Rheology of Complex Materials
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Lecture – 12
Kinematics for simple flows

So, therefore, in the last class we saw that how qualitatively simple shear flow uniaxial extension or biaxial extension can be described and visualise qualitatively.

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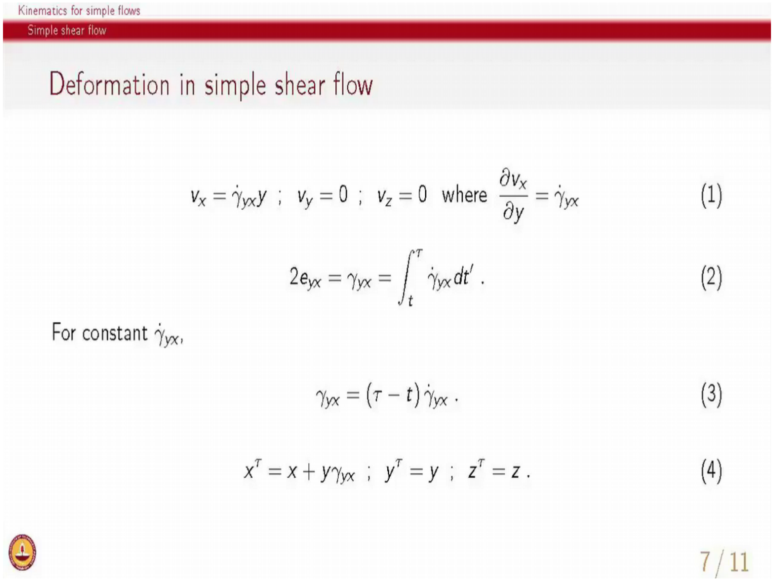

Now we will try to describe it quantitatively and the description is based on the flow that velocity is basically given as only in x direction and both v y and v z are 0 and of course, the velocity gradient exists and that is nothing, but gamma dot y x. And we can define deformation measure which is can be indicated using a symbol e y x or gamma y x.

Now, again in our course we will use e as a symbol for talking about strain and also the strain which is infinitesimal strain tensor. So, e is infinitesimal strain tensor.

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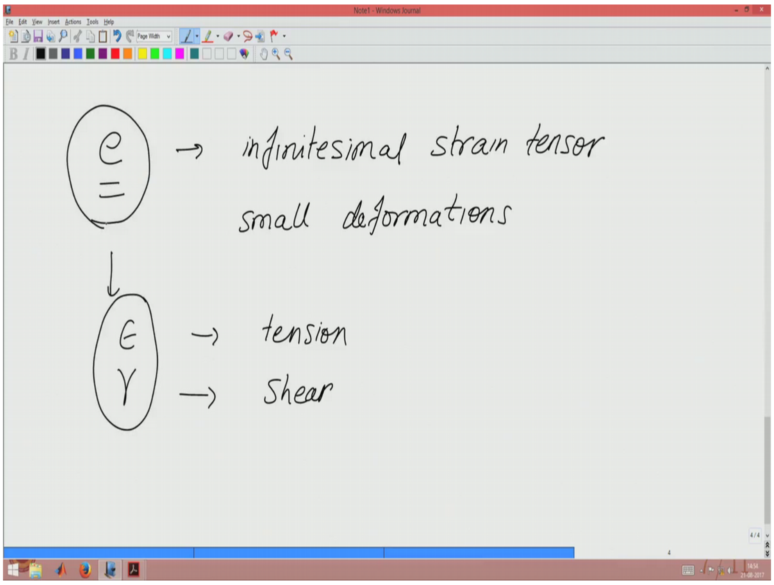

So, as the name suggest this is valid for small deformations, in solid mechanics generally this is indicated as epsilon when it is tension or compression it is indicated as gamma when it is shear. Again this epsilon and gamma and all are commonly used terms, but when you want to describe the complete details you must use a tensor. So, which is the strain tensor. So, that is why we will go back and forth between these symbols which are commonly used symbols, but the correct and complete term which is the strain tensor itself.

So, that is why given that a material is deforming at this gamma dot y x you could define deformation which is gamma y x or e y x, which is basically the amount of deformation which has taken from present to anytime tau. Whenever it is time in the future we will have strain being positive because the basis is present time when it is in the past then the strain will be different sign. So, since the basis is present time we can define it like this.

So, now let us just look at how do we describe the position of each and every material particle.

