Rheology of Complex Materials
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Lecture – 11
Kinematics for simple flows

So, given that we have already looked at stress tensor and the strain rate tensor, we are going to deal with materials in this course which will have both solid like and fluid like characteristics. And we saw that for fluid like its sufficient to look at strain rate and stress, but for solid like material it is important to look at the deformation itself. So, therefore, we should also define how do we characterize strain in the material. So, what we will do is look at qualitatively features of strain measures, and then we will also try to understand strain and strain rate for two simple flows.

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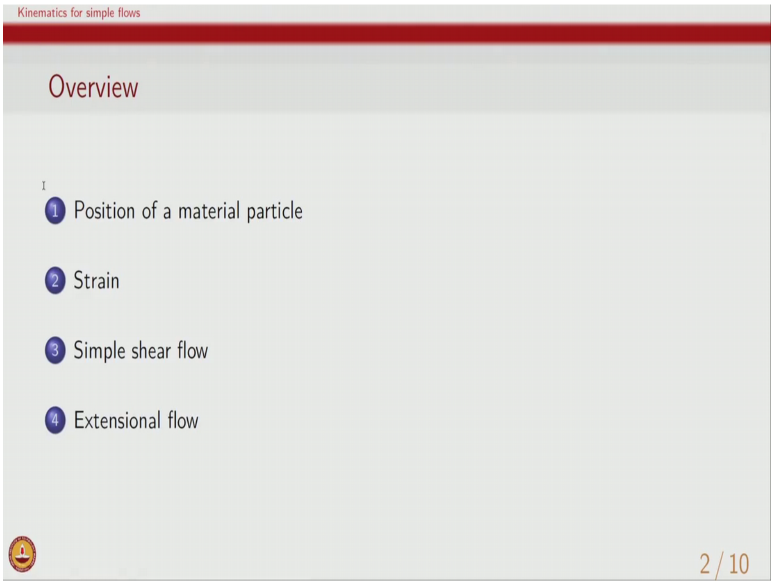

So, our plan in this lecture is to look at how do we specify the position of material particles, then we look at what is meant by strain qualitatively. Today we will not define the strain and complete in their definition, but then we will look at given that we know how to specify your material particle can we then specify the two simple types of fluids flow.

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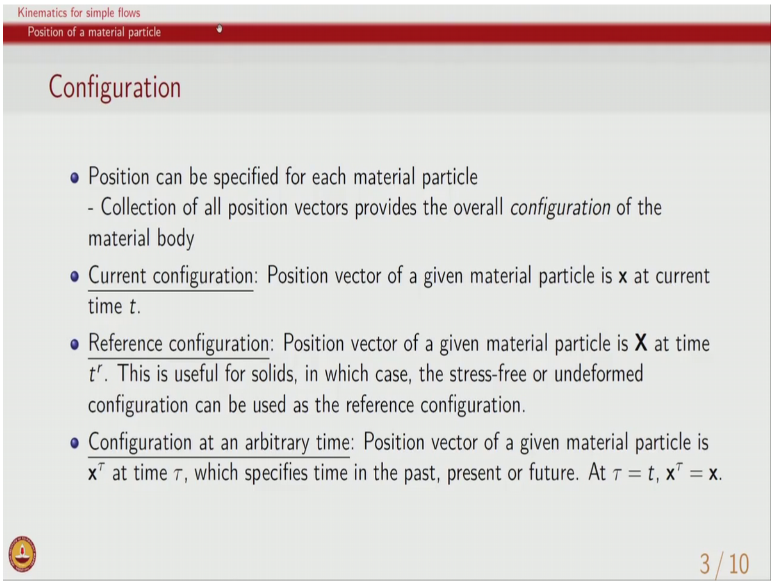

One is simple shear flow the other one is extensional flow; as these two flows will be important for our rheological analysis. So, if you look at the material we had said that its continuum and it is a collection of material points. So, therefore, each and every material point, we can specify its position. And the collection of all such positions is called configuration.

So, therefore, configuration of a material implies that specifying each and every position of the material particle that the material consists of. And there are of course, what we usually do is we think of specifying the material particle positions in three different ways. We always define a current time so that we will call the current configuration so that we say that at the present time we will specify where is each and every material particle. And then we will also have possibly sometimes a reference configuration.

