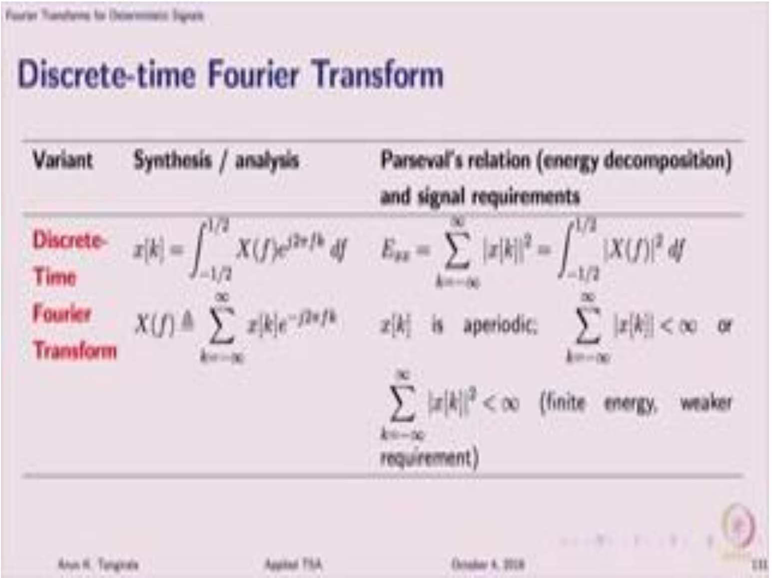
Applied Time-Series Analysis
Prof. Arun K. Tangirala
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 66
Lecture 29B - Fourier Transforms for Deterministic Signals 8

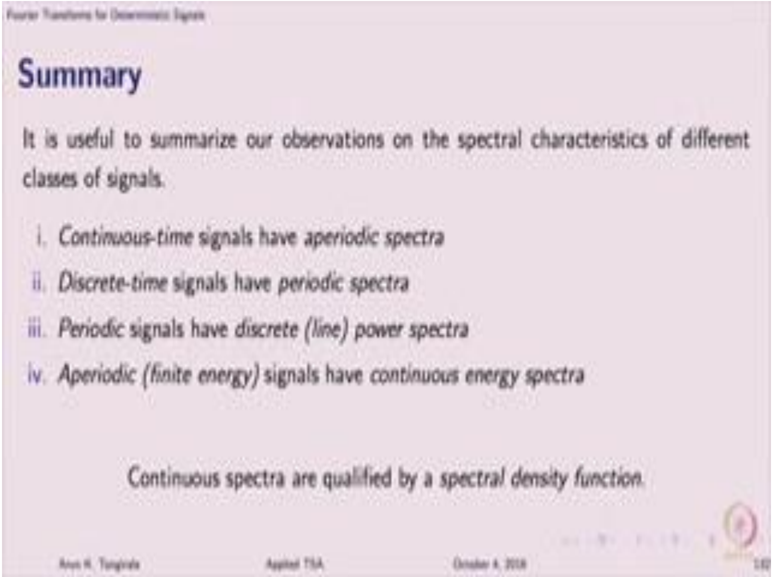
Let us move on now. And just summarize the discrete time Fourier transform that we have learnt.

(Refer Slide Time: 00:18)



A very quickly you have this synthesis and analysis equations and it tells you under what conditions the discrete time Fourier transform exist and also tells you what is the energy decomposition equivalent corresponding to the signal decomposition. You should not forget this in this world of transforms we are not only looking at signal decomposition, but also energy or power decomposition as the case may be.

(Refer Slide Time: 00:46)