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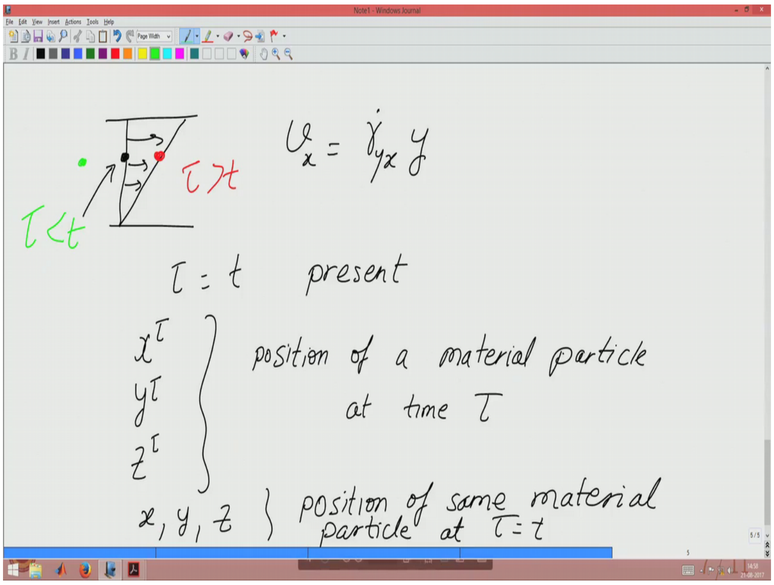

So, this is the overall flow and we saw that the velocity is gamma dot y x times y. So, now, let us try to describe the position of a material particle as a function of tau, because tau is equal to t means present right, but we would like to find and the position of a given material particle at any tau.

So, we will indicate x tau y tau and z tau as position of a material particle at time tau and of course, we would like to express it given that we are tracking a material particle that x y and z is position of the same material particle at what time at tau is equal to t which is a present time. So, how do we relate this? So, if you take a material particle which is here arbitrarily right any material particle which is there how do we now describe it is position as a function of tau.

So, again conceptually it should be easy for us to say that at present time if the particle is here, then somewhere in the future when tau is greater than time the particle would be would have moved right and similarly if I take some time where tau is less than time then the particle would have been to the left.

So, now the other thing which I have drawn automatically is the y position of the particle has not changed at all right because a particle is moving only in x direction and similarly the z motion is not at all described because z nothing changes it is an infinite plate in the z direction. So, therefore, now can we summarise this. So, clearly we can first write down y tau will remain y and z tau will remain z.

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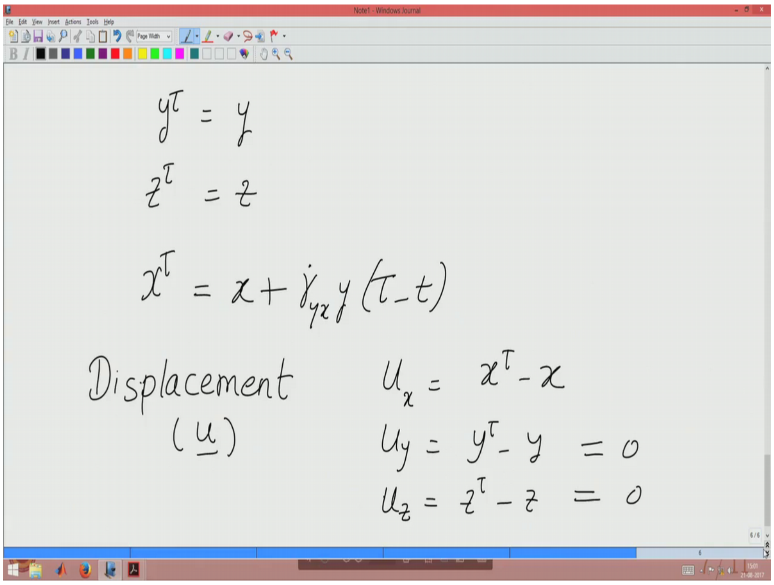

Now how is x tau related to x what is the velocity of the particle at this point given that it is y from the bottom plate right it is velocity is given by this. So, what will be the position of red particle with respect to position of black particle given that this is at tau and this is at t.

So, we can write gamma dot y x into y into tau minus t right will it be tau minus t because that is the amount of time for which particle has moved at a velocity and this will be x tau plus does that make sense.

So, if tau is equal to present time then what happens is this term whole this time goes to 0 and therefore, x tau is equal to x. So, of course, clearly a tau is equal to t x tau is equal to x y tau is equal to y and z tau is equal to z and any time in the past tau minus t will be negative and therefore, the x tau will be x minus something and. So, this way this is the complete description of the flow actually.

So, now, using this we can actually calculate the strain and to calculate strain we usually use a variable which is called displacement. So, it is again a vector because displacement can happen in 3 different directions. So, in this case u x x tau minus x, u y y tau minus y, and u z which is z tau minus z and this is the displacement vector which we will denote using u. So, clearly of course, u y and u z or 0 and this is not a surprise that material is. In fact, not getting displaced in y and z direction it is only getting displaced in x direction.