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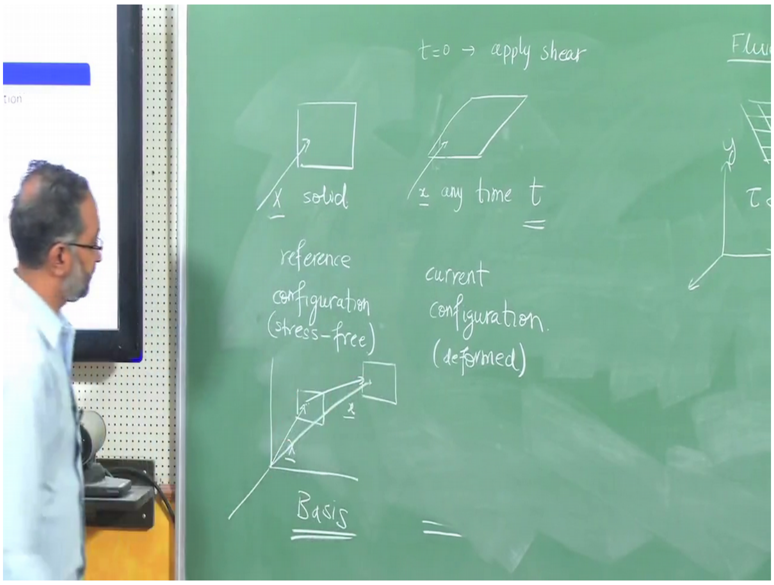

So, for example, in case of solid let us say I take a block of solid. And I apply a shear deformation on it. And then at time t is equal to 0, I apply shear. So, what will happen is of course, the material will get deformed right simple shear; and then after some time I can say what is the present configuration. So, therefore, that will be called the current configuration, so at any time t, so any time t.

So, in this case, it is it also makes sense for us to say that let me adopt when the material was un deformed, and it was stress free whatever were the positions of material particles, I will call that reference configuration. So, then will become the current configuration, and this becomes the reference configuration. I need these two to specify whether there is deformation in the material.

If let us say this solid body is only translating, then in that case what I can say is let us say if this is a coordinate system and then I have a solid body and I can specify the material particle each and every material particle. After some time the mater solid block has translated. So, of course, each and every material particle now position has changed, the overall configuration of the material has changed, but I know that there is no deformation in the material. So, I need these two configuration then I need to compare the configuration for each neighbouring particle.

I need to ask the question that if I take two neighbouring power point see here, and they had some positions, the same two points in this new configuration what are their relative positions. And if the relative positions do not change, I will say oh it looks like two configurations. So, in case of solids it is relatively easy to say that I will choose stress free configuration un deformed configuration. So, this is in case of solid like materials.

Now, when we look at completely fluid materials, current configuration is again easy to justify saying that so whatever are the material particle positions at the current time. What about can we define something called stress free configuration for fluids?

Student: Static fluid (Refer Time: 05:41).

Yes static fluid, but unfortunately there are multiple ways of being static fluid. In the sense I can put the fluid static in a big trough then also its stress free and I can put it in a long beak jar and again it will be stress free. So, there is no unique stress free configuration for fluid. So, then how do we what do we talk about? So, how do we specify.

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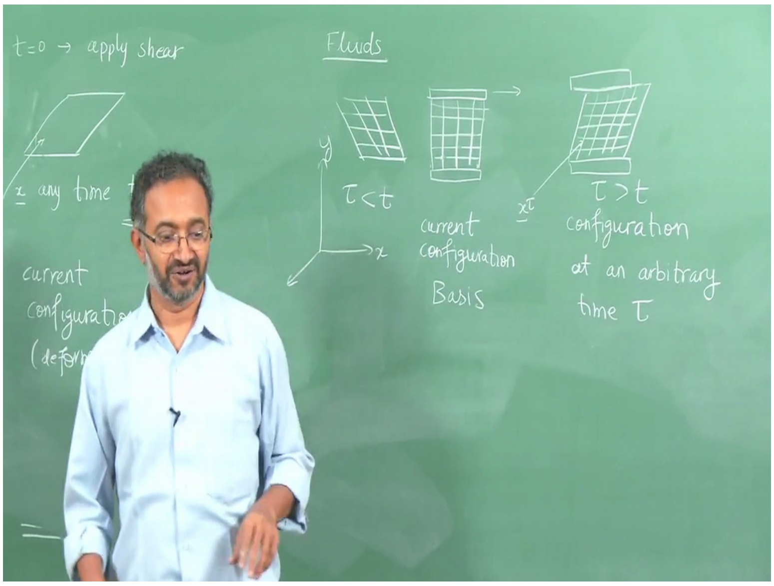

