we will look at the use of linear momentum balance, and navier stokes equations for some of these example flows.

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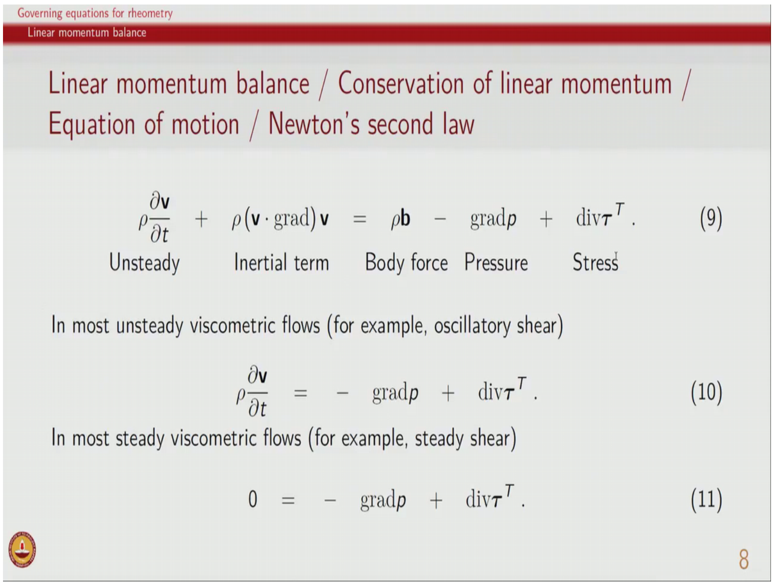

We know that inertial terms will be insignificant for most of rheometric flows, and depending on whether it is an unsteady or steady flow, we have the time derivative of velocity being 0 or non zero and. So, quite often we are mostly interested in looking at the divergence of stress tensor, we remind ourselves that we do not really know in some cases how the fluid stress depends on various kinematical quantities such as strain and strain rates. And in fact, the purpose of doing rheological analysis and characterization using a rheometer is to first try to characterize, and see whether try to observe how do some of the properties change as a function of strain; strain rate, and other kinematical variables.

So, therefore, what we will do is to look at a couple of examples of how the linear momentum balance simplifies in a given case.

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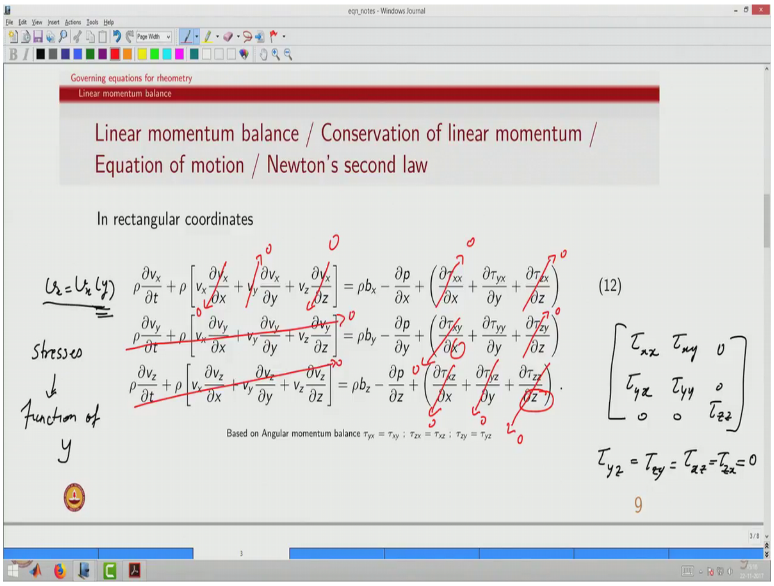

So, now this is the linear momentum balance for in rectangular coordinate, we remember that angular momentum balance ah is ensures that the stress tensor is symmetric. So, that tau y x is same as tau x y tau z x is same as tau x z and tau z y is same as tau y z, and given that the linear momentum balance is a vectorial equation we have 3 components the first component is the x component.