Now, the deformation exists if there is relative displacement 2 material particles which are next to each other. So, the one which we have drawn as this black and the other particle which is grey right are they getting relatively displaced with respect to each other yes right, because in sometime in future this particle would have gone little bit further down and the other particle would have. In fact, be further to the left, because the top particle is moving faster while the bottom particle is moving slower.

So, therefore, there is clearly relative displacement and relative displacement can be used to measure the and therefore, strain tensor and this is what we had qualitatively discussed also that strain measure is related to displacement gradient.

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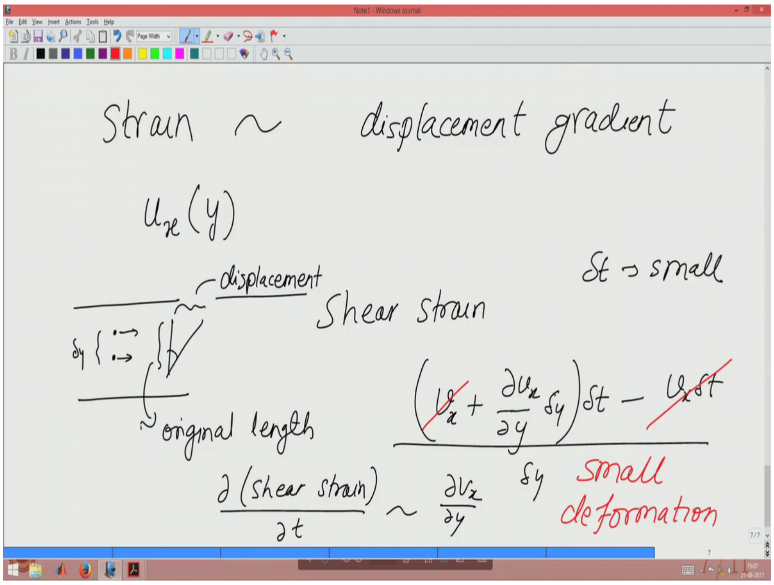

And what is displacement of function of u x is a function of x y z which one u x right displacement is in x direction, but is it a function of x or is it a function of y or is it a function of z. It is a function of y right displacement see displacement is function of y because x tau itself is a function of y right x tau is a function of as we saw it is a function of y.

So, therefore, u x is a function of y. So, again when we do displacement gradient we will again see that diagonal terms will be 0 and of diagonal term and only. In fact, x y terms will be non-zero. So, this is very similar to the velocity gradient terms only thing is velocity gradient is strain rate while displacement gradient will give us strain.

So, right now we have not defined is formally. So, that we need not do right now we will do this measurement of definition of strain later on because for most rheological purposes strain is useful for only solid like materials. So, if we have rubbers if we have materials which are more solid like then we use strain. Otherwise many other materials we will use strain rate and the differential form quite often we will see that we will have an integral form of constitutive relation.

So, there we may need to use the deformation or strain. So, since we have not defined it we can again try to see the relationship between the strain and strain rate by just intuitively looking at what is happening in this flow. So, we have the top plate and the bottom plate and if we take a look at any 2 points which are let us say delta y apart and we would like to know what is the shear strain right using our older knowledge of basically the amount change in length divided by original length.

So, in this case the original length will be in y direction and the displacement is in x direction, that is how the angle comes in right the shear is measured using this angle because this displacement is there and this is the original. So, this is the displacement and this is the original length measure of original. So, since we know that this particle is moving at some velocity and this other particle is moving at some other velocity we could write this as velocity at y plus delta y and velocity at this point.

So, it will be let us say v x plus del v y by del x into yes del v x by del y into delta y. So, this is a velocity and let us say some motion has happened for some amount of time, this minus v x into del t and divided by del y right this is the measure of shear strain. And if we take delta t out and take delta t tend to 0 then that will be rate of shear strain or strain rate.

So, that will be velocity gradient because v x delta t and v x delta t will cancel out. So, we will have basically v x and v x delta t cancel out and then we can say that del shear strain by del t is nothing, but del v x by del y.

So, for our present purposes we will do this there where partial derivative of time itself is actually given in us velocity gradient. So, the strain rate and strain are related to each other through a simple, but all this is possible when delta t is very small. All in other words this is all small deformation for an arbitrary amount of deformation what we will need to do is we will need to define rate quantities with respect to tau.

So, that is something we will do in future right now we do not need to worry about it, but del by del tau of a given quantity of quantity of interest this at tau is equal to t will be the rate of change of quantity.

