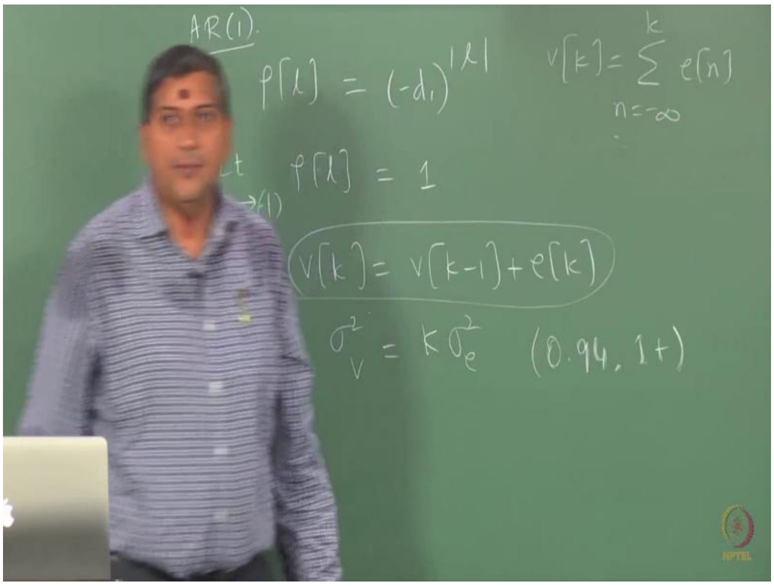
Applied Time-Series Analysis
Prof. Arun K. Tangirala
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 57
Lecture 25A - Models for Linear Non-stationary Processes 5

Discussed is how do we detect the presence of integrating effects and what cautions one has to exercise when differencing the data. We already know that differencing is the solution to handling the integrating effects. Now we have we have discussed this earlier that the ACF for example, of an integrating process has a slow d k and I have also argued this before that when you take an AR 1 process you know already that the ACF is a theoretically minus d 1 rise to mode l.

(Refer Slide Time: 00:42)



And now we can evaluate this ACF in the limiting case in the sense that when in the limit as d 1 goes to 1 in rather minus 1 in our case because of the sin that we have taken

The ACF actually goes to 1 and this is not be taken very regressly, but we know essentially qualitatively also that if I have an integrating process then the observations are highly correlated so 2 observations. In fact, the best prediction of this observation is the previous one. In other words the if you take an integrating process and look at its ACF it should have ideally no d k, but on the other hand whether this is the one that is ACF of the integrating process not really we have just saying in the limit as d 1 approaches one the reason we say that this is not the ACF of the integrating processes is because the integrating process is not a stationary one.

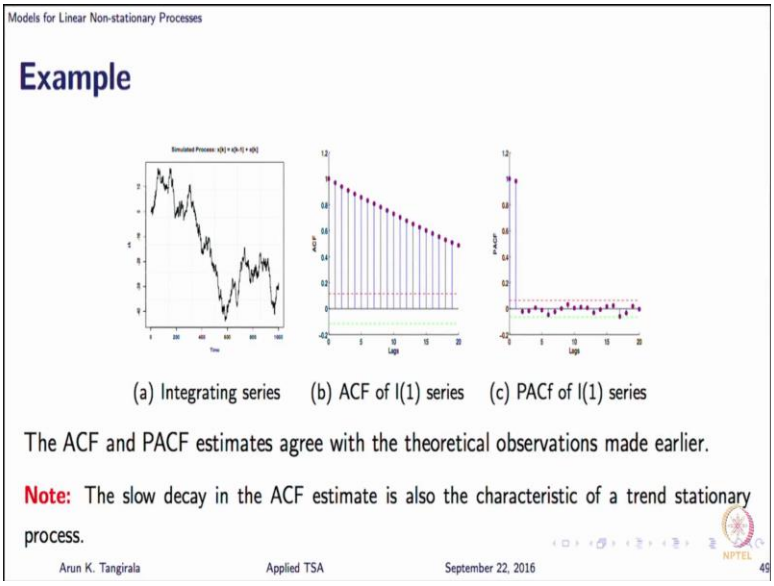
To be able to say that this is the ACF is not correct why because we have assumed in deriving this expression that the process is stationary. So, strictly speaking this result does not apply to the integrating process because both the variance. And hence the auto covariance are changing with time that if you recall the variance of an integrating process changes with time the sigma square v what is the expression that we derive do you recall you should be able to even re derive it using the moving average form of expression that we had why is it so difficult? Friday is you do not answer is it no, what is it, why is it? So, difficult now you can you can use the moving average from you derive this before you looking at me for answer, I am looking at you for your answer; somebody has to break the ice now.

