Rheology of Complex Materials
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Lecture - 15
Viscous response

In the previous lectures we become familiar with kinetic as well as kinematic measures quantities such as stress, strain rate, strain. And we also spend some time talking about the rheumatic flows which are useful in the analysis of rheological response and so now, we are in a position to start discussing how do the complex materials respond, what are the different types of responses that complex materials exhibit. And as part of this discussion on various types of responses we will begin with viscous response and how we will do this is initially we will look at some of the introductory concepts and then quickly review the Newtonian fluid which is the most common viscous fluid that me know and to quantify the viscous response of complex materials we will define some material functions.

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And finally, we will end up discussing non-linear viscous fluid response by looking at some of the mechanisms which lead to the non-linear viscous response as well as some of the models which are useful in describing the viscous response.

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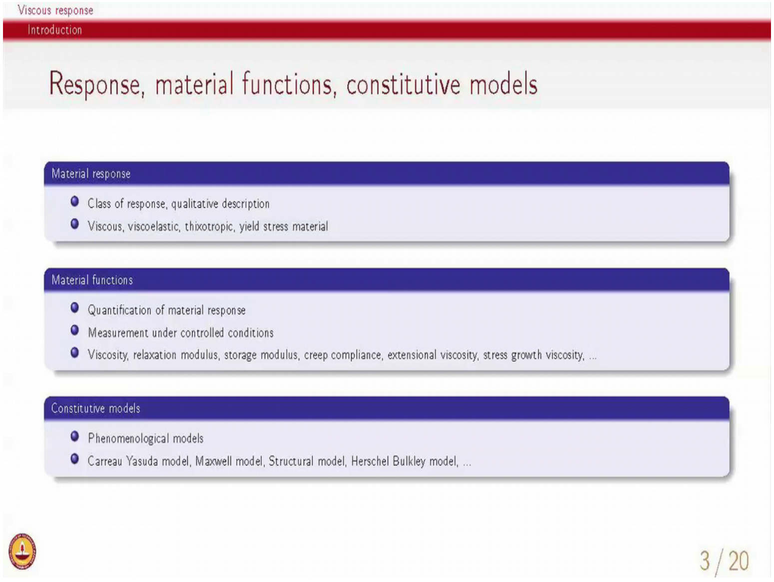

So, let us begin by looking at the overall framework in which we will do this. Since we are interested in looking at the response of complex materials we will first find it helpful to look at them as different types of classes and describe them using qualitatively. So, therefore, we will use terms like viscous viscoelastic thixotropic and yield stress material. So, of course, this lecture we will discuss viscous and in subsequent lectures we will discuss other class of material response.

Once we do this it becomes easier for us to identify what are the key features of each and every type of response and if a new material is being investigated whether it belongs to one of these different classes or does it showed responses which can be in multiple classes is type of analysis that we can do. To help understand better the qualitative descriptions we need to quantify the response and there that is where material functions become very useful. Material functions are defined based on measurement under controlled conditions for example, constant stress or constant strain or constant strain rate and also simple shear flow or uniaxial extension. So, these are all what I what is meant by controlled conditions.

So, we measure the properties under these control conditions and the properties or quantified in terms of material functions. So, for example, viscosity which we know as a material constant for Newtonian fluid in today's lecture we will see is actually material function which characterizes the overall viscous response of a material. Similarly later on we will define material function such a storage modulus extensional viscosity and so on. Additionally if further need of quantification is there in terms of having suitable models because these models are phenomenological they will incorporate terms which give us physical insights about how the response of complex materialises. So, it is important for us to not only define material functions and quantify the response we should see whether by adequate representation of the response in by mathematical means can we capture the behaviour which is described the material functions. And to this end we will look at some simple models so that we can go back and forth between the response the material function and the model.

So, today for example, will look at Carreau Yasuda model sometime in future we will look at different models which all correspond to different classes of material response.

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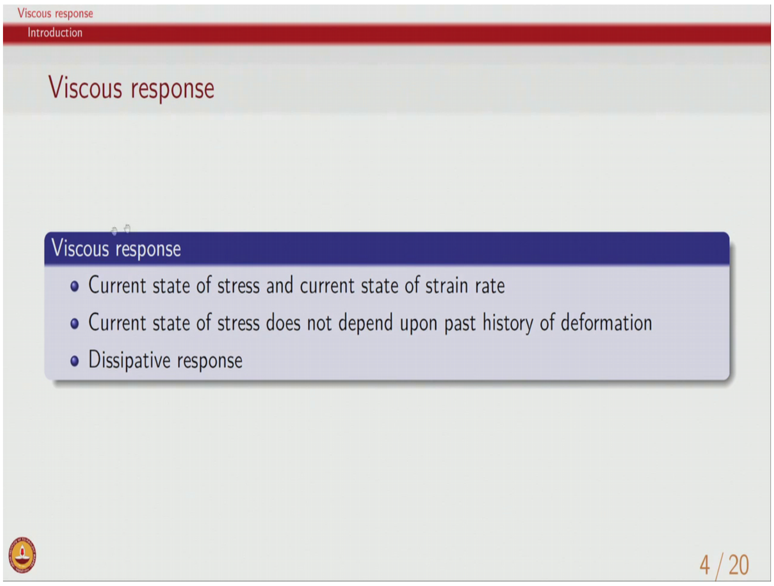