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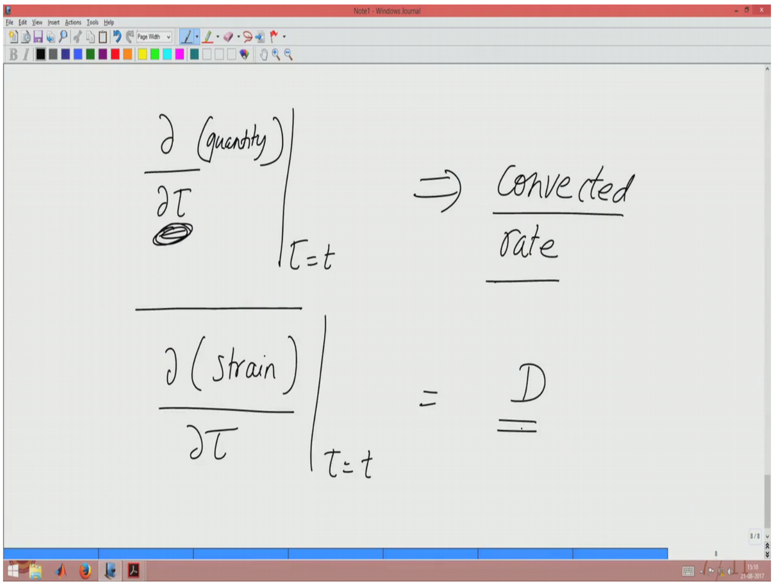

So, remember I had said earlier that strain at present time is always 0 right because we are using the current state as the basis, but this quantity will not be 0 because we are calculating the quantity in this running time and this is called a convected rate. We are moving along with the material and evaluating the derivative in the continuing time and if you are interested in the rate of deformation at present time then we just substitute tau is equal to t we will get the present time. And so what we will see in fact, is let us say del strain measure del tau at tau is equal to t will be the strain rate tensor.

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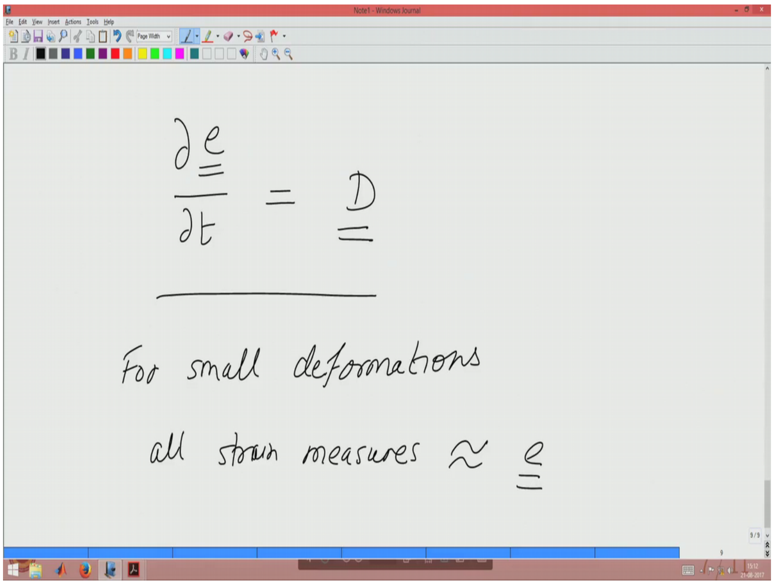

However, for small deformations is D it is partial derivative itself is fine, just the way in fact, that is what we showed here this that we showed is partial derivative of shear strain is just equal to the velocity gradient that is in general not true. We have not discussed this, but in one of the future classes we will derive first of all we have to define strain right and since we need it only for more solid like materials we can postpone.

So, lot of discussion in next set of lectures we will only worry about small deformations we will only look at material deformation which are small. So, in those cases we can use this relationship where strain and strain rate are. In fact, derivatives of each other and simple derivatives, but in general for a complete description we must take the convected rate of strain to get the strain rate. As a measure of deformation also we should use a strain measure which is not small e, but it is so there are several names like finger strain tensor, green strain tensor, so some of these we will define again there are multiple options unfortunately or fortunately.

There are multiple options in which way we can define strain, but all of these strains will reduce to e as for small deformations. So, this is again point which we can emphasize for small deformations I think we have said this before, but we will just remind ourselves that all strain measures will reduce to e. So, in the beginning of the course we will continue to use e.

So, this is what is summarised here that you can write this tau minus t as gamma y x another variable which is indicative of the strain and therefore, this is what we derive that x tau is x plus y gamma y x for if any time in the future gamma y x is positive any time in the future tau minus t is positive. And therefore, gamma y x is positive and therefore, x tau is more than x which means the fluidal particle has moved to material particle has moved to the right or in the positive y direction.

