

Rheology of Complex Materials
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Lecture - 16
Viscous response

So, in the previous lecture we saw that what is the non-linear viscous fluid?

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Viscous response
Non-linear Viscous fluids

Non-linear viscous fluids / Generalized Newtonian Fluids

Linear viscous fluid; Newtonian fluid

- Stress proportional to strain rate
- Steady shear viscosity μ a material constant (only a function of temperature)
- Extensional viscosity is 3μ , a material constant

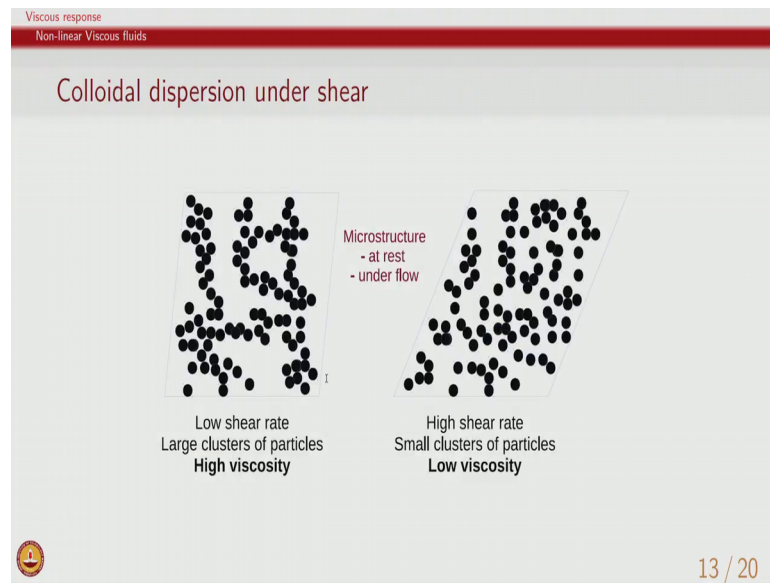
Non-linear viscous fluid; Generalized Newtonian fluid

- Stress as function of strain rate - rich variety of behaviour shown by complex materials
- Steady shear viscosity η a material function (a function of shear rate)
- Extensional viscosity is 3η

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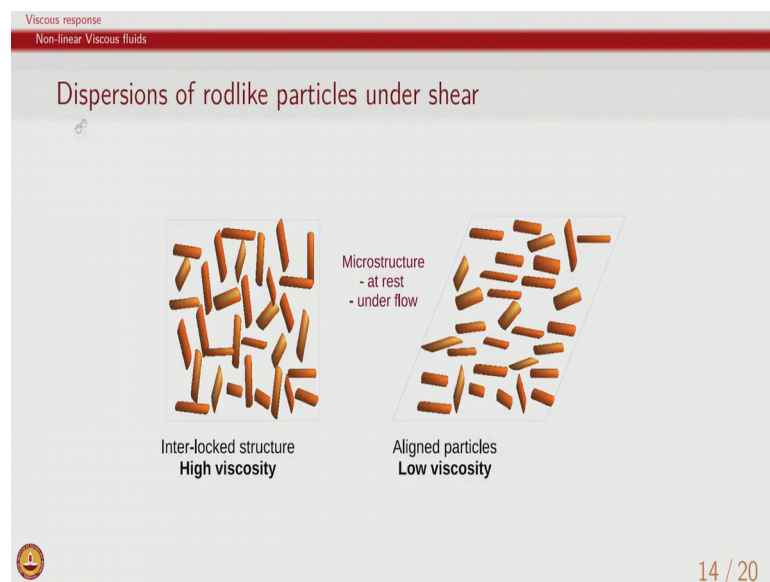
They are similar in most respects with Newtonian fluid in terms of completely dissipative response; however, the stress is a complex function of strain rate.

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The microscopic origins of viscosity being a function of strain rate completely depend and we looked at two different materials colloidal dispersions under shear and dispersions of rod like particles under shear.