So, we have this is for flow in x direction, and the second component is related to flow in y direction. So, we can see that the velocity here is y. So, this is a rate of change of velocity in y direction or acceleration in y direction. Similarly the 3rd component is related to the velocity in the z direction and therefore, v z is the velocity involved. The body force the similarly components in x y or z direction, and similarly the gradient in x direction, gradient of pressure in y direction, in gradient of pressure in z direction and these are the terms which are related to the stress the divergence of stress, and we can see that given that we are looking at ah the overall force balance in x direction or Newtons second law in x direction we have 3 stresses which contribute, and these stresses are on 3 different surfaces this is the stress in x direction, but on x surface stress in x direction on y surface, stress in x direction on z surface.

And how they change as a function of the position within the fluid will make sure, that if this is non 0, then it has to be balanced either by pressure gradient arising in the fluid or body force is sub the fluid is subjected to or the fluid will have to accelerate and be unsteady. So, based on these different terms which are basically related to force balance then we can try to analyze a given rheometric flow.

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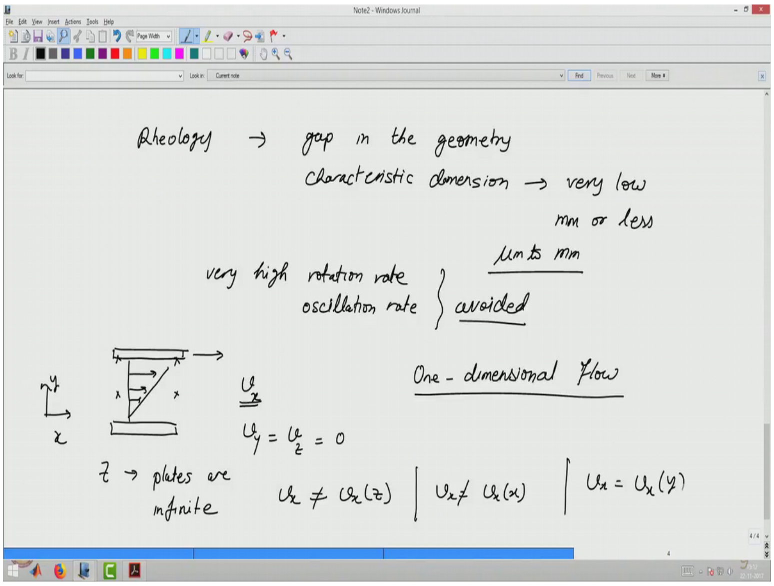

So, we can begin with simple example that we have seen many times is case of parallel plate flow, where we have the coordinate system y and x, and what we have is the top plate moving with certain velocity and of course, instinctively we have been drawing the overall velocity profile which is linear. And so, given that the plate is moving only in x direction we expect velocity or in the x direction to be present, we say that in the y and z direction there is no motion. So, this is an example of a 1 dimensional flow that we have seen when we were reviewing the rheometric flows. So, given that this is a 1 dimensional flow and v x is only present we can also say that since, in the z direction we consider the plates to be infinite. So, z direction plates are infinite and so, there is no dependency on z direction. So therefore, v x is not a function of z.

And similarly v x is also not a function of x because we are assuming that at this point in plate or at this point in plate velocity is similar, similarly at this point in plate and this point in plate velocity is similar. So, this leads to the overall conclusion that velocity is only a function of y.

So, given this overall flow situation of rheometric flow in which we have a 1 dimensional flow, and velocity is only there in x direction and it is only a function of y, we can now try to see what is the consequence in terms of linear momentum balance so, let us look at the overall linear momentum balance for this case. So, we can see that since v y and v z are 0 we have basically this whole term going to 0, because v y is 0 and v z is also 0 . So, therefore, we do not really have any terms which are associated with v y and v z.

Now, similarly since we have just to remind ourselves we are saying that v x is a function of y only. So, in that case what we have is terms like this also going to 0, and this also going to 0. Now v y itself is also 0 so, we can see that the inertial terms identically followed to 0 given our assumption that we are looking at 1 dimensional flow with y being the shearing direction we do not have any other terms, we cannot say anything right now about the overall stress components. In a another lecture we have pointed out that the most general state of stress, that can be there for a such flow is where there are only 4 components of stress which are possible to be non zero.

