**Applied Time-Series Analysis**
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**Lecture - 06**
**Lecture 03B - Motivation and Overview 6**

So, we have talked about this in practice we have only a single realization and hopefully now you have realized the challenge of realization. That is; we have only a single realization and we are supposed to draw inferences about the ensemble. So, there are two questions that will trouble us throughout and of course, you know people have thought about it and come up with some very nice answers over the last several decades. So, the first question is how good are the estimates that I draw from a single realization; that is number one and is a single estimate sufficient, which means that is a single realization sufficient. We will not answer these questions; I am just posing this question so that you know these are the questions that will look at, not initially though in the later part of the course when we talk of estimates, we will try to address this.

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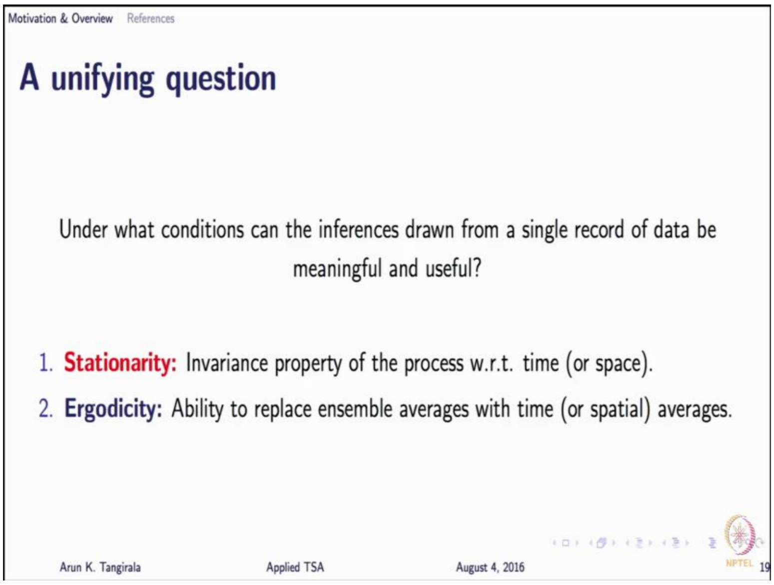


Now, the two related questions can we actually combine into a single unifying question which basically says or asks under what conditions can the inferences drawn from a single record of data be meaningful and useful. So, that is exactly that kind of summarizes what we have been discussing and the question that you have also asked right and you have also asked the same question, if this thing were to change with time she has put it in a different way, but the point is if the ensemble average where to be changing with time, then would the time average will be a good estimate or not. Now that is where we run the concept of stationarity and nonstationarity, so there are two related concepts now.

When it comes to drawing inferences about a process from a single realization and the two related properties are stationarity and ergodicity. Now one often gets confused with these two concepts, but let us quickly understand and what these concepts are and we will dwell on these concepts later on in detail when we talk of random processes settling. So, one concept is stationarity and stationarity is a kind of an invariance property of the random process itself; that means, whatever process we are talking of here there is got nothing to do with the measurement process or you can club the measurement process with the process of interest, whatever it is.

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Essentially the process that you are looking at; remember that here is a process and then you have a sensor right which introduces. So let us say here this process here is some y k so that we have things as general as possible. So, instead of the process giving a constant signal here the process is giving or is producing y k which I am observing with a sensor, which axis measurement uncertainty w k and the net result is v, this is what I have. I do not know y, I do not know w alright.

Now stationarity typically is talking about this, but not necessarily only about that. You can think of this v k itself as coming out of a; you can now define a new process which clubs. So, you can say just to keep things different I am going to write in caps here, this is slightly different from the random process in the sense that this includes also the measurement uncertain, so I do not really distinguish between what the process produces and what the measurement, the sensor produces; I have clubbed everything and I see the entire process and the measurement process together as one entity and call that as this random process of interest, that depends on whether I have a prior knowledge of the sensor characteristics.

Right now let us forget that there is a sensor, there is a process; there is a random process which is producing v k and I want to draw inferences about this process from the signal that I have and stationarity is telling us basically under the conditions under which I can use the inferences that I draw from a single realization over a period of time. What I mean by this is; let us go back to this example, the question that the two of you had asked, if the signal is constant, everything is fine in a sense I can use a time average to estimate the truth which is c, but if that constant c were to change with time; it is no longer constant, it is changing with time then there is no question of even thinking of time averages because the moment I move to a different time, the truth has changed.