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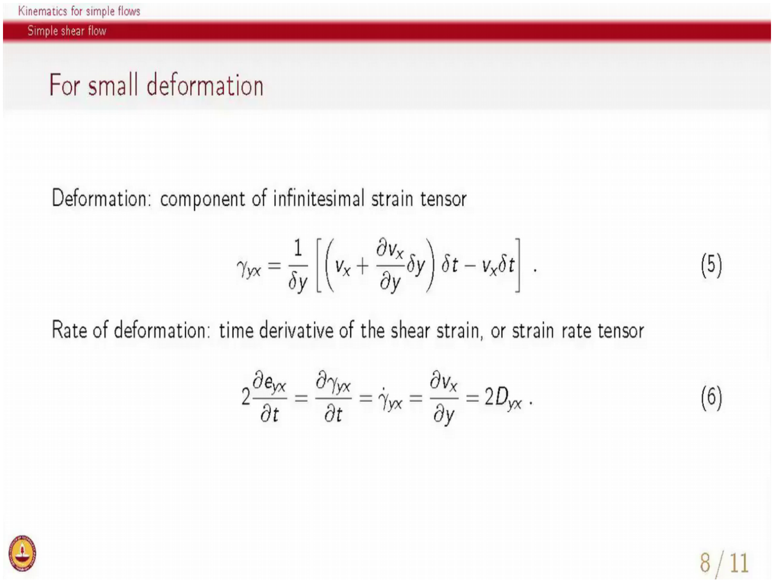

We also did this that we define the shear strain as small incremental amount of time and this is only valid for small deformation and of course, we will be using e e y x or gamma y x interchangeably, but all of these are for small deformation and of course, it can be written in terms of the strain rate tensor also, because these are derivatives with respect to time.

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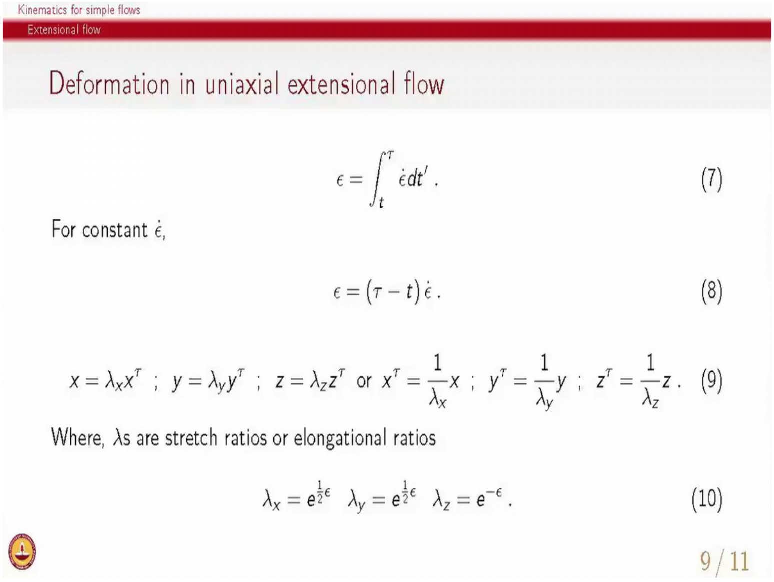

So, the same thing can be done for uniaxial extensional flow also where we can define again the position at any time with respect to the present time. So, x tau, y tau, and z tau and how are they described as a function of x y and z, but before we do that first let us try to describe what is the flow. So, how do we what is the velocity field just to remind ourselves again this is floor right.

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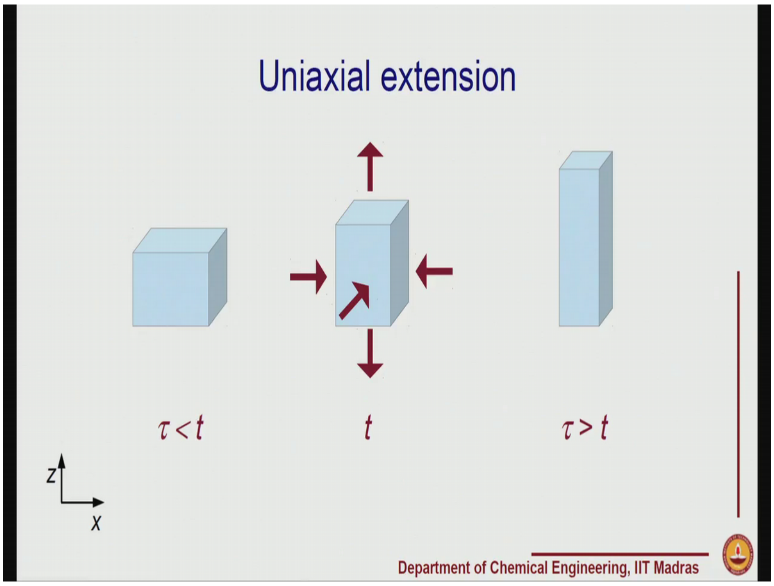

So, it is getting extended in z direction and getting contracted in x and y direction what components of velocity are non 0 all 3 right and what are they functions of. So, we can see that the shape completely remains the same which means there is v z, but it is not a function of x and y. Similarly v x is not a function of y and z and so on.