So, by definition we know based on material symmetry argument that tau y z tau z y of course, it is a symmetric tensor. Similarly tau x z and tau z x are 0. So, therefore, we have only the 4 components of stresses which are possibly non zero. So, in that using this therefore, we can then set some of these additional terms to also 0 so, tau z y is also 0. So, now, looking at the overall governing equation, we have the following set of terms which may still arise. Now we can use another fact the given that things are at most a function of y, since velocity is only function of y we would expect the stresses also to be only a function of y. So, we can given that there is no other dependency. So, we have stress also as only at most it can be a function of y.

So, therefore, any derivatives with respect to x and z will also can be set to 0. So, this is derivative with respect to z so, this will also be 0. Similarly this is derivative with respect to x. So, this will also be 0, and we also have this to be 0 and so now, if you look at the overall governing equation we have now simplified the overall governing equation to be as follows.

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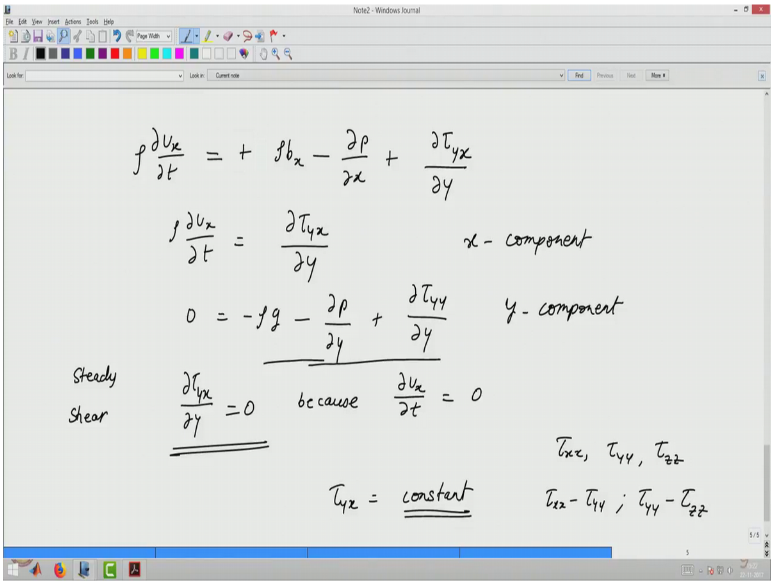

So, let us write it down in the governing equation for this parallel plate flow is del v x by del t rho is equal to plus rho b x minus the gradient in x direction plus what we know is we do not have any information. So, therefore, we only know del tau x y by del y.

So, del tau y x by del y and let me just go back and modify we know that the stress tensor is symmetric, but just to be consistent with our other sets of notes that we have used we will continue to use tau as tau y x. So, these are the only 3 terms which are possible on the right hand side, when we say that we are looking at simple shear flow between parallel plates. And now we can make some additional statements regarding what is the type of flow we are imposing, if we maintain the gap between the plates as very small. So, if the gap between the plate is very small, this by the way we also have to ensure for Reynolds number being small and so, and given that there is no flow in y direction and we let us say assume that gravity is in this direction we can ignore the effect of gravity on flow. So, the body force which is subjected to the is all in y direction. So, therefore, this and this is 0.

So, therefore, we do not really have any body force in the x direction because gravity is in the negative y direction. So, now, the other term which can ah arise is the pressure. So, we will assume that the flow in the flow it is only caused based on the plate motion and since plate is moving the fluid is moving. So, this is an example of a (Refer Time: 11:24) flow as we have defined earlier poiseuille flow is where pressure gradients are also there, since we are looking at only (Refer Time: 11:29) flow example, we can assume that there is no pressure also gradient. So, pressure is uniform everywhere. So, therefore, in this case we will have even these terms going to 0.