So, now let us summarise viscous response. We have seen this multiple times before that current state of stress and current state of strain rate is what defines viscous response. So, the past deformation is of no relevance. Similarly the amount of deformation in terms of the strain is not really relevant what is only important is what is the current stress in the material and current strain rate and both of these are related to each other and therefore, the viscous response is completely dissipative response. So, these are the basic description which we have already discussed several times in the course before. And mathematical model constitutive model which captures this for a class of fluids is called

Newtonian fluid, in which we are very familiar with in which case the total stress tensor is the pressure and deviatory stress tensor and its deviatory stress which is related to the velocity gradient as we discussed earlier.

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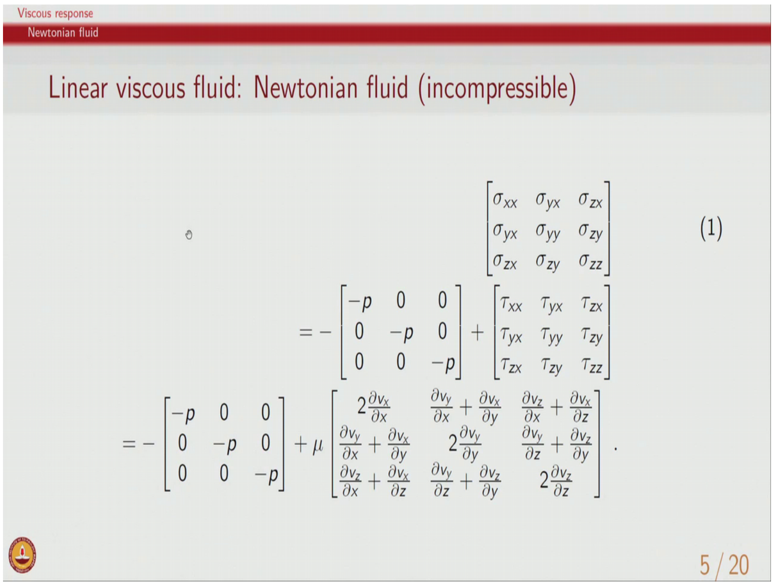

And the proportionality constant is viscosity, and depending on the type of flow one or more of these velocity gradient strain rate tenses components will be non-zero and therefore, the components of deviatory stress as well as components of stress will be zero or non-zero. So, this equation for Newtonian fluid which is incompressible is used very heavily in fluid mechanics and is an example of viscous response.

Let us look at what are the main features that the Newtonian fluid has. So, in Newtonian fluid in simple shear basically let us look at the rotational parallel plate and just to remind you we have basically a top plate and a bottom plate.

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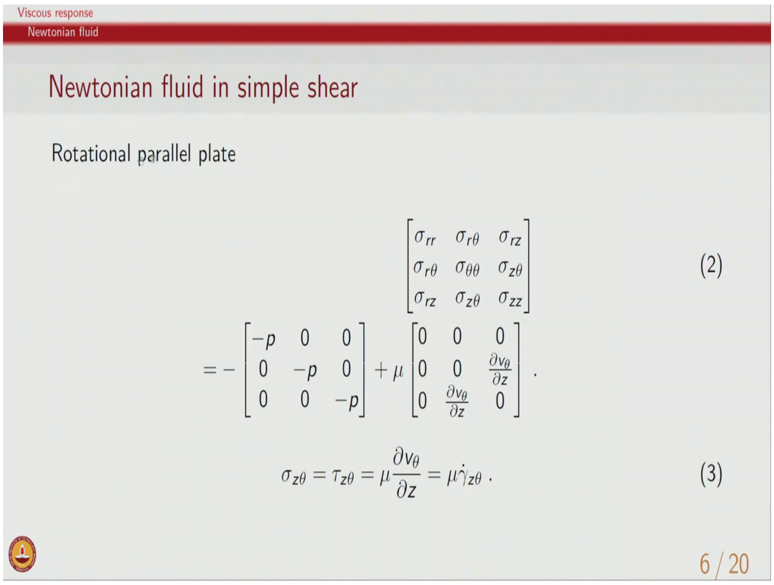

And the top plate is being rotated and the fluid is kept in between and, since we have this simple shear flow we have velocity in theta direction and it is only a function of z. So, therefore, the only component of velocity gradient which will be most relevant in this case is del v is theta by del z and that is what we have stated here.