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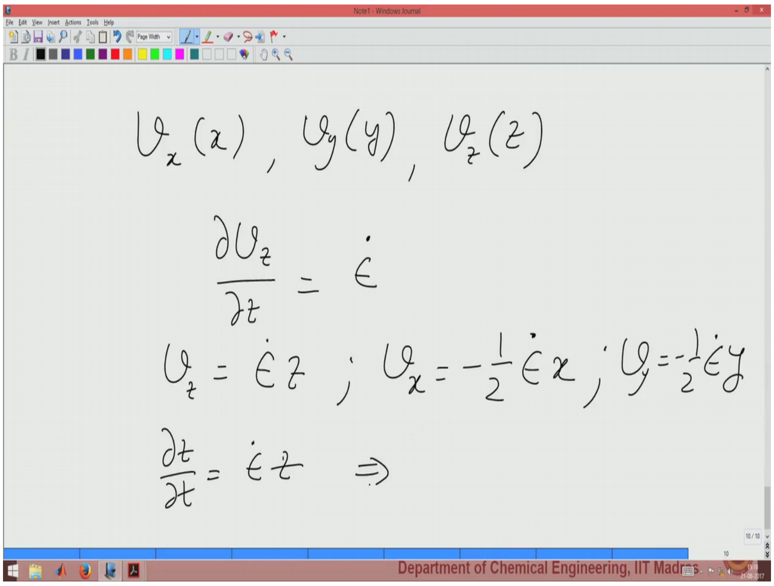

So, in general what we can write is v x is a function of x v y is a function of y and v z is a function of z and what we also said is we can have this pulled at a constant rate. So, this rate is constant the rate of pulling in x z direction. So, therefore, we can say del v z by del z is constant we will denote that using epsilon dot. So, velocity v z is epsilon dot times z what about v x and v y they will be negative and v x will be minus half epsilon dot x and v y will be minus half epsilon dot y.

So, which implies that at the centre of the cuboid velocities are all 0 x equal to 0 v x is not there y is equal to 0 velocity is not there and z also velocity is not 0. Now can you look back and try to justify what I had said earlier that position will be changing exponentially in extensional flow. The stretch ratio we will define which is with respect to position right epsilon dot is velocity gradient and strain rate down the stretch ratio is defined in terms of relative displacement or in terms of strain. In fact, that we will do, but that will be exponential because v z can be written as del z by del t epsilon dot z.

So, how does z changed as a function of time log of z will be epsilon dot times time. So, therefore, this so that is what is described here where lambda x lambda y and lambda z are the stretch ratios, which are relating to relative position if lambda x lambda y lambda z are one that implies that there is no deformation this is a stretch ratios. We have multiple symbols being used we will have a set of problems which you can work with. So, that you can become familiar with some of this notation because here onwards when we start discussing rheological properties, we will start specifying epsilon dot is constant epsilon dot is varying as a function of sine time and so on or gamma y x is constant or gamma dot y x is constant because our interest will be in studying the material behaviour under those conditions.

So, we have to remember in the back of our minds that how these quantities are related to each other and conceptually when we say simple shear what is happening to the material so, that we should be able to understand the material behaviour.

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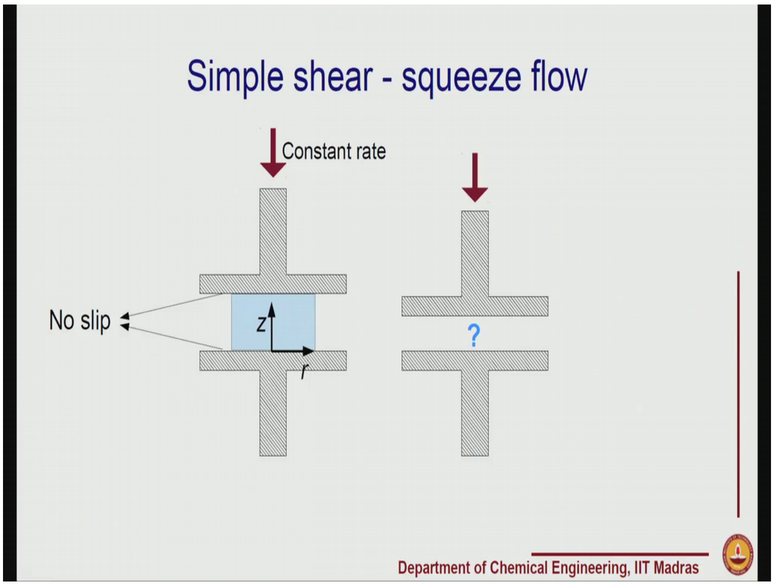

Just as a last thing we can stop by looking at this we looked at lubricated squeeze flow what if now again the same rotational rheometer we use and I take the fluid between the 2 plate and it is like earlier and again like earlier I squeeze it, but now this is not lubricated squeeze flow. So, which means there will be no slip at the top and bottom plate and top plate is only moving in z direction bottom plate is completely stationery.