So, therefore, now if we look at I should be careful here in terms of not having del p by del 0, because we have gravity in the y direction, we will have pressure gradient in the y direction. So, the only thing that we can say is there is no pressure gradient in x direction, and there is no pressure gradient because we are not imposing anything in that direction. So, therefore, del p by del x is 0.

So, now, coming back to our overall governing equation using all these statements therefore, reduces to rho del v x by del t is equal to delta tau y x by del y. So, this is the overall governing equation that most generally we can just for completeness sake write the y and z component also, in y case we have 0 is equal to rho g we can just write since gravity is there in the negative y direction instead of now writing the overall body force in terms of some general b we will write it as g, and we will say that it is in the negative y direction minus del p by del y plus del tau y y by del y.

So, this is the most general statements that possible for a parallel plate flow, and then z direction all the terms have are gone to 0. So, therefore, we have only the x and y component. So, this is the x component of linear momentum balance, and this is the y component of linear momentum balance. Now we also make a further statement that if the flow is steady, and in that case what we can see is the delta tau y x by del y is 0, because flow is steady and v x does not change as a function of time. And if at all let us say velocity itself is oscillating for example, when we apply oscillatory shear the velocity will be a sinusoidal function of time it will increase and decrease and so, it will vary like sine or cosine function. So, in that case it is easy to see here also therefore, we would expect the stress also to be very similarly, but in for a steady click case steady shear we have this is the governing equation.

So, this is the final governing equation for steady shear flow and therefore, based on linear momentum balance, we arrive at the conclusion that the stress in the fluid is constant. And so, we cannot say at this point the other what is the value of other components and how do we measure them is a clearly dependent on pressure for example, if depending on how pressure varies we can then look at tau y y.

So, variation of how stresses are there in such geometry will depend on the measurements of pressure, and this we will see regularly that the measurement of normal stress differences will depend on the measurements of pressure. And so, if we are only confining ourselves to simple shear flow and the analysis of only the shear stress, then the overall conclusion is that the shear stress is constant. And so again we can justify it and rationalize it by looking at the overall flow diagram.

So, what we have in this case is if i look at any let us say fluid layer at a given y location what we have is the fact that this fluid layer is basically being subjected to stresses.

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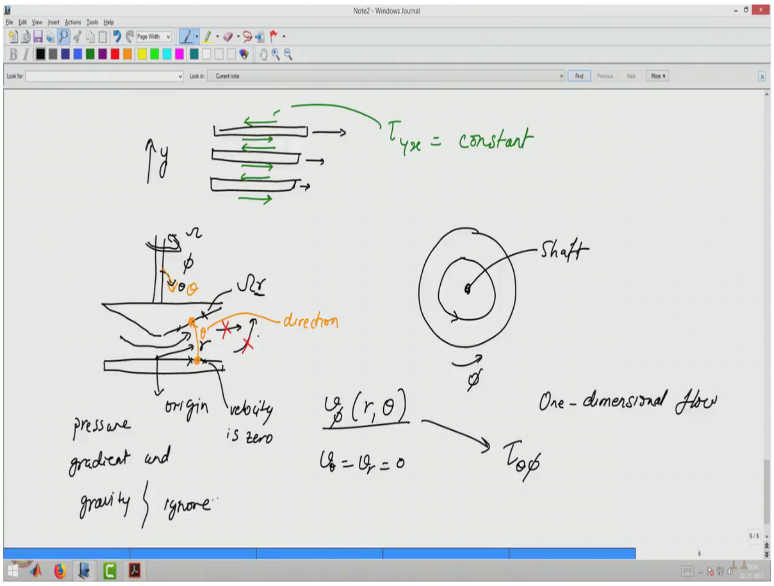

So, this is a fluid layer which is moving at some velocity, the fluid layer above is moving at a slightly higher velocity. And the fluid layer below is moving at a slightly lower velocity. So, all of these are balancing each other by having a set of stresses and what we are saying is all of these stresses are constant.