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So, we should remember that we are assuming a viscous response what we do is we are only focusing on viscosity as a material function.

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Viscous response
Non-linear Viscous fluids

Assumption of Viscous response

- Useful for steady flows in geometries that change gradually
example: pipeline transport of coal slurry
- Underlying mechanisms of shear thinning / shear thickening may incorporate viscoelastic mechanisms
example: cluster breaking and formation
particle alignment

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The underlying mechanisms which are let us say cluster breaking and formation or particle alignment are the mechanisms which also lead to viscoelastic response. However, for many applications it is useful to just assume viscous response because it is far more easier to describe only one characteristic of material which is viscosity. For example, if we have pipeline flows. So, pipeline transport of coal slurry. In general we would expect that the pipeline transport would imply constant flow rate of coal slurry which is being delivered from one point to the other. From time to time there may be shut down from time to time there may be an increase, in the requirement decrease in the requirement, but generally we would expect that the rate of required of coal slurry to be delivered will remain roughly the same.

Similarly, the pipes which are used in between some places there may be joints, in between few places there may be some decrease and increase, but in general it would remain a pipe line of constant cross section. And this is what we had also described during our introductory lecture that when geometries are not changing for example, there is no abrupt contraction or there is no sudden thin region in the geometry then in such cases where there is steady flow as well as geometries that change gradually we are in general interested in only steady response of material. So, given that we are interested in only steady response of material we can therefore look at only the viscous response.

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The image shows a digital whiteboard with handwritten mathematical and conceptual notes. At the top, the Debye tensor D is defined as a 3x3 matrix with diagonal elements $-\frac{1}{2}\dot{\epsilon}$, $-\frac{1}{2}\dot{\epsilon}$, and $\dot{\epsilon}$, and off-diagonal elements 0. To the right, it is noted that $\tau_{32}, \tau_{xx}, \tau_{yy} \neq 0$. Below this, the Trouton ratio is calculated as $\frac{\eta_e}{\mu} = \frac{\eta_e}{\eta} = 3$. Under μ is written "Newtonian fluid", and under η is written "All other viscous response". A box labeled "Viscosity ?" is also present. At the bottom, a flow diagram shows "Steady \rightarrow engineering applications" leading to "viscous response", which then leads to "unsteady \rightarrow viscoelastic response".

$$D = \begin{bmatrix} -\frac{1}{2}\dot{\epsilon} & 0 & 0 \\ 0 & -\frac{1}{2}\dot{\epsilon} & 0 \\ 0 & 0 & \dot{\epsilon} \end{bmatrix} \quad \tau_{32}, \tau_{xx}, \tau_{yy} \neq 0$$
$$\text{Trouton ratio} = \frac{\eta_e}{\mu} = \frac{\eta_e}{\eta} = 3$$

Newtonian fluid All other viscous response

Steady \rightarrow engineering applications
 \downarrow
viscous response
unsteady \rightarrow viscoelastic response

Viscosity ?

So, this is something which we can keep in mind that for applications, engineering applications that require generally steady response then it is to assume viscous response. Of course, the counterpoint we are making that if we have an unsteady situation which is very important in an engineering application then we must look at viscoelastic response.

So, this gives us a key idea again that a material by itself need not be one type of response. So, a material itself may not be just viscous. We can choose to look at the material only as a viscous material because that is what is most relevant for the application and this is something we should always remember while looking at rheological responses of different category.

So, in steady geometries that change gradall; the underlying mechanisms of shear thinning may incorporate viscoelastic mechanisms may have aspects of recovery; however, it is not relevant to analyse them for a simple situation where we are only interested in flow rate versus pressure drop. And in such situations we can though we are aware that the underlying mechanisms are quite complicated we will still describe the overall response only as two material functions. So, most often whenever we assume viscous response, viscous response would immediately imply we are only looking at viscosity. So, this is the only material function of relevance whenever we are looking at material response as a viscous response.