So, what we have in case of fluids, so let what we do is we say the current configuration is completely known, and that is the one which then becomes the basis, because we need two configurations to define deformation for defining strain measure. In fluid like case what we will say for example, let us say this is again a simple shear flow, so this fluid is again moving with our simple shear velocity profile. So, we say here that the current time and we will say that any other time, whether in the past or in future, we will compare the configuration with respect to the current configuration, because there is no point talking in fluid about some stress free or reference configuration only the current configuration can act as a basis.

So, what will happen let us say in the past, and what will happen in the future? So, let us just look at what I can do just as a example, I will say that this is a plate, and the plate is moving, and therefore, because the plate is moving the top layer of fluid is moving and then slower, slower and slow on till this plate when velocity is 0. Now, I define no present time I take a snapshot, and I put my coordinate system, and label each and every material particle. So, this becomes my 1, 1 point this becomes my 2, 2 point and so on. So, there are so many material particles in each of them now I have labelled.

Now, what happens to these material points in future, how will they look like? Something like this, something like this, it would look something like this, so that the fluid particles keep on moving. And then if I now compare these two then I can say that yes deformation has taken place in the material. If the whole fluid was moving at a rigid body then what I will see is again between two points, there would not be any relative displacement, so therefore, deformation was 0. So, we start saying that there is current configuration, and then we say configuration at any time tau, configuration at an arbitrary time tau. And the current time we will specify as t, while tau is a time which is with respect to the present time either in the past or future. When tau is equal to t, what do we mean, current time.

So, and similarly in the past also, you can see right. So, in the past how will the grid look like, it will be basically some grid like this. Here is there are three configurations current configuration which is present time t; reference configuration which is useful for solids where it is clearly defined in terms of an undeformed configuration. And then configuration at an arbitrary time which is the arbitrary time is tau. When tau is more than t, we know its future; when tau is less than t we know it was past. So, we also call each of these, so in this case, these reference points these are given name capital X. Here these are given small x. And here these are given; this is the position vector of a given material point.

Student: This is valid for only (Refer Time: 10:48).

No, this right now what we are discussing is completely general; it is not in fact restricted to also shear or any such thing right. I am drawing it for shear and then trying to motivate, but generally our reference configuration where each and every material particles position vector is known, then there may be a current configuration in which each and every material particle position vector is known, and similarly future at arbitrary time tau. So, all of these it is a completely general no assumption regarding Reynolds number no assumption regarding type of deformation nothing, it is a very general statement. In fact, whatever quantities we define whether its strain or strain rate should be for very general situations.

You can define strains right. So, two ways that we have talked about the fact if I solve problem one way I am answer. And that will happen, but only thing is sometimes it becomes more convenient to use this and some other time it becomes convenient to use this. And clearly what we have said is in case of solids, it is more convenient to use this; in case of fluids, it is more convenient to use this, but given that we are dealing with materials which are both solid like in fluid like we can use any one.

And therefore, if you look at literal literature in Rheology, you will see unfortunately for us to understand immediately is difficult because some people will use this approach some people will use this approach, so that depends on their historical learning and their historical interest. Therefore, they will either if they have come from the solid site to look at complex materials they might tend to use this approach if these approaches are used.

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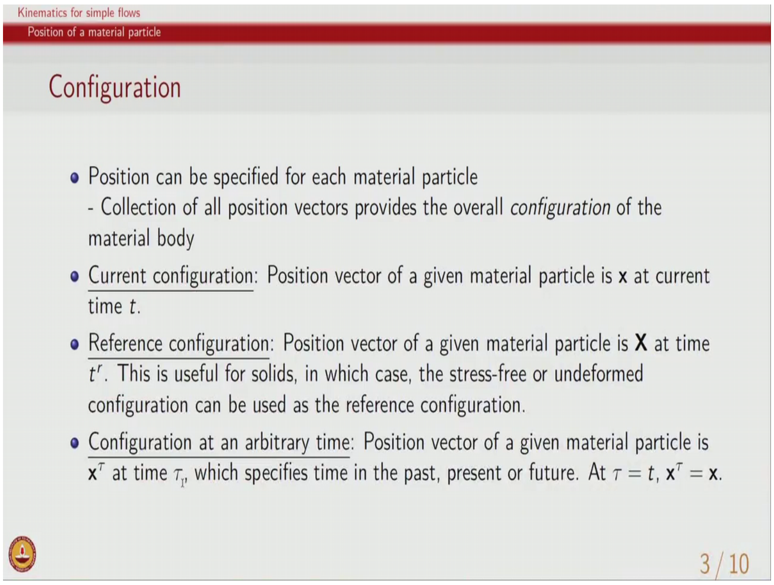