So, each layer exchanges the shear stress with the other layer, and the value of the stress does not change as we travel in the y direction and so, this is based on the linear momentum balance for a simple shear flow in rectangular coordinate system. So, with this example we have seen that how a linear momentum balance can be used to simplify and analyze the flow situation in a given rheometric flow. Now going on further we can look at a similar analysis for cone and plate flow, in which case we use the spherical coordinate.

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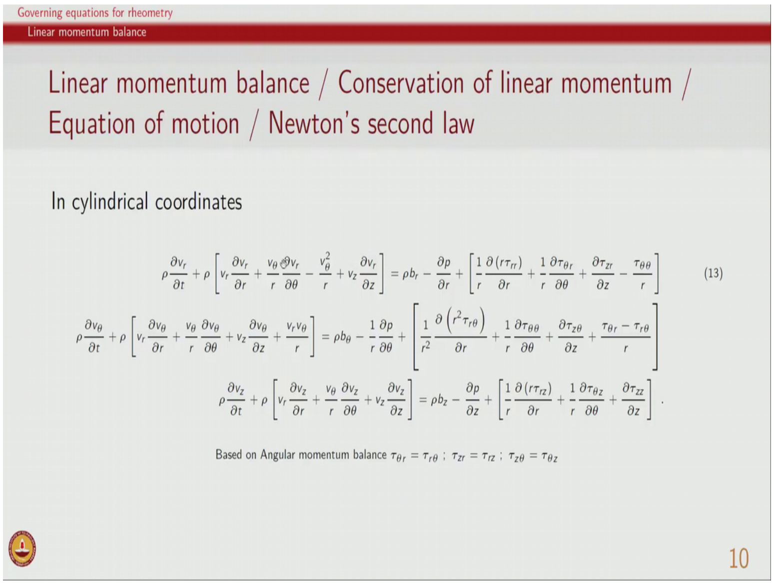

And as again we see in cylindrical coordinates, as well as spherical coordinates there are many additional terms which are involved.

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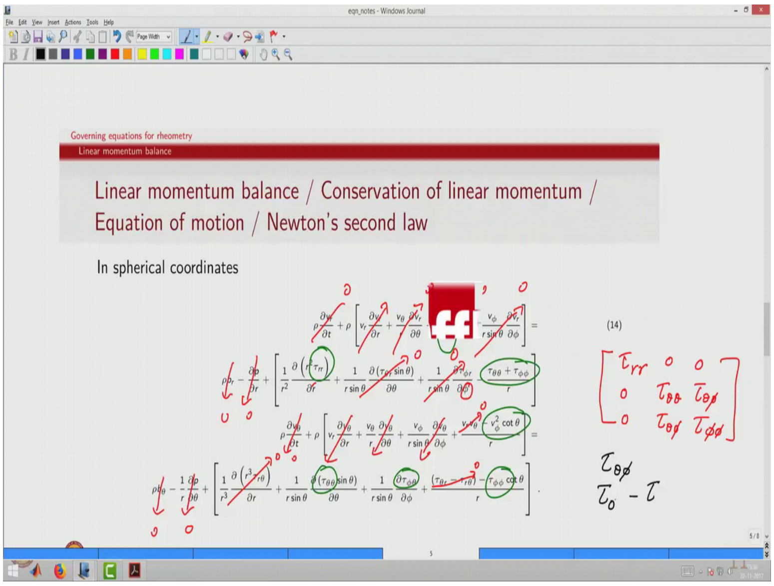

So, when we use the curvilinear coordinates spherical or cylindrical coordinates.

