Applied Time-Series Analysis
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Lecture – 75
Lecture 33C - Spectral Representations of Random Processes 5

This is the last in sense of the spectral discussion; of course, there is one last, last which is spectral factorization, which is kind of reaching the peak, beyond that there is a valley, anyway so, we have talked about random aperiodic processes until now, but before earlier in many other lectures, we have said that there are random processes that are also periodic.

Now, if it is hard for you to imagine a random process that is periodic you take number of examples that you see in real life bus schedules, abroad not here, but or you know any other systematic schedule that can be followed, India we have a number of reasons why we cannot really have some buses do indeed come on a periodic schedule, but if you take a bus arrival schedule and if someone says that on a average bus comes every 15 minutes that is kind of a periodic process.

Why? Why is it not aperiodic? In the deterministic sense because it does not come exactly every 15 minutes, there may be some deviation from the 15 minute schedule to the left or to the right, but on the average it stays on the 15 minute schedule that is not how we define a random periodic process, but that should tell you that there are processes that are periodic and also have some randomness in that.

Now, there are many other process examples also, but the question is what kind of mathematical models can model the behavior of such random processes there are a number of models and the classic model is a sign embedded in noise. So, I say that yes there is an underlying periodicity, but there is noise riding on top of it which does not make a periodic in the deterministic sense that is one way of looking at random periodic processes.

Now, is there any other model for a random periodic process and the answer is yes, there is yet another model for a random periodic process, but before we do that let us first define what are random periodic process and we will use the term harmonic to denote random periodic processes. Once we know the definition of a harmonic process then we can turn to models for harmonic processes again the, what is the idea here? The idea here is to detect periodicities in random processes right and a number of meteorological phenomena are like that your El Nino effect or any other climatic phenomena have number of periodicities embedded, you take sunrise for example, right there is a periodicity you take temperature variations in a day it will have some periodicity and so on.

How do we detect the periodicities and we need for that purpose we need mathematical models and for that purpose first we need to define what a harmonic process is so you see, theory is very deep is very demanding, but it helps you because you know the framework in which we are operating. So, I will go past the motivation and straightaway go to the definition of a harmonic process, it is good to compare this definition with that for a deterministic process for a deterministic process periodicity is very straight forward if I have a deterministic signal then it is periodic.

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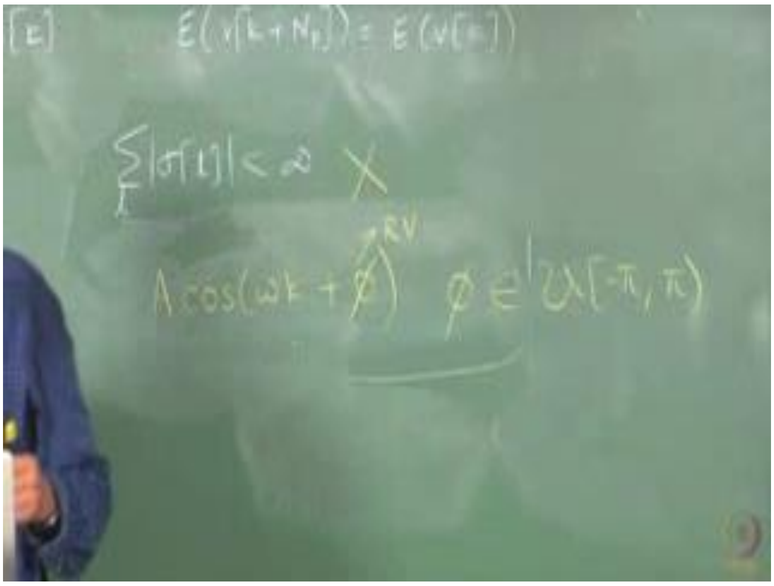

The period t p if there exists in fact, we will use n p if there exists an integer n p such that x k x of k plus n p is x of k right and n p then is said to be the fundamental period.

So, the definition attaches itself to the signal itself whereas, the definition that you see for a harmonic process here is does not attach itself to the signal, why is that? In the case of random processes, why are we not defining in terms of the signal?

Student: (Refer Time: 04:46).

Yeah because then it will apply only to that realization perhaps for another realization may be different the answer is always for a random process we are looking at a collection of signals it is an ensemble that we are looking at. So, it makes sense to define periodicity as an average property like we have always done before even for the spectral densities.