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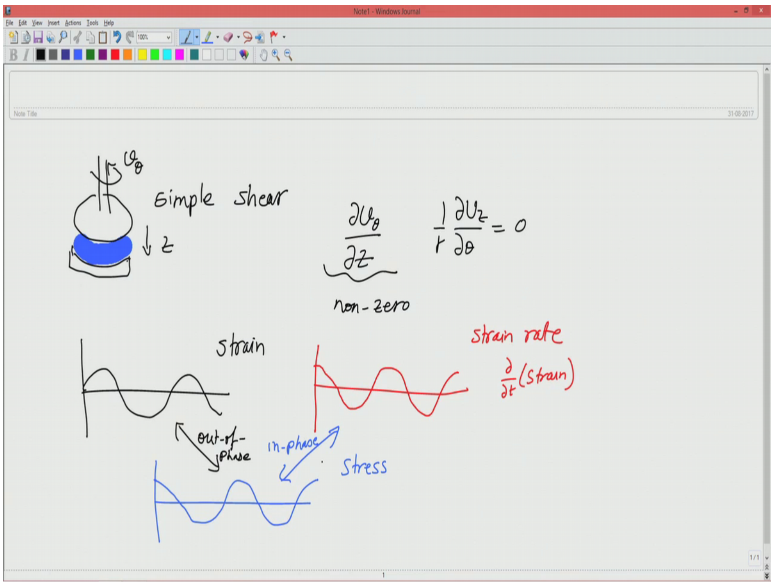

The fact that the total stress tensor for a Newtonian fluid will be pressure which is uniform everywhere and then the viscosity times the velocity gradient. So, the del v z by del theta term which is the other part of the strain rate tensor that is zero since we do not really have a velocity in z direction.

So, del v z by del theta this kind of term is actually zero. So, only this is the non-zero term. And so, in the sigma z theta there for which is this component plus this component and this component plus this component is related to basically the velocity gradient times the viscosity and we will represent it using this symbol gamma dot. So, that implies the strain rate in this case and this is the usual learning that we do in our earlier courses that stress is proportional to strain rate and proportionality constant is viscosity. And quite often of course, since we are discussing simple shear we will say shear stress is proportional to shear rate and the proportionality constant is this viscosity and sometimes the therefore, this is also referred to as the shear viscosity. But it should be clear to us that even the normal stress components are related to normal strain rate components to the same coefficient mu. So, therefore, it is the viscosity which captures the Newtonian fluid response in any type of flow be it shear be it extension or be it any combination of shear an extension.

So, broadly when we have this kind of a Newtonian model we can summarise the features like this for simple shear flow there is only one shear component of strain rate tensor D which is non-zero.

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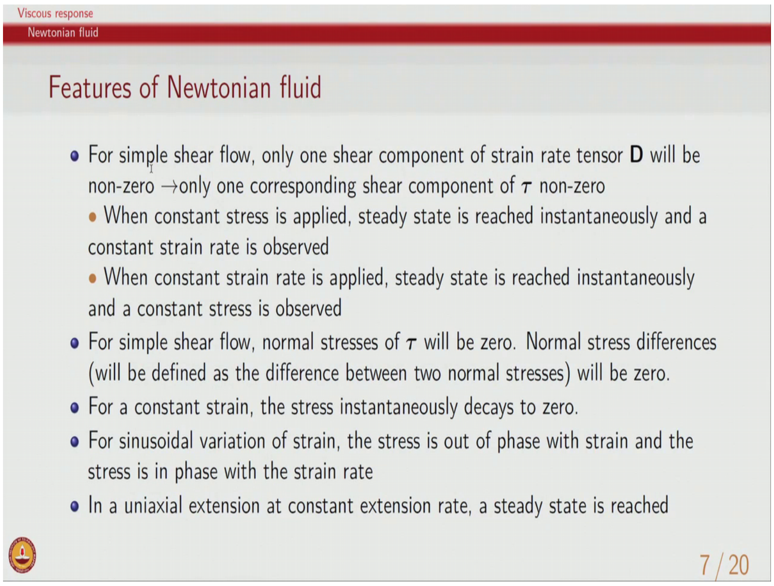

So, as we saw in the previous slide there is only one component this is of course, always a symmetric tensor. So, there are 3 plus 3, 6 components which are of relevance. So, therefore, only one component in this case z theta or theta z which is non-zero. So, only one corresponding shear component of tau or sigma is also non-zero. So, the stresses also we find similarly tau z theta or theta z or sigma z theta or sigma theta z non-zero.

Now, in there are two situations and these also we have discussed. So, I will only summarise them here quickly that if a constant stress is applied a steady state is reached and this is reached instantaneously because if you look at the governing equation as soon as we apply your constant strain rate the stress has to be also become constant. If this is the function of time this will be a similar function of time, if this is increasing this will also be increasing with time, if this is zero this will also be zero. So, there for as soon as this is constant both stresses will also become constant and therefore, in this case we have a constant rate of dissipation because stress is constant and strain rate is constant.