So, now what will be the velocity profile in this case one thing is clear is fluid will moved out in the r direction it is getting squeezed out, but what is being forced is the r velocity on the top and bottom plate has to be 0 and therefore, you would expect that the velocity is to be possibly something like this right well this is r direction and this is z direction

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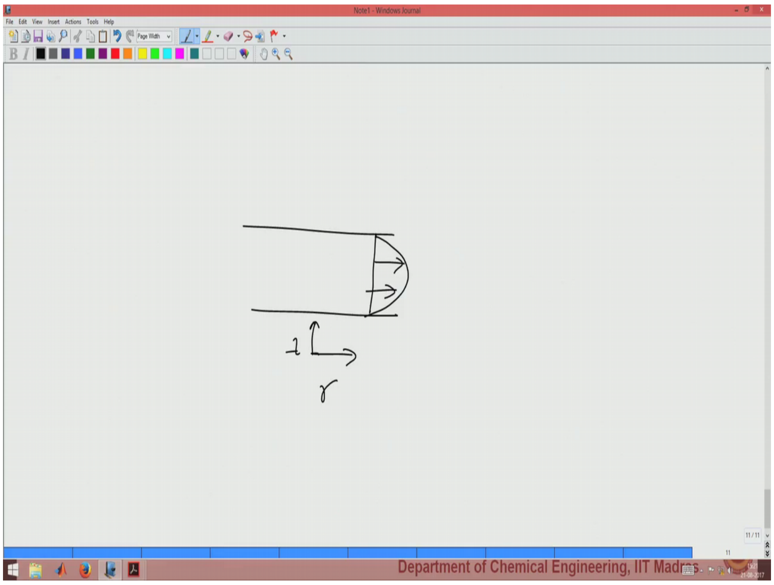

Because you are squeezing fluid and now this is an example of a shear flow see. So, that is why based on the rheometric experiment that we are doing we first will always need to identify whether this is a shear flow or shear free flow or a combination. And more importantly we will at times have to understand the fluid mechanics of the flow, under what condition does it make sense for me to write a velocity profile like this right now we have not worried about fluid mechanics at all.

So, at least for the first month of the course we will not we will somehow say that somebody else has done the analysis and made sure that the flow profile is like this, but there will always be some squeeze rate, some conditions, some separation, which will ensure that fluid mechanically the assumptions are correct. For our first section of the course we will assume that somehow fluid mechanics information is managed in such a way that we achieve a flow which is well described as a practitioner of rheology we will have to worry about when I am doing rheology are these conditions being met or not ok.

So, therefore, understanding of fluid mechanics of rheometry is also important, but that we will look at some time later.

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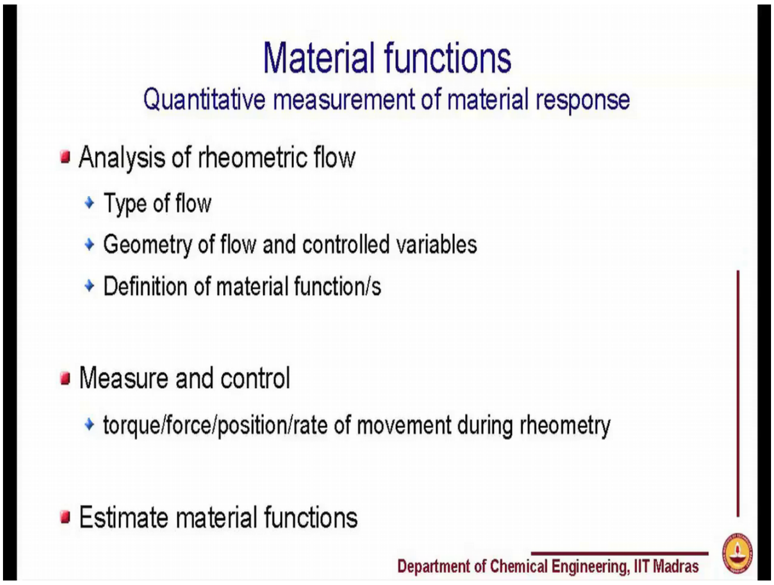

So, as we have emphasized that fluid mechanics of rheometry is very important and assumptions related to the rheometry will have to be satisfied for us to get good rheological data, which in the next few sets of lectures we will assume that such conditions are met. In general looking at this slide here we will define what are called material functions and these material functions enable quantitative measurement of material response, because we are doing analysis of rheometric flow correctly because we have assumed the type of flow because the geometry of a flow has been designed appropriately and the variables which have to be controlled are being controlled perfectly. So, based on all these assumptions and achievement of rheometric flows we define and characterize the material function.