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Viscous response
Non-linear Viscous fluids

Carreau-Yasuda model

- An example model
- 5 parameters

$$\eta = \eta_{\infty} + (\eta_0 - \eta_{\infty}) [1 + (\lambda \dot{\gamma}_{yx})^a]^{\frac{n-1}{a}} \quad (6)$$

$$\frac{\eta - \eta_{\infty}}{\eta_0 - \eta_{\infty}} = [1 + (\lambda \dot{\gamma}_{yx})^a]^{\frac{n-1}{a}}$$

Useful for response of polymer solutions / colloidal solutions

- Constant viscosity at low shear rates - zero shear viscosity: η_0
- Shear thinning at moderate and high shear rates
- Shear rate at which the onset of shear thinning is observed is related to largest relaxation time of the material

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So, now let us look at a simple model which can be used for describing the viscous response. As we said stress is a complicated function of strain rate and we have again denoted for simple shear flow here and gamma dot y x again implies that this is a simple shear flow where the simple shear flow which we have stated quite a few times in our class so far.

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The diagram shows a shear flow setup with two parallel plates. The top plate moves to the right with velocity U , and the bottom plate is stationary. The shear rate is denoted as $\dot{\gamma}_{yx}$. Below the diagram, a graph shows the viscosity η as a function of the shear rate $\dot{\gamma}_{yx}$. The graph is labeled $\eta(\dot{\gamma}_{yx})$. Two points on the curve are marked with $\lambda_1 > \lambda_2$. A note indicates that a lower value of $\dot{\gamma}_{yx}$ will lead to a higher value of η . The graph shows a power-law relationship $\eta \propto \dot{\gamma}_{yx}^{n-1}$ for $n < 1$, which is characteristic of shear thinning. The zero shear viscosity is denoted as $\eta = \eta_0$ and is associated with the equilibrium microstructure. The high shear viscosity is denoted as $\eta = \eta_{\infty}$.

So, it is a simple shear flow in which case gamma dot y x is the only relevant strain rate which is available. So, for such a case we have viscosity as a function of 5 different

parameters we have η_0 and η_∞ and we have λa and n are the 5 parameters. So, you can see that it is a fairly complicated algebraic expression and η is a function of strain rate.

So, therefore, the material function that is being analysed here is η as a function of $\dot{\gamma} x$. This is not a function of time nor is it a function of anything else not and only a function of the strain rate that is being applied and this is this model is quite useful for a variety of polymer solutions and colloidal solutions with low concentrations of colloidal particles and the main feature that this model has is the fact that if this term is very small. So, basically at very low shear rate if $\dot{\gamma}$ is small, then we can ignore basically this term this term can be ignored at low shear rates and therefore, then η minus η_∞ and this left hand side is equal to 1 and therefore, you can see that η is equal to η_0 .

So, therefore, the zero shear viscosity as it is called or it is the viscosity which is valid at low shear rates and, so this is an important characteristic of all complex materials. So, therefore, η_0 is what we should keep in mind and it is called zero shear viscosity. What we also see is as the strain rate increases this term is raised to power $n - 1$ and n is usually less than 1. So, because of this when this term becomes higher and we are raising it to a power which is a negative power because n is less than 1 what we will see is the viscosity will decrease therefore, the material has shear thinning at moderate and high shear rates which implies shear thinning means less viscosity as this rate strain rate increases.

This is the example we saw when we looked at the particles getting aligned or clusters breaking. In both cases there is low viscosity observed as shear rate is high. Here also we saw that there is a low viscosity because of shear and material is under flow. So, that is the same thing we can observe here. So, shear thinning at moderate and high rates.