There are a number of definitions that are equivalent for harmonic process one of the standard definitions is that a signal v k is said to be periodic, if the expectation of the mean square error is 0. Sometimes the temptation is can I replace this with this for a random signal can I say it is periodic if this is satisfied what do you think? First of all it has to be stationary that is required for a harmonic process we say it is the minimum requirement it should be stationary, generally the temptation is to define a random periodic signal in this world because we say you know for a deterministic signal it is sufficient to look at one realization because that is the only possibility.

Now, I want to look at for a random process I would look at a collection of realizations and use the expectation property right or the average property can we use this definition for defining a periodic process yes or no? What do you think?

Student: (Refer Time: 06:19).

Yes sure, think about it there is a trap there this is satisfied by any stationary process. So, by this definition every stationary process is periodic am I right? Why should this be satisfied for by any stationary process mean is invariant with time, correct. So, any stationary process will satisfy this. So, this cannot qualify to be a definition of periodicity even if you were to argue that I can come up with this definition.

Of course you know we can do deeper into why this definition holds.

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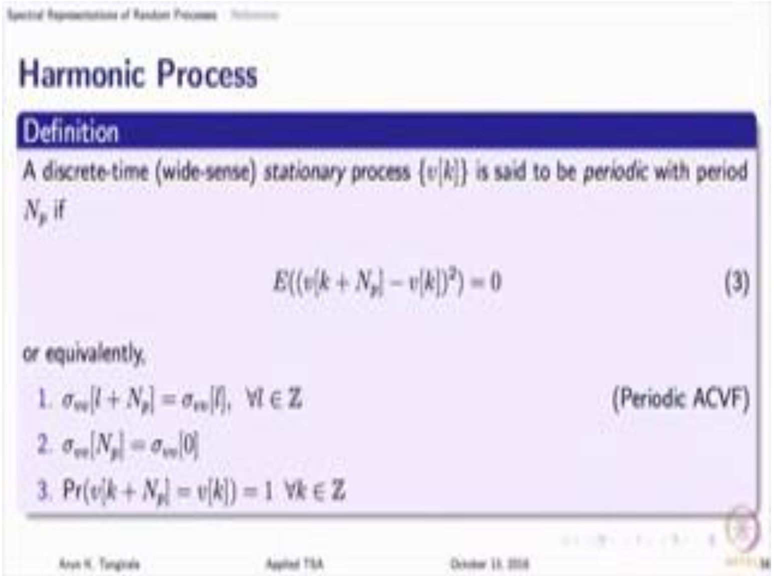

But at least I have shown you that a natural extension of this does not result in aperiodic in periodicity of random process, a natural extension in fact, I would say more than the definition that I have given you in equation three it is the corollary one that is the equivalent one not corollary the equivalent condition that I have given you which says that the auto covariance is periodic is a lot more appealing. You say when the correlation structure has a periodicity in it then it I can think of the random process to be periodic right and of course, there are two other definitions, but now let me ask you why do we have to consider this periodic process separately from aperiodic process in terms of spectral analysis, what is so special about this?

Student: (Refer Time: 07:59).

Yeah, it is, it does not satisfy that is why when I erased everything I left this alone; this is not satisfied by random by a harmonic process right because the auto covariance function does not decay it is periodic same thing that we discussed for periodic signals we said a periodic signal is not if you take a periodic sequence, it is periodic it stays forever it does not have a Fourier transform. In fact, it does not make sense to think of a Fourier transform because when I have a periodic sequence.

Now, we are only talking of sequences I can think of it to be synthesized from sins and cosines of the fundamental period and harmonics only why should all frequencies produce, so that is why here for a harmonic process we do not talk of spectral densities just like we did not talk of spectral density for a disc deterministic periodic sequence correct. So, once you reapply and recall the theory that we have learnt for a deterministic periodic sequence you can apply that to the auto covariance function. So, now, the auto covariance function would have a Fourier series expansion.

Now, one of the models in fact, we will continue this discuss discussion tomorrow I will just take one more minute one of the standard models for harmonic process we have looked at the definition now and it suffices to remember that a harmonic process is one that is stationary and whose auto covariance function is periodic.

Now, what kind of mathematical models can explain such phenomena? One of the standard models that is used is a sum of sinusoids with coefficients being random, but not any randomness you cannot have arbitrary because one the requirements is it should be stationary if you take the expectation of v k what does it amount to if you take the expectation? What happens? Does it come out to be 0? You have to be careful.