Truth is no longer invariant with time, so stationarity is a very important assumption that we make about the process so that whatever inferences I draw from a single realization can be used to draw inferences. Now the question is whether the process truly satisfies this property; that means, whether the truth is actually constant with time or not, can we determine a priory from data whether the truth is changing with time. Yes and no depends on the way changes, but by and large we begin by assuming the process is stationary, if the truth were remain to be invariant with time, the truth may be referring to the mean, may be referring to the variance, may be referring to some other statistical property; we are not worried about it, we say that all statistical properties of this random process that I am dealing with should remain invariant with time, if at all I have to be able to use this data that I have obtained in time because if the truth keeps changing in time, then it is going to be a problem.

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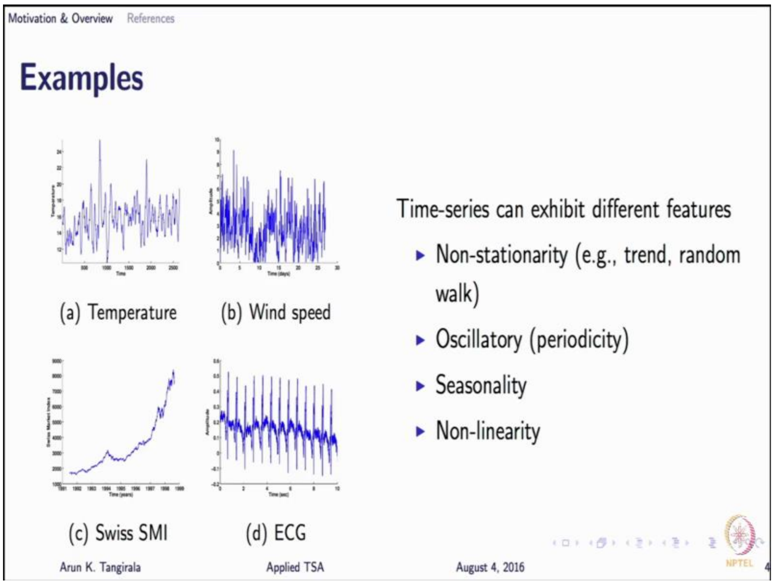


We saw an example right; the Swiss stock market index had such kind of a behavior. The averages were increasing with time, maybe some of you have forgotten, but we can actually go back and again look at that right. So, if you see at the bottom left lot, you see the Swiss stock market index series and you see there that the truth is changing with time, it is very glaring. So, can I use whatever methods I learn in this course to handle such series; yes.

But first we learn to deal with the series that are of the kind that you see in a; of course, you can argue how do you know that a is coming out of a stationary process. I do not know neither and not do you know, so the question is now whether you will believe me or not it is not about that, there are actually statistical tests that will allow us to test for non stationarity of a particular kind, there is no test out there that will actually tell you confidently yes the process is stationary; there is no test out that; if you assume that the non stationarity is of a particular type then there are tests which will tell you whether the hypothesis holds or has to be rejected.

So, there are non stationarity which can be detected just by visual inspection like in this Swiss stock market index, there are many others which are hidden there you cannot simply determined it by looking or by a visual inspection of the signal. We learn all about that, but the point is we will assume that the statistical properties of the process will remain invariant with time and it is a very strong assumption; we will slowly look at relaxed versions of such assumptions, we will not talk about it right now. At the moment we are saying that the process is strictly stationary, at a later stage; we will talk about week stationarity or second order stationarity and so on; around which we will build all the models.

So, the second property that will allow us to use the inferences drawn from a single realization to say something or the process is ergodicity. So, stationarity is talking about the process itself whether its statistical properties remain invariant with time; a yes.

Student: (Refer Time: 09:18).

Very good question; well I do not know in what context you are talking of time invariant systems. So if you are referring to the deterministic world, yes there is a very strong parallel between a stationary random process and a time invariant deterministic process. Some of you must have actually gone through courses on linear systems theory, where people talk about linear time invariant systems, but the notion of the linear time invariant system is slightly different from the notion of stationarity. You can say that the mapping of the time invariance property in the deterministic world to the stochastic world is stationarity.