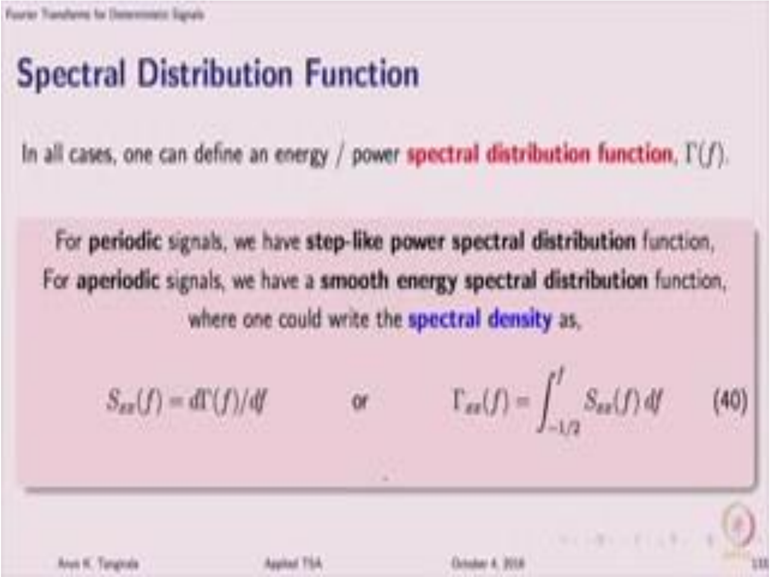
So, let me summarize now the confusion that you had until now, this is a systematic arrangement of the confusion hopefully it clarifies certain things.

Continuous time signals right have a periodic spectra; when I say continuous time signals here continuous time signals which are a periodic also, what about periodic is that also true work right. So, I do not have to specify periodic or a periodic. The moment I am looking at continuous-time signals the spectrum never repeats, that is because of the nature of the sinusoids which make up the family of building blocks. Discrete-time signals regardless of rather there periodic or aperiodic, they have periodic spectra whether it is energy spectra or power spectrum they are periodic.

Periodic signals regardless of whether there continuous or discrete have always line spectra that we have seen whether it is a continuous time periodic signal or discrete time periodical signal, you always have line spectrum why because only fundamentals plus harmonics can participate others do not have any role in the family. Aperiodic finite energy signals have continuous energy spectra. So, do you notice the duality there right the line spectra can be thought of as discrete you can say. So, when the time axis is continuous, spectrum is not periodic; when the time axis is discrete then spectrum is periodic correct when in time signals are periodic again it is a dual, you can see that the spectrum is discrete and when in time the signal is aperiodic, the spectrum is continuous.

Now, of course always continuous spectra whether you are looking at discrete time or continuous time signals as long as a spectrum is continuous, they can think of a spectral density which is what we have done.

(Refer Slide Time: 03:06)



Now regardless of whether a spectral density can be defined or not; we can always think of a spectral distribution, which tells me how much energy or power is contained in a certain frequency band; if you are confused you can actually now draw upon the analogy of probability density functions, when the random variable is discrete valued what happens? Do you have a density function no; you only have a mass function which is a line right F of x if you this small f x if you have to draw, it would look like a line like a similar to line spectrum that is the case of your periodic signals.

Whereas when the random variable is continuous valued we can offered to think of a probability density function correct, but in both cases I can always think of a probability distribution function, which we call as a cumulative distribution function. Same story here, whether we can think of a spectral density or not, we can always think of a spectral distribution function and a spectral distribution function is defined in the same way as the probability distribution function, if you look at the expression that I have given it is the cumulative one of course, I have only given this for discrete time signals because we are only going to work with discrete time signals by enlarge therefore, the limits of integration on the right hand side expression in the equation there, the bottom limit begins from minus half the lower limit and goes up to f; if you recall we had similar expression for c d f which is probability of x taking on less than or equal to x, but here we assume that the left extreme is taken care of.

(Refer Slide Time: 04:56)

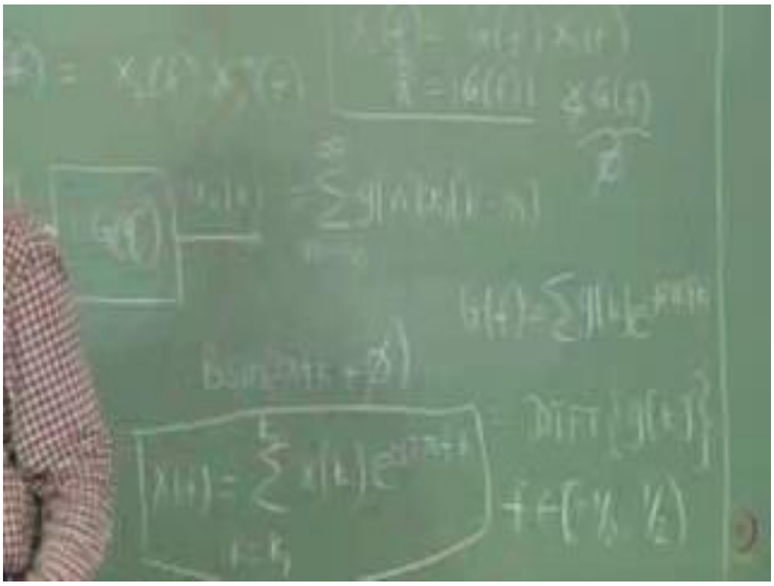
The left extreme in the case of discrete time signals for frequencies, the left extreme is minus half or you can say that the spectral density is the derivative of the spectral distribution, but you can say that only for continuous for aperiodic signals. For periodic signals how does the spectral distribution function look like? Sorry how did the probability how did the c d f, how does it look like for discrete valued random variables? Step like same story for the spectral distribution of periodic signals as well right for periodic signals there are there is no frequency component between two frequencies, it is either fundamental and then or harmonics. So, when you add up you would get a step like shape. So, story is a same fortunately that becomes easy to remember.