Now, another important aspect of this is the shear rate at which the onset of shear thinning is observed is related to largest relaxation time. We have not in the course yet discuss the concept of relaxation time this λ here is similar to a relaxation time. What you can see here is that this combination is unit less. So, λ has units of time $\dot{\gamma}$ has units of 1 over time. So, rather than you the strain rate itself being involved in describing how η varies what we have is a group. So, it is the product of

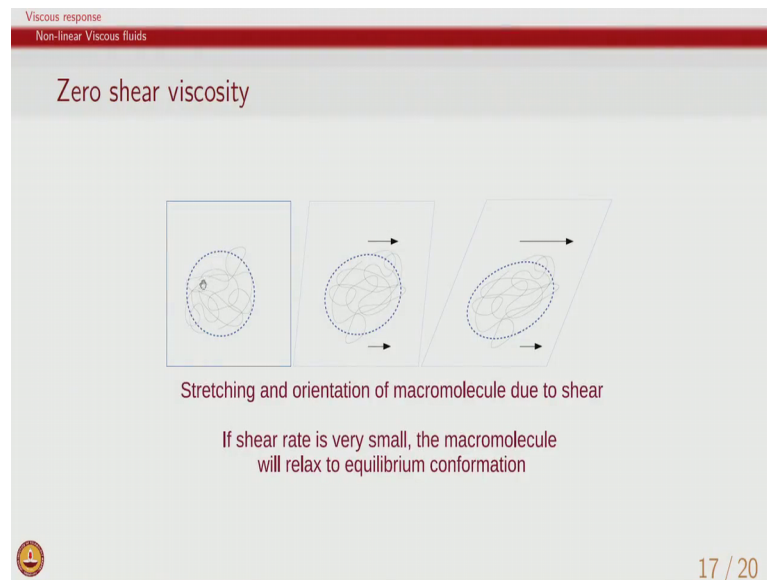
shear rate and stray relaxation time which is describing the overall behaviour. So, λ times $\dot{\gamma}$. So, this is second, this is 1 over second and so you can have two materials for which if you have λ_1 and λ_2 as the relaxation times then what you can see clearly is at lower strain rate itself.

So, $\dot{\gamma}$ x lower value will lead to the product being higher value because λ_1 itself is high or conversely in because λ_2 is low the product itself will be lower value. So, you can see that two different fluids can have similar responses provided we adjust the shear rate differently and so one of the key features that we will see is the onset of shear thinning given that at low shear rates there is a constant viscosity at high shear rates there is a shear thinning. So, therefore, in between there is a transition and that is what is called onset of shear thinning and this onset of shear thinning happens at a given shear rate. This will be an important characteristic because basically what we are saying is under what condition does this term start contributing. At very low shear rates this term will not contribute, at very high shear rates what will happen at high shear rates we can pretty much ignore this term and therefore, η will be only equal to; at very high shear rates we will have η is equal to η_{∞} .

So, in both the cases what we see is the fact that the two extreme when $\dot{\gamma}$ is very low we have η is equal to η_0 and when $\dot{\gamma}$ is very high η is equal to η_{∞} because at very high strain rates since this term is very large one can be ignored. So, this one can be ignored whenever the strain rate is very high and in that case we will have η is equal to η_{∞} . At low shear rates we have η equal to η_0 and in between is there is a transition which is called the onset of shear thinning.

Now, let us try to look some of these in qualitative way by first looking at what is the meaning of this zero shear viscosity.

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We have already seen that when you have a molecule and it is a fluctuating molecule which undergoes segmental motion and when we apply shear the top some section of it starts moving faster compared to some other section and therefore, you have overall stretching and orientation of this polymer molecule. And the higher the shear rate the more will be the stretching and orientation.