Essentially, it is saying that the process should remain whatever it is, but it is not a single realization collectively that is ensemble should be stationary alright. So to answer your question; yes stationarity is some kind of a time invariant equivalence that you see in the deterministic world and we will revisit that equivalence later on. So, if coming to ergodicity quickly; ergodicity is more of a feature of how you measure, there are many examples that you will see on Wikipedia and many other resources and let us take one such example that you will see commonly.

Suppose I want to know the park or let us say in modern times theatre; that is visited the most in a city. Now when I want to do that, a natural way of doing that is to actually freeze the day or maybe the show that I am looking at and look at the number of people visiting all theaters in a city right; that is how I should be doing, but suppose I do not have access to such a measurement process, what I would do is actually maybe appoint a person to actually do this for me over a period of time.

I have maybe one or few people who would do this for me, but over a period of time now one way is to freeze time, freeze a show time and then look at all the theaters.

Let us change the (Refer Time: 11:39) to the most frequently visited it is a kind of indicative of maximum itself. So, if I want to know what is the most frequently visited then I would either follow the first method or the second method where I would now hopefully have identified a person who goes to theaters quite often and then follow the pattern over a year or two years and figure out, but there is a very important assumption underneath when I use a second method, what is that; that assumption is stationarity that is whomever I am using, whomsoever schedule I am following, pattern I am following to figure out what is the most frequently visited theater.

First I will assume that this person does not change, have a change of mind or develop a bias over a period of time; will actually visit all the theaters at randomly, there is no particular bias and that there are other things that will not change the scenario for me. So, stationarity is the first assumption that I will make. Then comes a second assumption which is I am hoping that this person actually is a movie buff. Suppose I have selected the wrong person who does not go to movies at all or who goes only to the theater near to his place of residence then I have a problem correct, my measurement process has a problem.

So even if the process is stationary, I can actually run into problems if I do not measure things properly, that is what ergodicity is all about it is essentially saying that, if I were to observe or if I were to construct averages in time then given sufficiently long time, I would be able to draw say that the time average is a representative of the ensemble average and that can only happen if the measurement process has no bias, nothing.

So, it is all about your measuring phenomenon rather than a property of the process itself, but typically we do not know. Suppose I am looking at a sensor and I am recording the temperature, assume that the process that I am observing is stationary then if I were to rely on the sensors reading, the implicit assumption first of all is at the process is stationary; that is important. Then comes the second requirement that if I were to let the sensor record the temperature in time and use that data then an important assumption for example is sensor does not develop a drift, it does not have a bias and so on.

So, that kind of constitutes your ergodicity property and ergodicity is a very hard condition to verify. Theoretically if you look at the definitions of ergodicity, it would simply say that if your process is such that; when we say process typically it is a measurement process that does not mentioned. If your process is such that the time averages that are computed using one realization, as the time goes to infinity, as a number of observations go to infinity; you will be able to say that this time average is representative of the ensemble average, then we say the processes ergoding.

Now, there is no mention of how you estimate that time average or anything; I mean simply just averaging in time, so you can say simple average in time for that matter. Ergodicity is a property that is very hard to verify in time or even in practice, you simply assume that the process is ergoding, but if you know something in your measurement process is going to violate this condition or requirement of ergodicity, then you will have to fix it up front. Basically you should make sure that there are no biases in your measurement phenomenon; of course, some part of ergodicity can be clubbed with non stationarity and so on, but the point to remember is stationarity and ergodicity are completely different properties and for you to think of even ergodicity stationarity should hold.

Because ergodicity is answering the question when can I say that? Whatever average a computing time will be suitable representative of the average in ensemble but for that the truth itself has to remain invariant with time. Therefore, whenever you examine the process for ergodicity; implicitly stationarity has to hold, for non stationary processes you do not even talk of ergodicity. So, that is something to keep in mind; we will revisit the notion of stationarity. Ergodicity is something that we do not really revisit, it is an assumption that we will make that whatever I estimate from a single realization will serve as a good estimate of the average in on some (Refer Time: 16:47).