The other type of experiment is where if we have a constant strain rate applied then again steady state is reached quickly and instantaneously we have constant stress is observed. So, there therefore, in both cases we reaches steady state and the constant dissipation is observed. Simple shear flow normal stresses will be zero, as we saw in the previous case the normal stresses are all zero because normal strain rate components are all zero. So, therefore, tau rr, tau theta theta, sigma rr, sigma theta theta are all zero.

We will see this that this is an important characteristic of viscoelastic fluid. So, any signature of elasticity in a fluid will be based on the normal stress differences which means in this case normal stress differences are zero because normal stresses themselves are zero, but for viscoelastic fluids we will see that the stresses will be non-zero and their differences will be a key indicator of the elasticity of the fluid. For a constant strain the stress instantaneously decays to zero which is means instantaneous relaxation.

For sinusoidal variation of strain the stress is out of phase with strain. What do we mean by this? That if you have a strain which is like this. So, if this is the strain, strain rate will be given by derivative of strain. So, strain rate is nothing, but del by del t of strain which means it is the time derivative and so wherever the strain was zero we will find that strain rate is maximum and, if you look at the stress itself stress will be in phase with strain rate. So, this is the stress. So, therefore, stress and strain rate are in phase while stress and strain are out of phase. We will see later on that viscoelastic fluids will again we can characterized them using this kind of a sinusoidal strain and strain rate. For an elastic solid the situation is exactly opposite the strain rate and stress are in out of phase and strain and stress are in phase. So, we will discuss this later on when we discuss viscoelasticity. So, keep this in mind.

So, in a summary what we are saying is sinusoidal variation of strain the stresses out of phase and the stresses in phase with strain rate and in a uniaxial extension at constant extension rate a steady state is reached. This is fairly important again from elastic fluids which are used in making fibres or sheets and so many for plastic materials are used to make where extensional flow is involved and many of those materials actually do not show a steady state.

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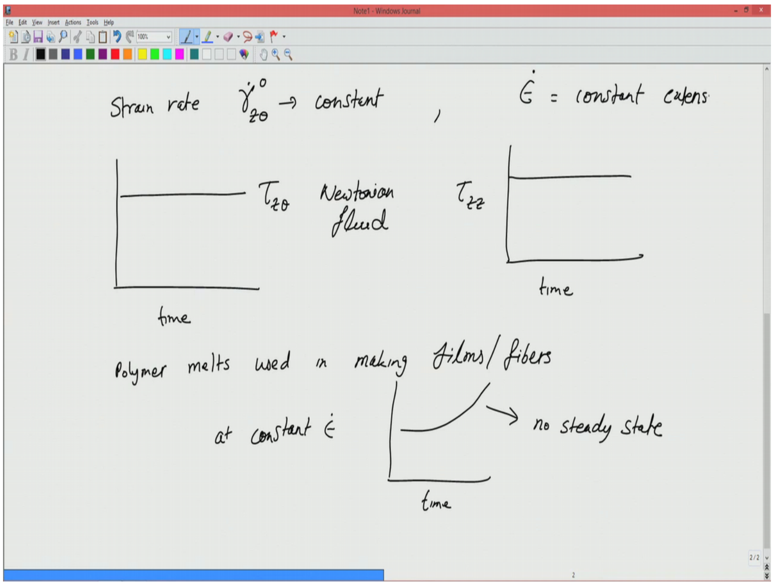

So, what do we mean by that? Steady state is, for example, when we have a constant shear rate applied or constant strain rate such as gamma dot z theta it is a constant strain rate being applied what we see is the stress itself is constant for a Newtonian fluid and similarly when we apply epsilon dot which is also constant and we look at tau zz again as a function of time we will see that it is a constant value. So, this is what we mean by a steady state is reached.

What we see in many of the polymer melts used in making films and fibres is at constant epsilon dot they show responses of stress as increasing. So, therefore, there is no steady state clearly for Newtonian fluid we have no such feature. So, Newtonian fluid shows a steady state both in shear as well as extension. And that is what we have summarised here, saying that in a uniaxial extension constant extension rate a steady state is reached similarly when a constant strain rate is applied for a simple shear again we reach steady state.

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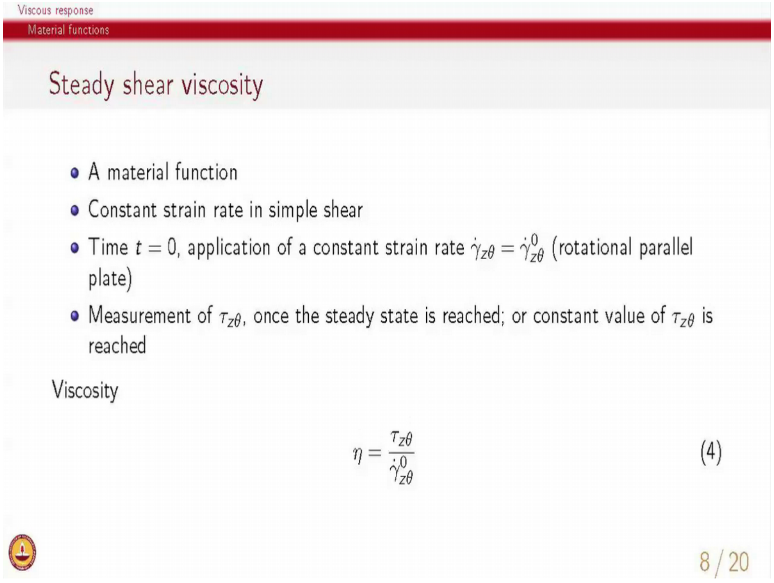

Now, we define a material function to quantify a response of material in what is called steady shear. So, this is called a material function and it will become evident soon as to why we refer to it as function and not a material constant.