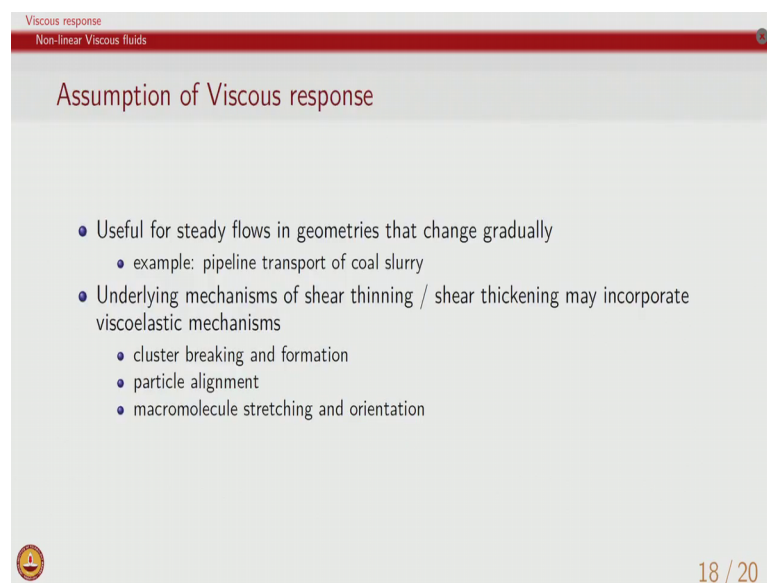
Now, if the strain rate is very small then what will happen is even though there is a tendency for this molecule to stretch and orient there is also a tendency for it to go back. So, there is a tendency for it to go back to this equilibrium state and therefore, if the shear rate is very small the molecule will relax shear or shear rates which are very low what we have is the difference between velocity here and here is so small that molecule pretty much maintains the equilibrium shape and therefore, this is the viscosity which is measured under these condition is called zero shear viscosity.

Clearly you can see that zero shear viscosity which is this is related to the equilibrium microstructure and it is the disturbance of microstructure from this equilibrium state which leads to viscosity being different. So, for example, at higher and higher shear rates since we have the polymer molecule which is stretched and oriented then it basically effectively offers less resistance to flow and therefore, we see that this has less viscosity. So, a polymer solution also will have highest viscosity when there is very low shear, shear thinning and again very low viscosity at very high shear rate. So, the feature which

was exhibited by the Carreau-Yasuda model in terms of the zero shear viscosity is very much shown by polymer solutions also.

All of this discussion we should always remember that the polymer molecule that is being discussed is always flexible and fluctuating because of the interaction with solvent molecules. There is always a tendon equilibrium shape, only thing is when shear rate is high this is not possible when shear rate is very small then the molecule can relax back to the equilibrium conformation.

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The slide is titled "Assumption of Viscous response" and is part of a presentation on "Non-linear Viscous fluids". It contains the following bulleted text:

- Useful for steady flows in geometries that change gradually
 - example: pipeline transport of coal slurry
- Underlying mechanisms of shear thinning / shear thickening may incorporate viscoelastic mechanisms
 - cluster breaking and formation
 - particle alignment
 - macromolecule stretching and orientation

The slide also features a small logo in the bottom left corner and the page number "18 / 20" in the bottom right corner.

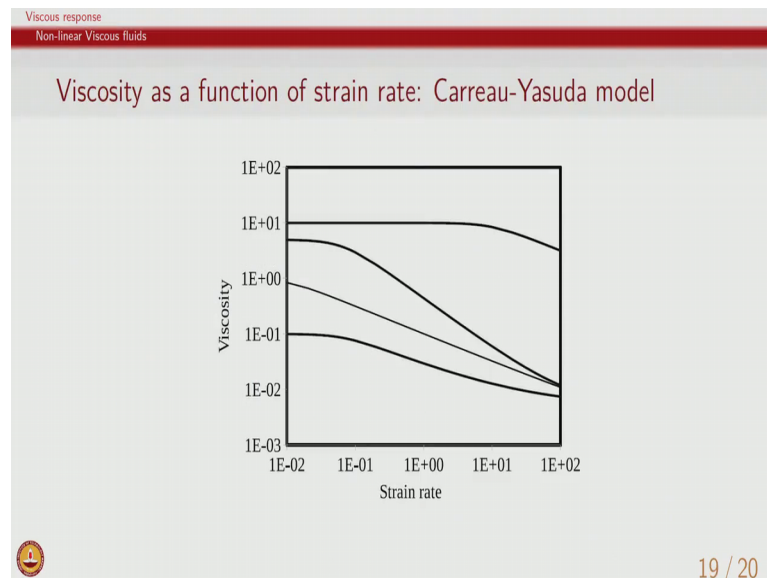
So, now again we have seen in this case of polymer also that the underlying mechanism is actually leading to elastic recovery and variety of other phenomena which we will study under viscoelastic response. However, because we are only interested in steady response we are ignoring the underlying mechanisms and focusing only on the viscous response of the material.