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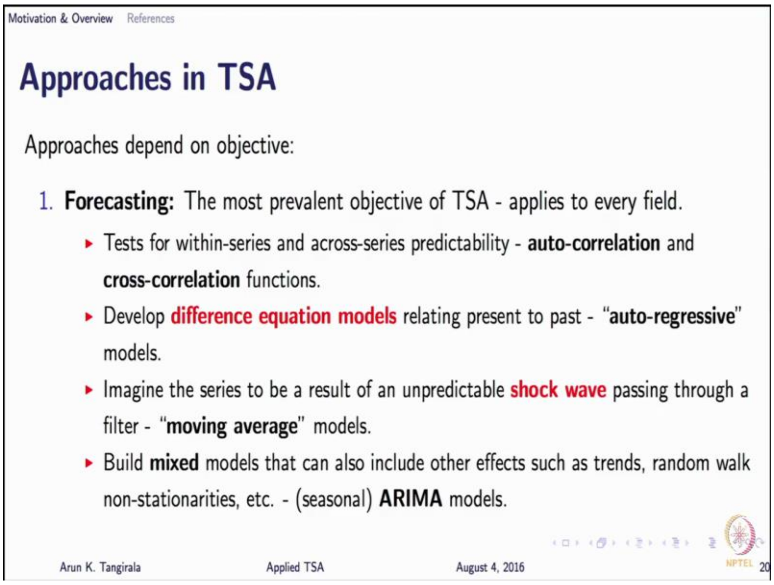


So, very quickly let me tell you what are the typical approaches in time series analysis and that depends on the objective. As I pointed out on day 1, there are different exercises one encounters in time series analysis until now we have talked about the core challenge of time series analysis. Now we will quickly get a glimpse in a couple of minutes as to what are the typical approaches in time series analysis and that depends on the kind of analysis that you are doing. I mentioned that forecasting is one of the predominant exercises that you will run into in time series analysis and when it comes to doing that, you are going to rely on the history; remember we said the strategy always is to rely on the history and then make a prediction hopefully that there is something in the history that will allow us to make a prediction, but it is possible that there is nothing in the history that will allow us to make a prediction.

So, an important step in forecasting that is in developing models for forecasting is to test for predictability. We should be able to test for predictability thus the historical data contain anything at all that will allow me to make a prediction and that is done by what are known as auto correlation functions or if you looking at bivariate signals that is if you looking at two signals, you are using one signal to predict the other then you are looking at cross correlation functions. There we run into concept of autocorrelation, cross correlation, concepts of white noise and so on.

White noise signal or a white noise process I should say is that process which does not offer any scope for prediction, even given infinite history. So, we want to make sure that the process I am dealing with is not white noise. Well ideally I would say when your solving assignments or exams you would hope that it is white noise and then say there is no score for prediction I am done with, but when you are actually sitting there in your job and you are asked to make a prediction, you are hoping that it is not white; it is colored and that there is some correlation of the past with the future or the present. So, that I can build a model, what kind of models do we build; well the kind of models that we build as you will see later on our difference equation models.

Now difference equations are something that we have seen before in the deterministic world, I am sure we have come across many difference equation models for example, Fibonacci series is one difference equation model explaining, what is the Fibonacci series explain. Are you familiar with Fibonacci series?

Student: (Refer Time: 19:44)

So it explains the rapid growth phenomenon correct, it is a difference equation what we mean by difference equation is same kind of a recursive equation. So, as you will learn later on, we will build models for random processes that offers scoffer prediction and these models as they are known as auto regressive or moving average or a r m a; ARIMA models or you know if you take it to the multivariate case vector autoregressive moving average models and so on, they are all nothing but difference equation models, you can think of them as stochastic difference equations, but if I use that term that may scare you, they essentially difference equation models that will build and what we will learn in this course and in fact, the core part of this course will teach you how to build ARIMA models, given time series. What are the steps that you would follow systematically and eventually help you in your placement alright?

So, that is the thing that we will follow in forecasting. There is yet another branch of time series analysis I have talked about which is got to do with detection of periodicities and I did mention at that point in time that this periodicity detection actually makes use of Fourier analysis.