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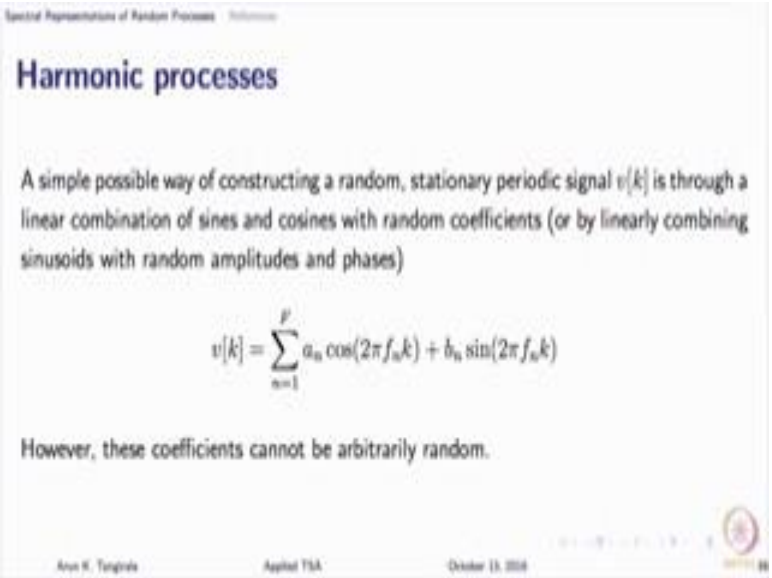

Remember cosines and sins have no randomness in them only the coefficients may have some randomness.

You want it to be stationary and one of the ways to assure stationarity is to have a and b this coefficients to be random variables with 0 mean. Any other value will produce a time varying mean so that is the first requirement, the second requirement is that a and b should be independent random variables we are not proving that. In fact, let me just tell you that another way of writing the standard way that you would see is this, this is another model for a harmonic process where a is random, a random variable and phi is a random variable or sometimes phi alone is random variable and a is not random. Phi in fact, you can show that if phi alone is random and what kind of random variable should? It be it should be of uniformly distributed random variable phi should have uniform distribution in minus pi to pi.

If this is the case then you have again when you expand what do you get when you expand this? You get this, right of course, I have written this for one frequency maybe omega naught, but if you have many such frequencies contributing remember we cannot have a continuum, it is a periodic process. So, when you have many such discrete fundamental harmonics you get the same model that you have here.

It is much easier to actually consider this model for a reason, I will tell you later, then this model although both are equivalent. You think about what is the difference between this model and the other theoretically there is no difference you can show they are inter convertible, but one prefers the second model that is that you see on the screen not on the one that on the board.

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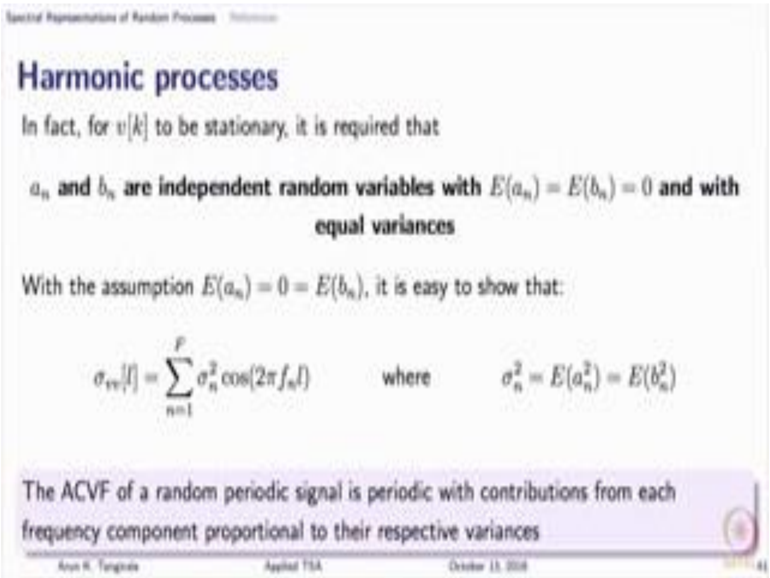

And you impose that a and b have 0 average and are independent random variables and then you can show that the auto covariance function as this kind of an expression. That is fairly straight forward, you start with the given time series model and show that. What you should observe here is that the auto covariance functions as been given a Fourier series expansion correct.

Why do not we have e to the minus j? We will answer that in adjourn because auto covariance function is symmetric, I can only explain expand a symmetric function in terms of even functions, not odd functions, these are common sense we are not learning every anything new, but it is just a interpretations and some subtle points. So, with this, we will close today's class.