So, therefore, whenever we assume a fluid to be viscous we realized that the non-linear viscous nature is due to mechanisms which are fairly complicated and it is just to describe these mechanisms well. Last 20-30 years of rheology research is actually to find out what these mechanisms are and explain what is the viscous response based on these mechanisms. However, in early times when polymer processing was done or paints or other complex fluids were analysed it was thought to be sufficient that can we capture the viscosity as a function of strain rate alone and only some heuristic arguments

regarding what might be leading to the shear thinning or the changing nature of viscosity. However, with time we find that such viscous response is only a very small window of what the overall response of this material is, and from recent times its very often we look at the underlying mechanisms and try to explain the viscous response rather than using the models like Carreau-Yasuda model which assumes to begin with a viscous response for the material because viscosity is the only material function which is being considered.

As we have stated already if we use Carreau-Yasuda model for describing the material response it will show instantaneous relaxation, it will show only dissipative response, a steady state will be reached either in constant stress or constant strain rate experiment, normal stress differences in a simple shear flow will be 0. So, therefore, in a nutshell all the response which is viscous. And of course, that is expected because we are only looking at viscosity as a material function. So, therefore, the viscous response is only a subset of what a material could actually be exhibiting. Like what we have seen in case of the colloidal dispersions as well as polymer solutions.

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Now, just to look at quantitatively what is the variation. If you see here the constant viscosity at low strain rates, these are two different behaviour being shown in both cases viscosity is constant at low strain rates and then beyond a certain strain rate viscosity decreases. What is also shown in the third curve is viscosity is a continuously decreasing

function, if you go back and look at the Carreau-Yasuda model in some range you can actually show that viscosity is only proportional to $\dot{\gamma}$ to the power $n - 1$ and therefore, this is called the another model which is called power law. So, in 60s and 70s many of the polymer processing was done using power law model.

So, in Carreau-Yasuda model the power law model is already incorporated where if this term dominates then a can cancel out and its $\dot{\gamma}$ to the power $n - 1$. So, that kind of a response is what is being shown here where viscosity continuously keeps on decreasing as a function of strain rate because n is less than 1, as I have said already this will lead to a viscosity which is continuously decreasing function of strain rate. And in fact, this also says that if $\dot{\gamma}$ approaches 0 η basically goes to infinity and that is what is depicted in this case that if you extrapolate it in this direction then viscosity will continue to increase.

For a very dilute dispersion sometime the change in viscosity is not very high. So, η_0 at this end and η_∞ are not very different and in between there is a mild shear thinning. So, Carreau-Yasuda model can capture all of these basic features which are observed by polymer solution and colloidal dispersions. In many of the colloidal dispersion also shows shear thickening which means viscosity increases as a function of strain rate and again this is related to basic mechanism which is there in the material and to understand that we would again need to go back to what the colloidal dispersion is made up of.

So, many times if the interlocking structure which is present at low shear rate as I showed in the two examples what we will see is such interlocking structures come into play only at high strain rates. So, therefore, we have generally a viscosity which initially is constant then it decreases and then it starts increasing. So, later on in the course we will look at some of these mechanism which lead to shear thickening in materials.

So, with that we will close the discussion on viscous response with a point that viscous response is a fairly limiting way of looking at complex materials, we only look at viscosity as a material function and use it for situations which are largely steady and with geometries which are gradually changing. As we know most engineering applications will involve unsteady situations as well as complex geometries. So, therefore, viscous response will not really serve in most of those engineering applications. So, the lectures

here on will focus on other aspects which are related to viscoelasticity of material systems.