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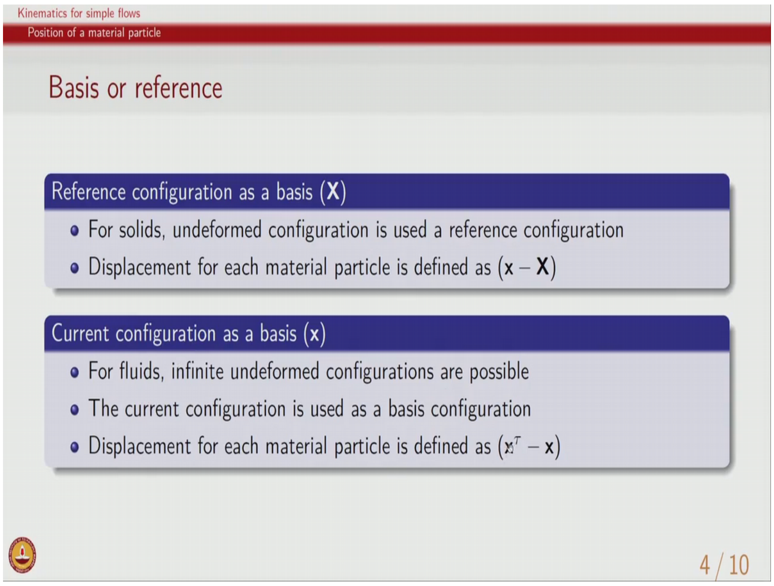

And given that these are fairly complicated, therefore we are not defining them now because for discussing initial aspects of rheology we do not need their detailed definition, but you still need to know that strain measure is an important quantity and the fact that they can be defined in multiple ways. So, as I said basis or reference, it can be X - capital X which is the reference configuration. And for solids it is a undeformed configuration is used as a reference configuration. And therefore, displacement for each material particle is what was the position vector article has moved. So, in this case, this is capital x, the same material particle here is donated by small x. And what is this vector mint is useful for us to find stream because it is not just displacement, but relative displacement of two neighbouring material particles, we will determine whether there is deformation or not.

So, the other option is to choose current configuration and which is used usually for fluids because there are infinite undeformed configurations possible. And the current configuration is used as a basis; and displacement for each material particle in this case is x tau minus x. So, what is the displacement at present time always 0. So, in fact, in fluid what we will see is since the basis is current time, the strain will be always 0 in the current time because that is the basis, does not mean fluid is not deforming so that is something we have to get used to the idea.

In this case, whenever solid is undeformed only strain will be 0, but in case of fluids fluid is continuously deforming and there is deformation rate and strain in the material, but you ask the question what is the strain at present time, answer will always be 0 because present time with respect to present time will always be 0. So, we will the way we set up the governing equations and the strain tensors, we will see that therefore, we will have to measure any arbitrary time with respect to present time and that will tell us. The other important thing that we will notice in case of fluid is even though current strain is 0, current strain rate will not be 0. Does that make sense?

Student: (Refer Time: 15:14).

Strain is 0, but strain rate is not 0. It is material is flowing right from here to here to here, there is continued strain rate, but at present time strain is 0. So, in fact, derivative of a constant quantity is.

Student: 0.

It is not 0.

Student: (Refer Time: 15:34)

So, strain itself is 0, but its derivative is so that is why it is not simple derivative, so that is a point another point we have made that we will tend to use convicted rate. So, we will see that convicted rate of a constant quantity will not be 0. So, this is something which we, so that is why going between strain and strain rate, we should always remember that these two quantities are not related as simple derivatives. We will see later in rate can be related as a partial derivative; otherwise they are more complicated the relation in terms of convicted derivative. Just the way we said that there is partial derivative, material or substantial derivative we also have convicted derivatives that we talked about earlier the strain rates in these kinds of deformations.