So, constant strain rate will be applied in simple shear and in case of rotational parallel plate the z theta component is what is non-zero. So, therefore, at time t is equal to zero a constant strain rate would be applied and this zero is to indicate that it is a constant value. And as soon as the constant strain rate is applied we start measuring the stress based on the torque that is required to rotate the parallel plate and, this stress and torque will be measured and we will wait till the steady state and when the study state is reached the value of tau z theta is noted and so constant value of tau z theta divided by the strain rate which was applied is called the viscosity. So, this is a material function which describes the response of material at steady shear. So, therefore, we wait for constant value of stress to be reached.

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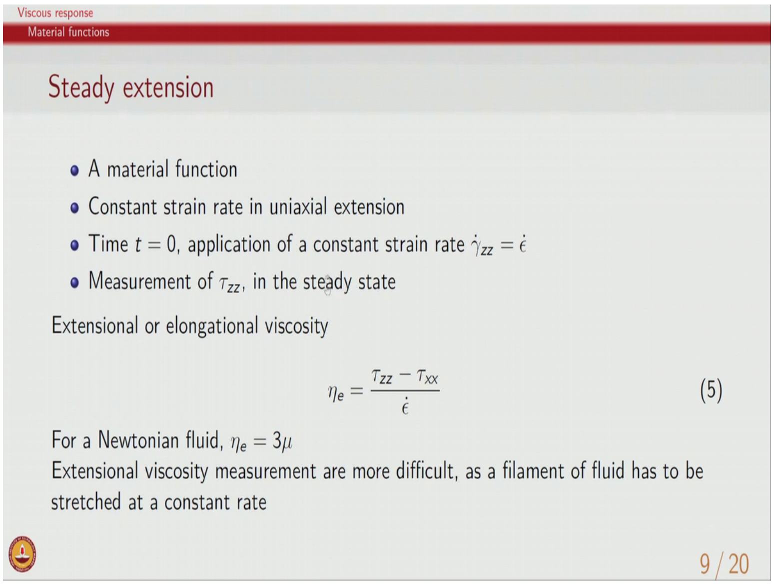

Similarly in study extension we have a material function defined called elongation or extensional viscosity again a constant strain rate in uniaxial extension is applied at time t is equal to zero we apply a constant strain rate which is in the z direction in epsilon dot. And while discussing rheumatic flows we have already discussed the type of extension that can be experienced in uniaxial.

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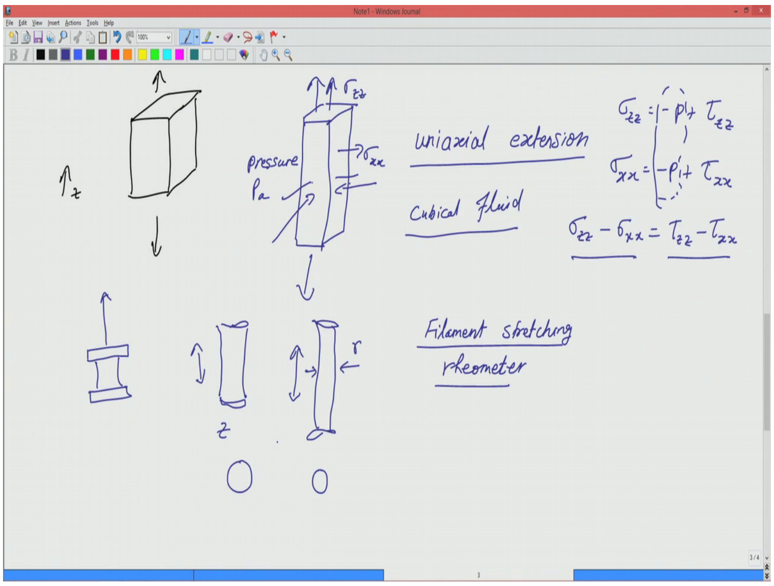

For example, if you recall we had discussed cuboid of material and if this is the z direction then we can pull it in this direction and we would expect that after sometime this material would become longer in z direction and it would shrink in both x and y direction. So, in x and y direction it will shrink while in z direction it will increase. So, this is the uniaxial extension.