Why is it so difficult to derive the variance? You knew it and you wanted to (Refer Time: 03:35) k time sigma square you derive these before and you have shown that hence it is non stationary any questions. So, the variance actually changes with time and. In fact, you can view the integrating process interestingly as a moving or average process whose length keeps changing with time of course, it is not characterized because you have n equals minus infinity, but even if you were to begin from some time and take the initial conditions into account, you can see that the moving average part of the integrating process, the order of it keep changing with time, more and more e s are being considered.

Anyway so coming back to the point the variance of the integrating process is changing with time and so will the auto covariance. So, strictly speaking this limit that we have evaluated does not necessarily apply to the integrating process, but imagine just for the sake of argument that d 1 is somewhere very close to 1, but just following short of 1 by epsilon, practically you would not be able to make a difference, I will show you an example practically you will not be able to make a difference and in general we work with a sample ACF and you can show that the sample ACF again decays very slowly by using similar arguments.

The bottom line is the ACF, in fact, more useful observations for this is a sample ACF, the sample ACF decays extremely slowly for integrating process, but the exact behaviour cannot be known and let us look at an example. So, here I have an integrating series and I am showing you the ACF of the integrating series.

(Refer Slide Time: 05:21)



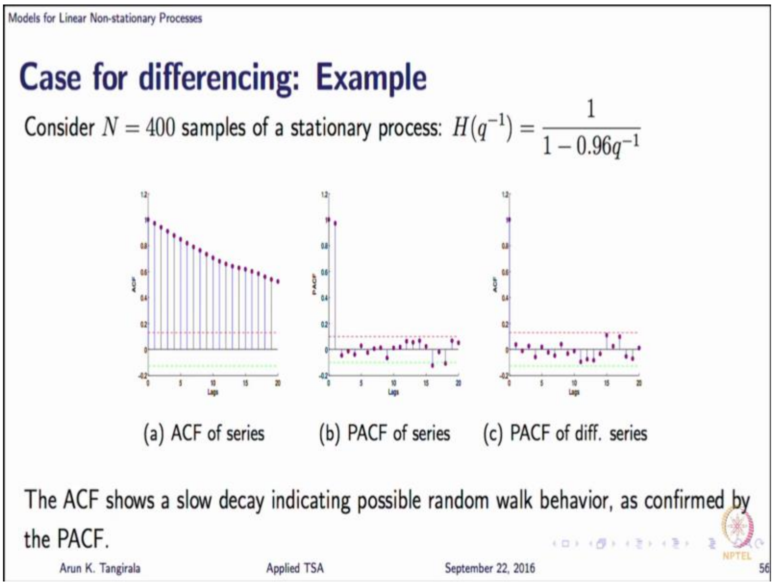
It is just a realization of the integrating series that I have and in this case, the sample ACF is decaying slowly in some cases, depending on the realization the sample ACF can be decay even for the slower, slower than this, but the general qualitative observation is all integrating processes have this kind of a signature, do you relay only signature to detect integrating effects formally? No, this is a good way of detecting integrating effects up front, formally you have to conduct unit root test and I will talk about it a bit later.

The other way of detecting the presence of an integrating effect is to fit an AR 1 model and see how close the pole is to the unit circular unity. In fact, the PACF shows you that if you look at the PACF of the series, you see that the value of PACF is very close to unity and we know that the PACF at lag 1 is the coefficient or with the science say is the coefficient of the AR 1 model. So, it says if you are to fit an AR 1 model to this process, you will find a pole very close to unit circle giving you an indication that there is indeed a pole on the unit circle. So, this is how informally you detect the presence of integrating effects. Now does this give you the write to difference series, let us say you have observed this kind of a behaviour do we actually go head and difference series is differencing really work the risks, what if the series does not I have pure integrating effect, but d 1 may be very close to 1.

For example, d 1 maybe minus 0.95 minus 0.97, by differencing what are we doing? We are forcing one of the poles in the model to go and sit at unity, that is a difference, if I give you a series and you observed a behaviour like this for the ACF, you have 2 options, one to believe that indeed there is an integrating effect and force fit pole on unity at unity or you say no I do not think necessarily that the pole has to be at unity, I will simply fit an AR 1 or whatever appropriate model and hope that the integrating effect is not there and even if I get a pole less then unity, I am happy with it, as I said there are unit root test that will formally help you detect even then let us say your unit root test actually gives you there result saying that there is no integrating effect sometimes it may be worth differencing the series. So, that is the point that will go through an example.