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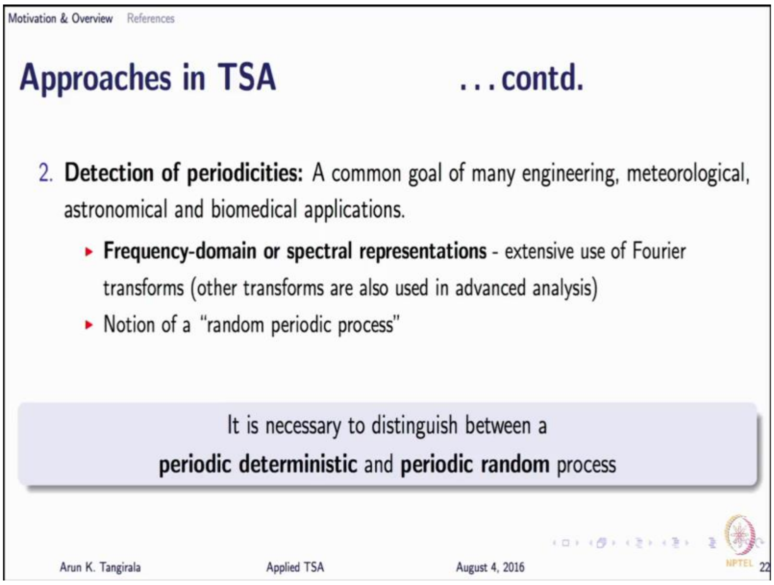


So, we move to frequency domain and so on. Here as well one has to worry about the distinction between a periodic deterministic and a periodic random process. Now on the face of it this so called periodic random process seems a contradiction, how can a random process be periodic, if it is periodic; it is no longer random right, but that is if you are still thinking of a periodic random process on the same lines as a deterministic process. For deterministic process; when is a deterministic process said to be periodic, when it repeat itself; if there is a finite time after which you see a repetition, then we say the deterministic process is periodic.

It is not the same definition that is used to define a periodic random process in fact a better term that is used for periodic random processes; harmonic processes. So, we will look at how to actually, what are the definitions of harmonic process and how to detect periodicities in such harmonic processes. There are different models, we will not worry about that, so when we move to frequency domain analysis, we will define first what is a periodic deterministic process, we will learn how to deal with such processes because many a times I notice that a lot of students are not familiar with even periodic deterministic processes and how to detect periodicities using tools such as power spectrum and so on.

So we go through that; that gives us the base to understand how to define a harmonic process or a periodic random process and then we will study two different models for periodic harmonic process. There are two different ways in which you can model harmonic process and how the notion of spectra or spectral densities comes handy in defining a periodic random process. At that point, we will extend this notion of spectral densities for harmonic process in fact for harmonic processes, there is no notion of density; we only talk of spectra, we move from other point in time from periodic harmonic processes to a periodic random process. So, we generalize we say for general random process can I do a frequency domain analysis and that is where we run into the notion of spectral density where we will also run into (Refer Time: 23:29) theorem and so on, they are very powerful results as much as you may be dreading to think of it.

Those where the cornerstone results and those still remain the cornerstone results for the entire theory of random processes that we see, we may be by in large developing ARIMA models and so on, but if you go back to the theory for example, a burning question that was addressed, that was studied for a long time was for a stationary process, when can I build in ARIMA model? Under what conditions can I build an ARIMA model and the answer came out in the form of results in frequency domain and that result is called a spectral factorization result, we will talk about that later on.

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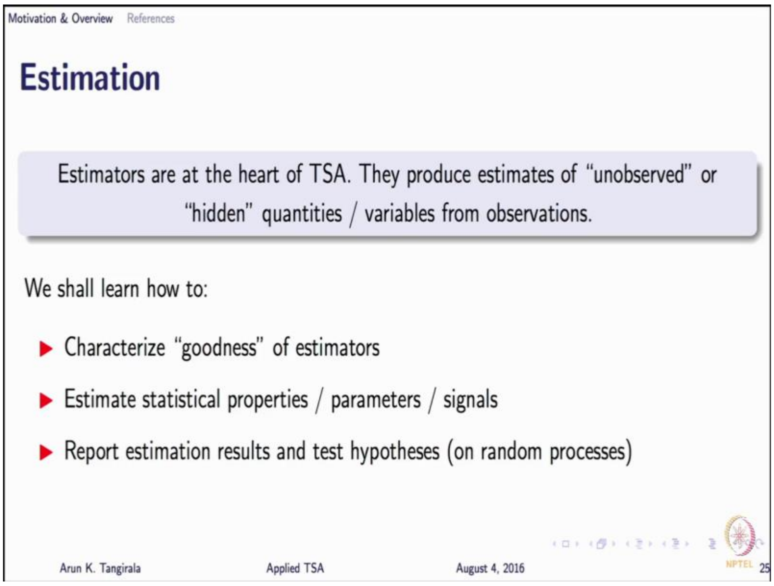