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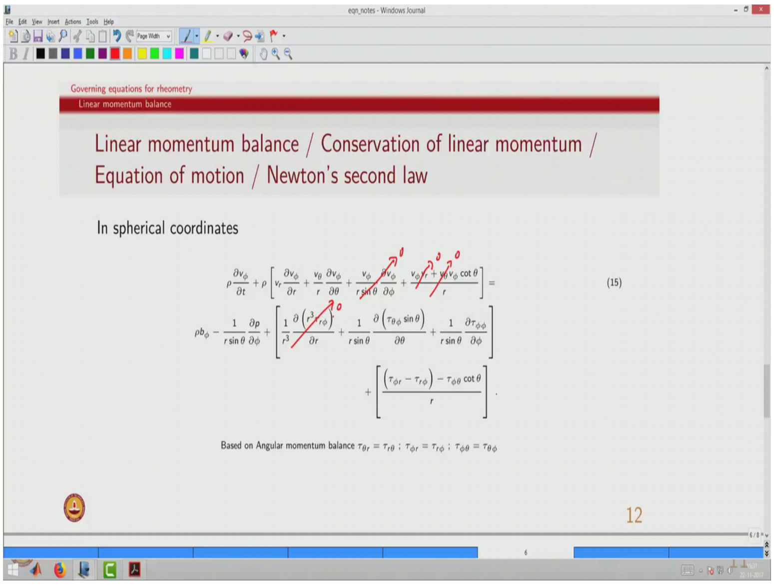

We should be very careful in terms of using. Now this term is actually the centrifugal acceleration that we are familiar with rho v theta square by r. So, this is actually a component r direction component, and in r direction we have of course, velocity in r direction and the stresses which are in also r direction though they may be on different surfaces, and we can see that in this balance we also have ah term which is due to the flow in theta direction.

So, this is that is why what we learn known as centrifugal acceleration. So, that if there is a motion in the circular direction there is a force in r direction. So, this is the inertial term and if Reynolds numbers are high, then this term will be significant. So, even if we are imposing the rotational flow on the material, and we have a radial flow to be negligible we should be careful that the flow in the theta direction should not be very high to cause this v theta square by r term to be very significant.

If this term becomes significant then we will have flow also in the r direction, and so a fluid which is moving in theta direction, and if we are using a rheometer geometry to rotate. If we rotate it very fast then we will have a tendency for fluid to get thrown out. So, fluid though we are moving it in theta direction fluid will get a motion in r direction.

So, this is very important when we do rheological analysis we should always observe what is the status of the sample when we are doing the shearing, and very often we will be able to see that if we shear it at very high rates the fluid actually comes out and . So, now, let us look at the cone and plate geometry for which we use spherical coordinates. So, in cone and plate geometry we have the top cone, and we have a bottom plate the top cone is rotating and this is in the phi direction, and we measure the theta direction from the vertical shaft and of course, the r direction is measured radially outward from the origin.

So, this is the so, again the question that arises is which velocity components are 0 and which velocity components are non zero in this case. So, we can see that we do not expect fluid to move either in r direction or in theta direction. So, fluid motion is in theta direction as well as r direction is not there, we expect the fluid to only flow in phi direction. So, we expect the fluid to rotate if I look at it from the top view where this is the top cone and this is the shaft, then I expect the fluid path lines to basically circular.

So, we only have a motion in phi direction, and there is no motion in theta direction or there is no motion in radially outward direction. So, therefore, we can say that v phi will be the only component which is present, and we also say that v theta and v r is going to be 0. So, there is no radial flow there is no flow in theta direction. Now v phi what can it be a function of. So, you can see that the bottom plate velocity will be 0 so, here velocity will be 0 because the bottom plate is stationary on the top cone the velocity is going to be omega r because the top cone is rotating. And so, we can see that as you travel in theta because this is the direction of theta.

So, this direction if we travel this is theta direction. So, clearly velocity goes from 0 at 1 point to omega r at another point. So, v phi will be certainly a function of theta. Similarly when we go at different r s the velocity keeps on changing so, v phi therefore, will be a function of r n theta. So, in this scenario we are looking at again simple shear flow and velocity only 1 component.

So, this is again 1 dimensional flow and only one component of velocity is non 0, and again we can for this flow also we can again look at what happens to the overall linear momentum balance. And we can see that again we will be able to very effectively set many of the terms to 0, and we may be able to for example, since v r is 0 we can say this overall term on the left hand side will go to 0 except 1 term.