If you have if you detect an integrating effect, your formal unit root test have given the word it that yes the reason integrating effect then go head and difference it, what happens if it does not and you still see a behaviour like this? What happens if your unit root test this no says no there is no integrating effect in there is no evidence to support that hypothesis should I still go evidence difference? What happens if I do not and what happens if I do is what we will discuss when there is no need, strictly speaking? So, let us look at an example understand this point with an example.

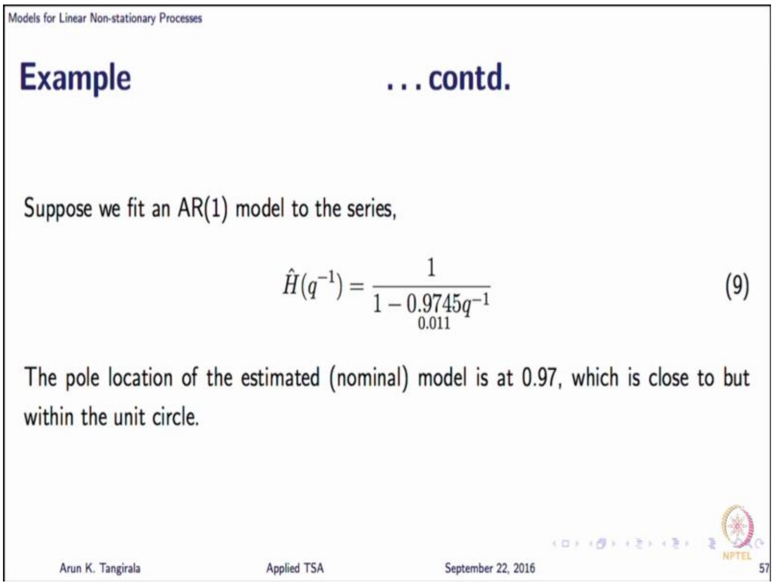
(Refer Slide Time: 09:09)



Here is the ACF of a series, I am not conducting any formal test here, it decays pretty slowly and the actual processes 0.9; an AR 1 with the pole at 0.96, this is not an integrating process correct.

And the PACF of series looks like this, it has when it says if you fit an AR 1 model the coefficient will sit, will go and sit very close to unity. So, should I believe now there is an integrating effect as I said you can connect the formal test, but let us say that I do not do it or even if I do it let us say that the verdict, it is not now I am just showing the PACF of the difference series.

(Refer Slide Time: 09:56)



But let see what happens now if I fit and AR 1 model without differencing, yes, the differencing has got an rate of all the correlations as you saw with the PACF, the PACF of the difference series shows that the difference series more or less is white, more or less is white in the sense we have we are basing or conclusion on the significance levels.

Let us say the difference series is for all practical purposes white. So, one option is to build a model like this you can say that this is the model for the given series or the other option is to fit and AR 1 model, if you fit an AR 1 model using any of the routines that we are seen in r, I am reporting the estimate that I have obtain for the series of course, the exact value will change depending on the realization and I am also reporting the standard error and now we can rely on the standard error because we know its AR 1 the residual should be white and. So, on and let us not get into that now the pole location is at 0.9745 roughly 0.97 very close to unit circle which is fine it is still stationary the model that I have identified is stationary.

However, if I look at the confidence region that is for the region in which the truth is present with it some degree of confidence, let us say I am looking at ninety nine percent confidence interval then roughly its point in the estimate plus or minus 3, 3 sigma, it is actually 2.58 sigma, but 3 sigma, what are the extremes that you would get, what is the interval that you would get? One extreme would be 0.94. In fact so you have 0.011 as the error. So, one extreme would be if you are to construct a 6 sigma interval then you have the interval running from 0.94 to 1 plus now what does it tell you all seeing to be last today.

Student: To estimate maybe equal.

So, the interval that you obtain is correct. So, 0.94, that is for the poles to some 1 plus.

What does it tell me that more than 1 is also possible possibility for the truth, I do not know what the truth is, but I believe with a very high degree of confidence in the truth is in this interval, it is not too wide, but now ideally what is 100 percent confidence interval for the truth, 100 percent is I ask if I ask you what is the interval in which you believe the truth is with 100 percent confidence infinity, but that is useless, that is of no use to me and that is why we are constructing 99 percent confidence intervals and sometimes you construct 95 percent and so on because as we up the degree of confidence, the width of the interval also increases, that is a standard result in confidence interval constructions.

The point here is all though the point estimate is giving me a pole that is within the unit circle, there is a risk of running into a confidence region that contains unstable or non stationary models and you cannot say that, how can you say that here I have a stationary process and then I am saying there is a confidence the possibilities also include non stationary models that is not acceptable you want confidence region that has the minimum characteristic required which is stationarity and this is always the case when you have a pole very close to unit circle even though your point estimate may be very close with the well within the unit circle satisfying the condition of stationarity the set of possibilities may not contain the stationary processes.