So, in general to do this the instrument will have to be physically manipulated in terms of controlling some variables and measuring some variables and of course, these variables could be torque force position or rate of movement.

So, for example, keeping position fixed would imply that way we will impose a constant strain if rate of movement is constant then we may have a constant strain rate. So, these are all the variables which we could measure or control depending on which type of flow and therefore, this combination of analysis of rheometric flow and measurement and control allows us for estimating and quantifying the material functions we will see that each of these material functions are called. So, because unlike material constants which was so basically in case of Newtonian fluid viscosity is the material constant. In case of conducting solid the conductivity is a property of the material and therefore, it is a material constant, but we will see in the case of course, on rheology that the material behaviour cannot be described simply based on a constant, but a material function.

So, we will see that for example, viscosity is a function of strain rate itself. So, therefore, we choose to call all these characterization in in terms of material function as supposed to material constants.

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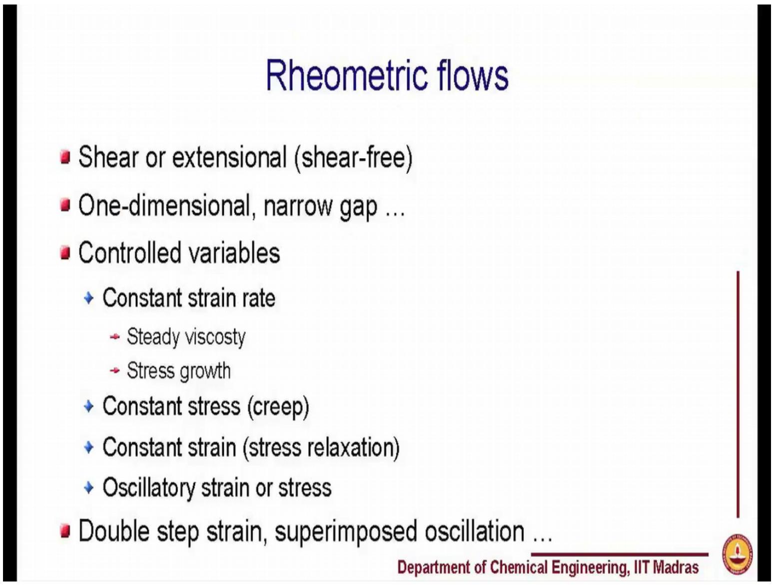

And so in the next slide we have summarised again that in rheometric flows our objective will be to achieve certain control flow, which could be shear flow or which could be a shear free flow. Of course, all the assumptions regarding fluid mechanics in terms of one dimensional flow narrow gap to ensure that Reynolds numbers are low or the flow is in a particular manner basically will ensure that we are able to do the quantification of material functions and in general when we do the rheological analysis we will see that there are 4 or 5 very common ways of looking at and measuring material functions. For example, if we keep constant strain rate then it is called steady shear and in that case we define something called a steady viscosity, but we could also keep constant strain rate, but look at how the stress changes as a function of time .

So, in this case this is not a steady state measurement we look at stress growth. Eventually of course, the stress will become a constant it may in certain cases not become a constant, but generally we will expect it to become a constant. So, therefore, stress growth is another type of measurement we can keeps stress to the constant which is called a creep measurement we can keep strain constant which is called stress relaxation and of course, we will also see that an very important part of rheometric analysis is to do oscillatory measurements, in which case either strain or stress or even strain rate are varied in a sinusoidal manner.

So, there is an increase decrease based on sin theta cos theta or another other sets of sinusoidal functions that we are familiar with in addition to this of course, there are more complicated deformations such as double step strain or superimposed oscillation; however, in this course we will not really look at these, but learn all the basics with the very standard set of rheometric flows which are achieved to define the material functions.

So, in each case for example, with constant strain rate and constant stress we will define steady viscosity as material function with constant strain rate and growing stress or changing value of stress we will define a stress growth viscosity. In case of constant stress creep we will define a creep compliance, in case of constant strain we will define a stress relaxation modulus, in case of oscillatory strain or stress we will define a complex modulus or a complex viscosity which implies that we will have an in phase out of phase components of these modulii and viscosities and we therefore, choose to call this a dynamic modulus or dynamic viscosity and we will have the real and imaginary or storage and dissipative parts of these material functions.

So, these are several ways of doing rheology and several material functions that could be defined based on this. In the next set of lectures we will examine each of them carefully alongside we will also look at some important models or some very simple models which can be used to understand the overall response quite easily. So, with that we have reviewed the basics of material be deformation and fluid flow which is required for us to look at rheometry of material systems.