So, this term goes to 0 this term goes to 0, these terms v theta is 0. And similarly we will we can again in this case also argue that we will only have some of the stress components 0 and some other stress components again we will have 4 dominant components of stress which are non 0. And so, based on that we can simplify and therefore, we can say that this term will be 0 because there is no phi dependence there is no theta dependence may be there, because we are looking at so, we have v phi as a function of theta will be the overall so the most dominant component will be tau theta phi.

So, we can then say that the stress in terms of tau theta r will also be 0, and similarly here we will have tau theta theta will which will not be 0 because this is a normal stress tau r theta will be 0. And similarly these terms will be 0 now phi phi may not be 0, and then if we look at the next set of governing equation, since v phi is not a function of theta not a function of phi this term will be 0 v r itself is 0, and v theta itself is 0, then we also have r phi as 0 so, these terms are also 0, and tau theta phi is the only non zero term.

So, we can see that many terms drop out based on the consideration of momentum balance. So, the terms which are significant and they are present are tau r r 0 tau, and again this will also be 0, and 0 tau theta theta tau theta phi, and 0 tau theta phi and then tau phi phi. So, these are again the 4 dominant terms and based on this given that v r v theta is 0 even this term will be 0 which I missed earlier. So, we can see that there are several terms which fall out and therefore, we can have a very limited set of terms which overall give rise to the governing equation for the so, if you look at now the terms which we cannot set to 0 are here, the tau r r tau theta theta and tau phi phi v phi itself is of course, non zero tau theta theta is non zero, tau theta phi is nonzero, tau phi phi is non 0.

And if we assume tau phi phi is not 0, tau theta phi is not 0, theta phi ah is non 0, and this term again will go to 0 because v theta itself is 0 and v r also is 0 and of course, so therefore, now we have terms which are this v phi is also 0. So, therefore, there are no terms on the left hand side the inertial terms. So, we only have rho phi by dou t, and then we have terms here which are related to theta phi and of course, terms related to phi phi and of course, we remember that angular momentum balance tells us that stress tensor is symmetric. So, theta phi is equal to phi theta tau theta phi is equal to tau phi theta.

So, we can see that there are several terms which have now can be neglected or they are identically equal to 0, now given that again we say that the cone and plate is a (Refer Time: 27:11) flow we can again ignore the influence of gravity and pressure gradient. So, in that case again we can then say that pressure gradient is not significant, similarly the overall gravity also is not playing a role, and gravity these effects can be ignored.

So, in that case we can again set various other terms also to 0 which are associated with the pressure and body force terms. So, we can see that several of these terms are 0. So, we can see that simplification is indeed possible, and the overall simplified equations though are still reasonably complicated for us to solve and so, the rheological analysis would try to use the nature of these governing equations to try to see, if we can measure the all the 4 stress component. And for incompressible fluid we cannot measure these 3 independently because pressure is an indeterminate value. So, therefore, what we do is we measure 3 quantities in a rheumetric flow we can measure tau theta phi, and we can measure normal stress differences.

So, which would be basically given that flow is in the y direction tau phi phi minus tau theta theta, because theta is the direction of the shear, and then we also have tau theta theta minus tau r r. So, these are the 3 quantities which can be measured. So, we will have to manipulate the rheological the overall fluid mechanical governing equation, and then try to see what will be the variation. Now since I said there is no pressure gradient what we need to remember, in case of viscoelastic fluid flow is that pressure gradients can arise even though they have not been applied.

So, therefore, even though we are saying that there is no pressure gradient that is being applied on the material we cannot therefore, set them to 0. So, what we will need to do is to say that we will need to measure the pressure gradient, and then based on that we can then look at what will happen to the overall governing equations. So, what we have seen are couple of examples of using the linear momentum balance for solving problems which are related to rheometry.

And in the next set of lectures we will assume that the flow profile, and the measurement of stresses is possible, and it is done by the people who are devising rheometers and the data is being collected and analyzed. So, that these fluid mechanical assumptions are valid, but assumptions that have been made while analyzing the flow for example, 1 dimensional flow, Reynolds number being flow. So, we need to be constantly aware of these assumptions. So, that while we are doing experiments, we make sure that our experimental conditions match the overall assumptions that are being used.