So, finally I will conclude with what we will come across in estimation and then spend a minute on the scope of this course. I have talked about estimation already; I will not spend too much time on what will come across. I have already said we will go over the estimation theory which will teach us first of all how to characterize goodness of estimators, given any method of estimation; how good is that method, you may come up with some other method tomorrow you should actually again question the same way and then of course, learn what are the different methods of estimation.

In particularly we will look at least squares maximum likelihood Bayesian estimation and something called method of moments, these are the four classes of estimators that we will learn and then of course, look at some fundamental results in estimation theory and hypothesis testing; hopefully gave you a preview of what we do post estimation. Once you estimate, you have to actually conduct certain hypothesis tests because the purpose of estimation is mostly to test hypotheses.

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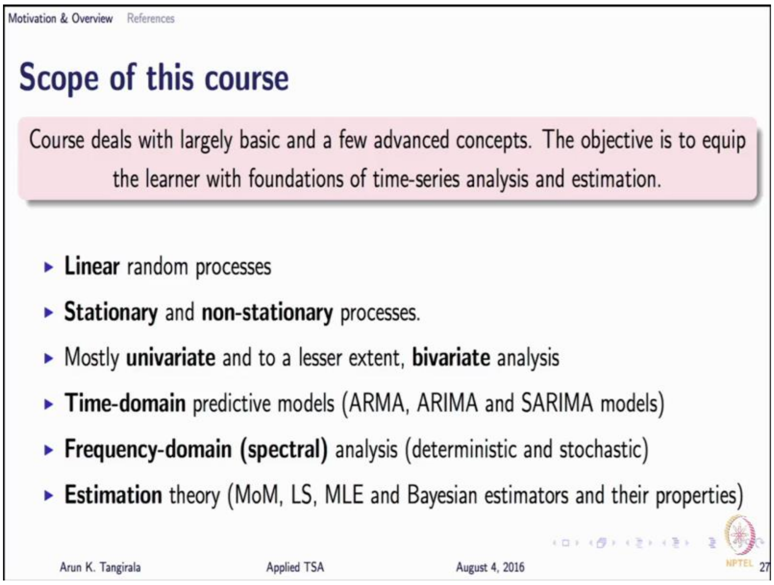


So, as far as the scope of this course is concerned we looked at linear random processes, we looked at stationary processes and to a certain extent non stationary processes as well and as I said mostly will restrict ourselves to univariate and as I have just said, we will go over the time domain models including ARIMA and seasonal ARIMA models, SARIMA models and then also learn how to analyze random processes in frequency domain, these are called spectral representations and then spend considerable time in estimation theory.

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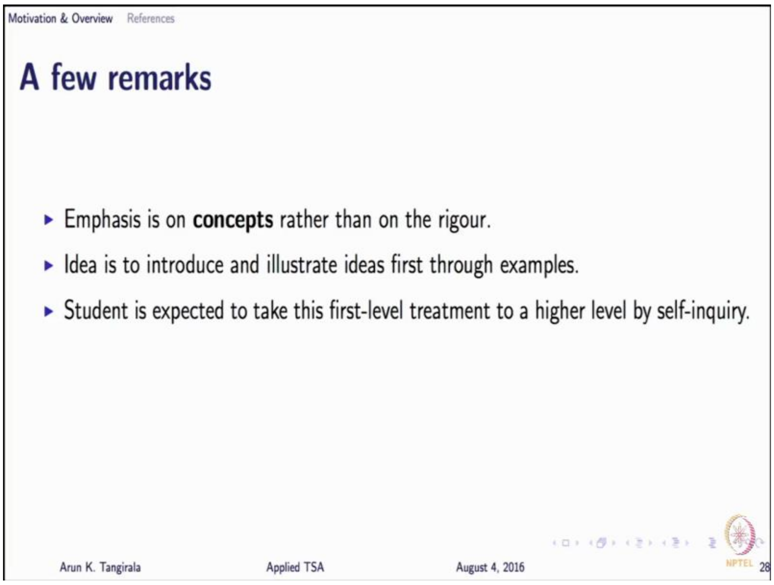