Of course, in a discussion or in a text book or in a classroom setting we are discussing it with cubicle fluid which is of course, not practically achievable. How this experiment is done, is actually you take a cylindrical body of fluid and these two plates are pulled apart and therefore, this z will actually with time become thinner and thinner and so on. So, therefore, there is a pull again in z direction and there is going inside in radial direction and from this side if you look the cross section of the fibre or filament keeps on changing. So, this is called a filament stretching rheometer. And we will discuss some aspects related to instrumentation and makeup of such rheometers in a class later on.

So, therefore, we are describing the uniaxial extension using this rectangular coordinates assuming a cuboid element, but similar definition is valid whether it is a cylindrical filament which is actually done in an experimental scale there again the stresses will be zz and rr while here we are just talking about zz and x x. So, the normal stresses in these two direction the subtraction is done so that the pressure which is unknown. So, sigma zz and sigma xx the pressure will be subtracted and that is the same as tau zz minus tau xx.

So, what we are doing in this case is the fact that there is sigma xx here and sigma zz here and of course, both of these sigma zz will be equal to minus p plus tau zz and sigma xx will be equal to minus p plus tau xx and what we are really interested in knowing is the stress difference between these two because this is anyway an unknown quantity. So, therefore, we define the extensional viscosity in terms of a stress difference which is what is written here. And it can be shown that if you look at the overall velocity profile if you recall in such an extensional flow we had written earlier that the extensional the strain rate is given by minus epsilon dot 0 0, 0 minus half epsilon dot 0, 0 0 epsilon dot.

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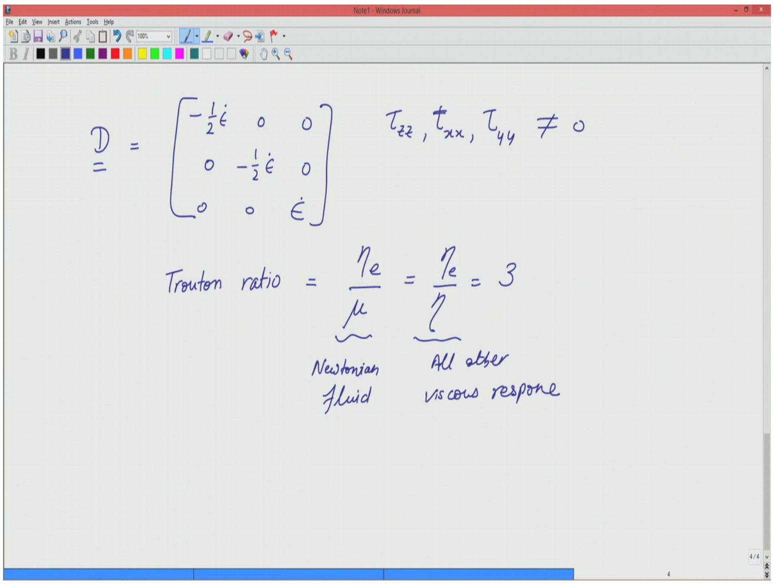

So, therefore, we have all three components tau zz, tau xx and tau y y are not 0 and we can manipulate algebraically and using boundary conditions we can actually find out what is the viscosity based on our equation. The boundary condition will be that at these surfaces pressure is given by atmospheric pressure. So, using this we can solve and get the result that the extensional viscosity for a Newtonian fluid is three times the viscosity that we defined earlier. So, this is also called the Trouton viscosity, the Trouton ratio is there for 3 and this is something which is discussed once in a while in literature where we talk about the Trouton ratio which is nothing, but eta e by mu or eta e by eta and it is equal to 3.

We will see that we will be using symbol mu as well as eta mu is generally used for Newtonian fluid and eta we will use for all other viscous response. So, whenever we use mu we will indicate by that it is a material constant and therefore, it is constant for a met it is only a function of temperature. So, given these two material functions that we have define one for steady shear and steady extension we just again remind ourselves of overall features of Newtonian fluids that they we reach a steady state in steady shear and extension the normal stresses of components are zero and also the in case of a constants strain there is an instantaneous decay.

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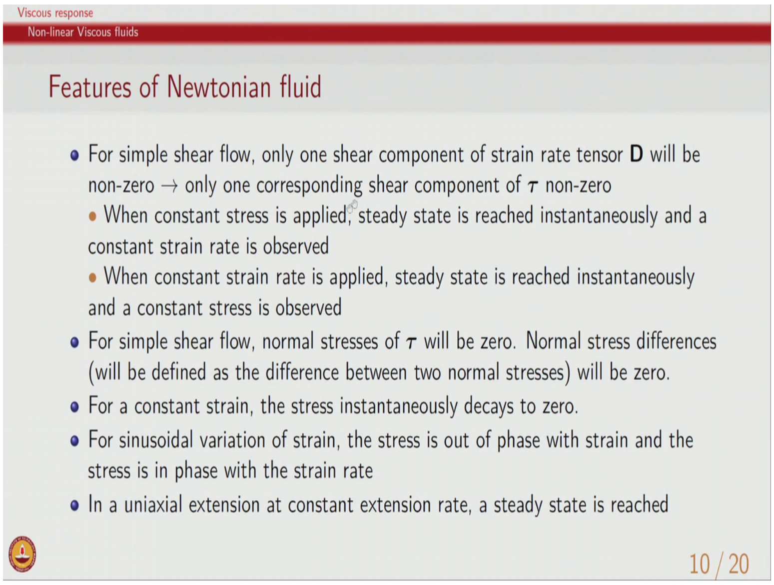

And why we repeated these is to highlight to you that in fact, these are exactly the same features which are valet for all classes of viscous fluids also. So, if you take any other viscous fluid also the response these features that we talked about are entirely identical.