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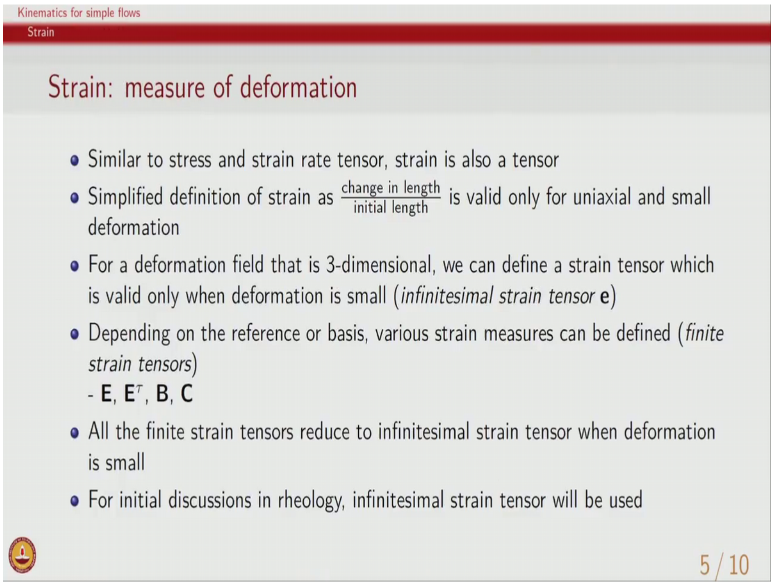

So, therefore, just to summarize qualitatively, as I said we will not define these strain tensors because for our some of our initial rheological discussion we will look at mostly fluid like materials and we may not need the strain, but some of these in ideas are important, therefore we should keep them in mind. So, the first thing is that given that stress and strain rate tensor have nine components, strain will also have nine components because displacement is in three directions and the relative displacement gradient in displacement is also possible in three direction. So, therefore, you have nine components.

And the simplified definition of strain has change in length to initial length this is valid only for very simplistic situations like the rod extension or a simple shear flow, but in general we have a three-dimensional field, where there will be displacement possible in all three direct directions. So, we will define actually a strain tensor which we will call infinitesimal strain tensor, which is only valid for very small deformation. In fact, those of you who have done solid mechanics course have only dealt with this tensor strain tensor and that strain tensor is not (Refer Time: 17:41) or when we talk of plasticity of steel. So, anywhere there are large deformations then we should we cannot use these strain tensor because it is valid though it is a three-dimensional tensor, it is only valid for very small deformation.

And then depending on the reference or basis as I said we can use either current configuration or reference configuration, there are several tensors which are defined which we are not going to define now in a more advanced class later in the course we will define these. Any of you have done solid mechanics; I mean finite elasticity finite deformations course large deformations? So, you finger strain tensor or

Student: Continuum mechanics.

Continuum mechanics. So, you have been exposed to the finite strain tensor. So, any time when we look at materials with arbitrary large deformations, then we need to define these as strain measures. And all the finite strain tensor of course will reduce to infinitesimal strain tensor for small deformations. So, this is a usual thing which we expect in any branch of science, for example, any complicated equation of state should reduce to ideal gas because we know under some condition molecular interactions can be ignored. So, a gas should behave like ideal gas, but in many other situations it may have lots of interactions. So, therefore, we need a more complicated equation of state, but the expectation is no matter how complicated equation of state under some condition it should reduce to ideal gas.

So, similarly here also no matter what definition since they are all consistent they will reduce to infinitesimal string tensors for small deformation so that is why in our solid mechanics courses, there is no need to spread define all these different bases and multiple strain tensor, because all of them will reduce to only infinitesimal strain tensors. So, there is only one strain tensor required; only when you have large deformations then all these multiple options become important. And as I said for our initial discussions, we will only focus on the infinitesimal strain tensor.