So, this kind of summarizes the theoretical Fourier transforms that we wanted to study; which has given as a lot of insides into how signals can be analyzed, systems can be described and so on, but now we have to worry about practicality; how do I apply these transforms when I am given some time series or when I am given a bunch of observations even for a deterministic signal? I am just given some n observations how do I apply? None of the definitions that we have learnt, allow me to apply in a straight forward way for example, the most practical one that I can think of is discrete time

Fourier transform why is it so practical? Because it assumes at signal is discrete in time; that means, you are dealing with sample signal, more over it assumes that the signal is aperiodic which may be the case.

Even what is the problem, what is the definition of your DTFT?

(Refer Slide Time: 07:09)



It says sigma the expression is x k times e to the minus j 2 pi f k; k running from minus infinity to infinity this is your x of f right this is the DTFT, what are the issues with using this some practice? The first obvious one is that is assumes the signal is infinitely long and I have full history and future everything which I do not have in practice. So, that is the first limitation. Let me say I give you that I tell you that the signal is 0 like our pulse that we saw, signal is non 0 only over a finite time and 0 otherwise let us say that is a signal that I give you. Is there any further problem in using this expression? So, let say that the signal exists only over some interval right some k equals k 1 to k 2. No problem even for such signals, I say there is still an issue with this expression, what would be that expression? I mean what would be sorry that issue?

Can you quickly think of what issued we may have here (Refer Time: 08:36) it finite duration as long as it is bounded, it is nice it is a finite length signal no convergence issue at all. Let say you want to code this you have to compute x of f, let us see you have to do this in our, what is the first thing that you will have to do? I give you x k from k 1 to k 2, I give you the values finite length no problem something very obvious may be your not actually stating it, if it is sometimes it too obvious it become difficult to state, you cannot compute x of f at every point right bit what is the interval for f? Minus half to half; can you give me x of f at every point? You cannot right it is like computing the value of a function at every point in a continuum, what we do? we choose to compute over a grid or we sample that exactly what we need to do here.

We need to sample now until now we have dealt with sampled signals in time, but now we have to sample signals in frequency domain; in other words put in very practical terms I can only compute DTFT for finite length and over a grid right now I do not know what should be the grid spacing, these are the questions that we will talk about tomorrow, but before we do that will just quickly go through the properties of DTFT there fairly straight forward once we do that tomorrow when we come to the class will talk of DFT the discrete Fourier transform where we drop the T also, until now we have been saying discrete time Fourier transform; why because we are emphasising the fact that only in time the signal is discrete, but in the frequency domain it is continuous.

But now we have decided for practical reasons I have to now discretize frequency as well therefore, we drop the T and we know simply say DFT; it is actually healthy for us one we do not have to actually at over one more sound right. So, we will look at the DFT, concept of DFT and periodogram, but before we do that we will just quickly parts through some of the useful properties of DTFT and we shall observe that DFT also preserves most of these properties except with some minor variations. When you use FFT in any software package, what your actually doing is your computing DFT. FFT is only an algorithm it is not it is stands for fast Fourier transform, it is only an algorithm for computing DFT. It is not a new transform yet again fortunately for us, it is only an algorithm that was a conceived by (Refer Time: 11:48) in mid 60s, until then computing Fourier transform as a night mar because of the number of computations that one has to perform. Today you have really very efficient algorithms or computing FFT. So, we will meet tomorrow and then go through DFT and periodogram.

Thank you.