So, in the sense for any viscous fluid also we will reach study states when a constant stress and strain rate is applied the normal stress will be zero for simple shear flow, for a constant strain the stress decays to zero which means this is perfectly dissipative response and for sinusoidal variation of strain the stresses out of phase with strain or stress is in phase with strain rate, stress strain rate are in phase and. So, if you look at the overall features of Newtonian fluid and features of viscous fluids they are entirely identical then what is the difference.

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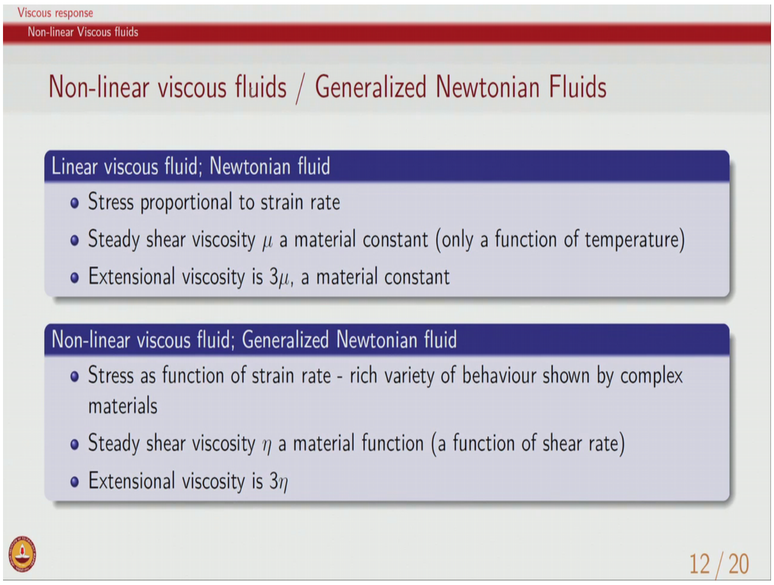

And that is what gives us an idea to describe what are called non-linear viscous fluids or generalized Newtonian fluids. So, they are called generalize Newtonian because as we saw many features of Newtonian fluids are sheared by these fluids. And they are called non-linear viscous fluids because unlike Newtonian fluid which is a linear viscous fluid these have non-linear relations between stress and strain rate.

So, summarise again linear viscous fluid which is also Newtonian fluid stress is directly proportional to strain rate they are linearly related to each other therefore, linear viscous fluid. The study shear viscosity mu is a material constant in fact, it is only a function of temperature and of course, pressure depending on if pressure differences are very high. In this course we are mostly concentrating on incompressible fluid. So, pressure dependence does not really arise; however, temperature dependence is viscosity is a fairly strong function. Many of the situations in this course we will discuss isothermal situation so therefore, again we will find viscosity to be a constant.

We also saw that extension viscosity is three times mu are the trouton ratio is 3 and again it is a material constant. For a non-linear viscous fluid or generalize Newtonian fluid again stress is a function of strain rate. So, like we describe for Newtonian fluid current state of stress and current state of strain rate only relevant. And only thing is instead of being related to each other through a linear relationship there is a very rich variety of behaviour shown by all types of complex materials, whether we look at ketchup as a material whether we look at any other sauces which we use in the kitchen whether we look at shampoos or any other personal care products, whether we look at polymer solutions which are used in enhanced oil recovery all of these show fairly different types of behaviour and so challenge in terms of generalized Newtonian fluid and non-linear viscous response is to try to capture the correct response that is shown by a real fluid.

The study shear viscosity is a material function because it is a function of the strain rate. Since stress is a function of strain rate we actually have the viscosity itself also which is the ratio of the stress and strain rate becomes a function of strain rate. And very much like Newtonian fluid the extensional viscosities still three times the viscosity; however, since this itself is a function of strain rate we will see a similar function for extensional viscosity also. So, if you see the difference between these two the overall dissipative response or the general viscous nature of it is completely similar and exactly the same. In fact, however, the only big difference is in terms of the relationship between stress and strain rate.