And I just want to say that emphasis always some concepts, you will feel that even in the exams i do not test you on your ability to remember a formula, but I will test you on your ability to apply a certain concept, you will come to know that even in assignments and of course, this is a introductory course but gives you all the principles required for understanding advanced topics and finally, again I want to repeat that time series analysis requires both the theory and some skill, some art that art comes through practice and for that practice, you need a computational software which is what I have talked about earlier.

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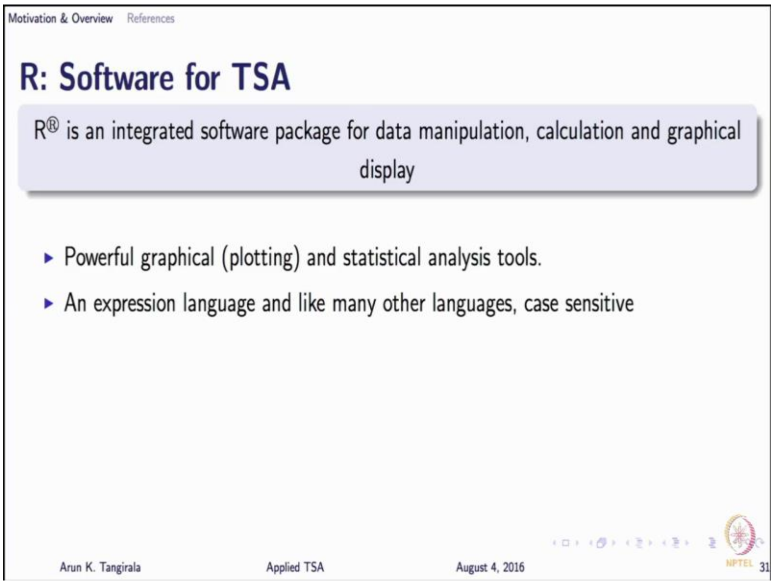

We will use (Refer Time: 26:39) and these are some of the references.

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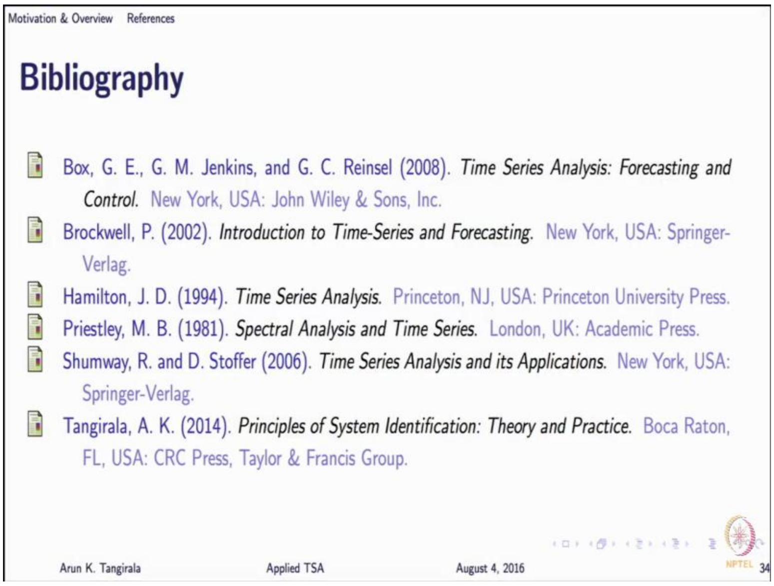


Next Wednesday onwards we will start writing all the necessary equations and will take the theoretical developments.

Thank you.