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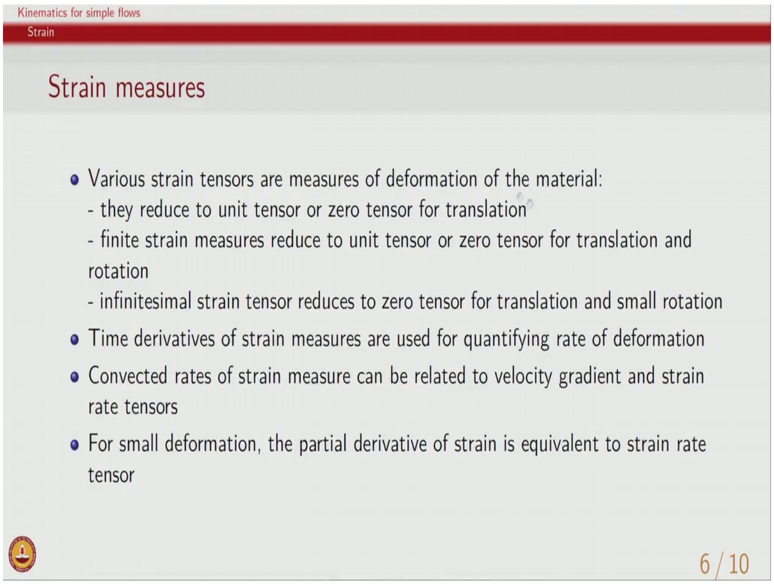

And just to again qualitatively continue the information so strain tensors are of course the measure of deformation of the material they reduce to either unit tensor or zero tensor for translation. Depending on what tensor is defined not all of them reduced to 0, our expectation intuitively will be and also should go to 0, but some of the strain measures will only go to unity you subtract unity from that you will get 0. So, we will see that some of the strains are defined as some quantity minus unit unity tensor, so that we get 0s. So, that is why some of them will reduce to unit tensor some of them will reduce to zero tensor or for zero tensor for translation as well as rotation, so which is again expected that once there is rigid body translation and rigid body rotation we should not have deformation measures.

And infinitesimal strain tensor actually all reduces to zero tensor for translation and very small rotation. So, we can show in fact, that if let us say I take a block like this and then I rotate it. And then I evaluate the infinitesimal strain tensor which we will define we will show that we will be able to show that if this theta is very small then it is fine, but otherwise we will get nonzero terms in this infinitesimal strain tensor. So, clearly it should not be used for large theta that is why it is always used for small deformations. It gives us correct results for only small deformations.

And as I mentioned time derivatives of strain measures are used for quantifying rates of now given that for fluid we only need strain rate for solid we only need strain, we have now materials where both of these are involved. So, therefore, we will need strain as well as strain rates. So, we will need to not only have a measure of strain we will also need its derivative and so we will have to find these derivatives. And we will see that convected rates of strain measure can be related to velocity gradient and strain rate tensor so that is where I mentioned that in you using current configuration as the basis strain is constant at present time, but its derivative will not be constant because it is a convected rate.

And for small deformations only the partial derivative of strain is equivalent to strain rate tensor. So, fortunately whenever we have small deformations, we can go back and forth between strain and strain-rate easily without taking care of strain, convected and other more complicated time rates. So, we can just take partial derivative or just quickly integrate and get from strain to strain-rate or strain-rate to strain.

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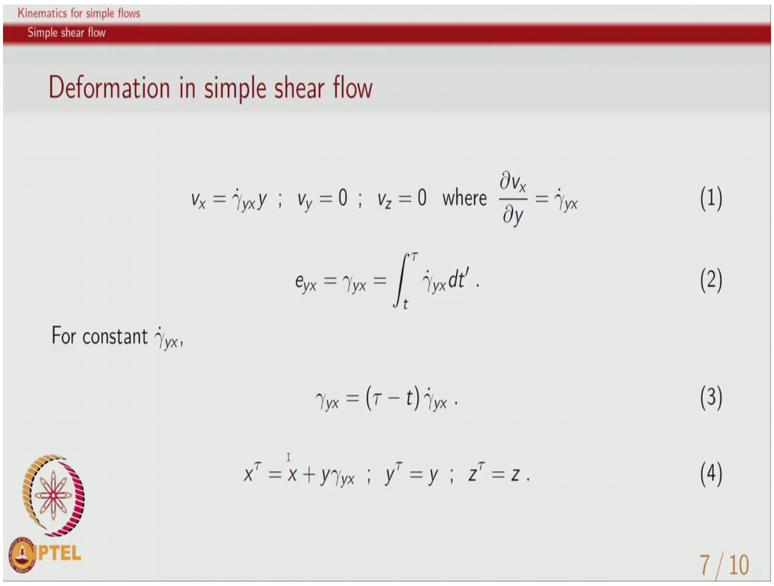

Now, we can look at two simple flows, and try to look at these quantities for those simple flows.