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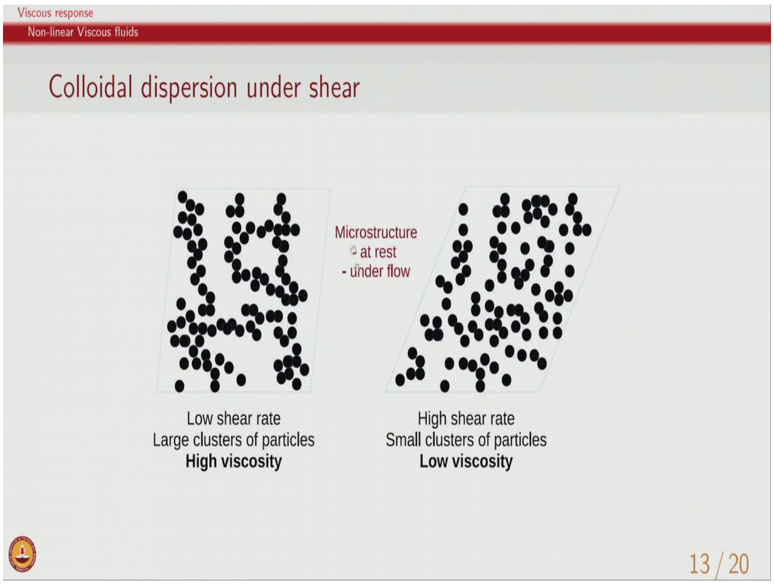

Now, let us look at what we already saw in the first class as to what happens to a colloidal dispersion under shear. So, for example the microstructure at rest would incorporate many of these particles agglomerated with each other and forming a network. So, when this network is disturbed at low shear rates since there are very large clusters the high viscosity is experienced. But if we apply a shear rate and that shear rate is reasonably high then what that leads to is these particles move at different relative velocities and the particle interaction in terms of attraction can be overcome due to the shear field and therefore, the large clusters can actually now breakup into smaller clusters and therefore, the viscosity can be lower. So, this is what we mean when we say here that study shear viscosity is a material function and it is a function of strain rate of the material.

So, in this particular case we saw that we are not saying that the viscosity would be lower when you increase the strain rate.

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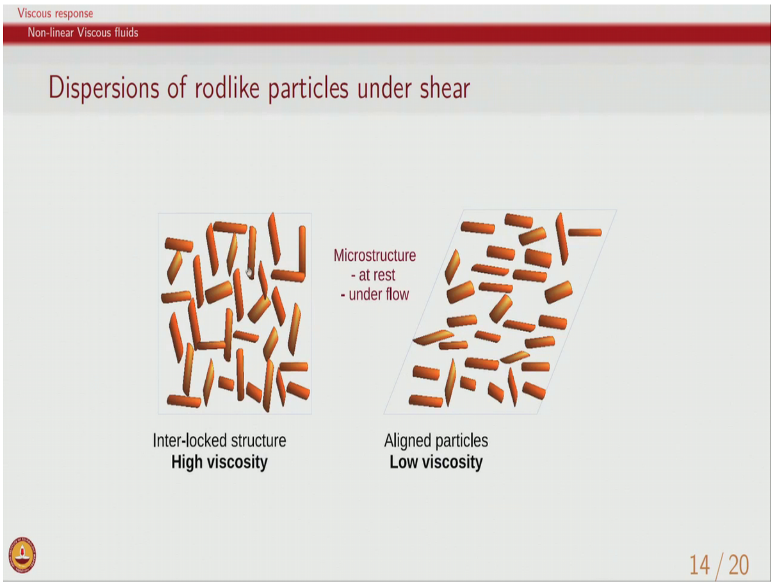

We can look at another example of how viscosity may vary due to shear and that is given by let us say dispersion of rod like particles. So, these rod like particles are oriented randomly and so when we have a microstructure at rest basically there is an interlock structure because all of these particles are impinging on each other and they basically block each other's motion. We should keep in mind that of course, there is some degree of thermal motion available. So, each of these particles overtime maybe a rotating, maybe vibrating moving about slowly, but in general if you look at there is no large scale motion because of this interlock structure.

Now, as soon as this kind of dispersion with these long particles is shear is applied what happens is there is an alignment. So, these aligned particles if you see here in this drawing most of the particles are depicted to have align in the direction of the shear.

Now, clearly because the no longer interlocking structure is there and in general particles will move easily because they are not interlocked with each other the apparent viscosity or the viscosity as a material function when we calculate the force required for this or the stress required for this arrangement will be much less. So, it is easier to shear this material when compared to this and so therefore, in effect we will have a low viscosity which will be experienced.

Now, both of these things one key feature that we must remember is there is a microstructure at rest and then there is a microstructure under flow and clearly there is an indication of recovery in this kind of structures. So, for example, if I stop this flow the particles would again start getting agglomerated and they would again form a cluster and therefore, material will come back to this. Similarly in these case also as soon as we stop the flow the particles again would start getting randomly oriented and they would again get interlocked. So, even though we are focusing on only viscosity and only looking at a steady state picture we should remember that the underlying mechanism may have some aspects of elastic or recovery features.

So, with this we will stop this lecture and in the next lecture we will look at another example related to polymer solution and we will also look at a model which can capture some of the viscous response that we have discussed so far.