In which case, one has to abandon this root because when you construct forecast and then the intervals for the forecast the interval for the forecast will be extremely wide what we call as prediction intervals because your saying the some of the possibilities include explosive series also that is what we are meaning to say when the confidence regions include non stationary processes I what we want to do always in estimation is not only obtain the point estimate, but guarantee that the confidence region includes non stationary parts as well. So, the bottom line is now when you find series that has extremely slow slowly decaying ACF then you may be warranted I am you have the kind of a right to difference series for these reasons.

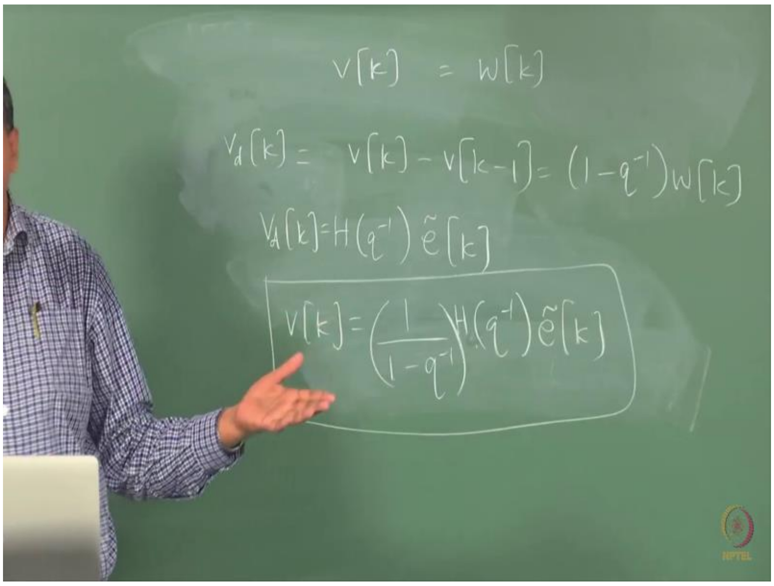
Any questions and that is an unfortunate part 2 and that is see the unfortunate part, here is the truth is it is not in an integrating process, but because of the finite samples and because of estimation errors one is forced to difference series to treat the series as an integrating process you can try this go back and simulate a process with 0.98 and AR 1 with pole at 0.98 and try to estimate, yes, the errors can are indeed very small, but if you look at the confidence region then it can include unstable effects now this is generally known it is a widely accepted fact that when you have poles very close to unit circle your presented with a lot of challenges when it comes to identifying it very precisely.

You want to very precise estimate of the pole as the pole close to goes close to unit circle now some of the methods that we have discussed for example, Yule Walker's method sorry, is known to perform purely when the data falls out of integrating or near integrating processes. So, one has to adopts some other method to estimate this poles the these kind of a models for these kind of series in general as I saying what I am trying to tell you here is even though the true process does not have a pole on the unit circle you may be forced to place the pole on the unit circle because of estimation reasons, but that is not the only remedy, that is one of the remedies such followed.

Now on the other hand, you may also end up over differencing. So, as I said sometimes even though the processes not an integrating process you will end up difference in the series there by force fitting a pole on the unit circle, but there is another scenario where in a bid or in for whatever reason you have over difference series more than necessary. So, as a simple example suppose, I am giving you white noise process you did not even look at ACF you heard somebody talking about differencing and you thought you should difference. So, you just difference series what happens. So, the difference series here now is no longer white it is correlated now we are talking about the harmful affects of differencing if you do not exercise caution.

What has happened here is I given you we have a series that is uncorrelated white noise, but over unnecessary differencing has introduced spurious correlation in to the difference series because now your new series is e k minus e k minus 1 and that could be true even for w k even if we e k, but to be replace by some other correlated process what your doing is your introducing 0 at the unit circle. So, over difference the general differencing actually definitely puts in pole at the unit circle, ultimately what you do? You put together when you fit a model for the given series and use the differencing method so you given v k.

(Refer Slide Time: 18:37)



And now you decide to work with some differenced series v d which is v k minus v k minus 1 and suppose v k actually given to you as w k itself which is in our example, w k is e k.

Suppose w is stationary and you have accidently differenced then you have the differenced series as one minus q inverse w and then you say ha you know now I just look at the ACF of the difference series it appear stationary the m a part does not spoil stationarity let say you go head and fit a model for v d some ARMA model, ultimately you will put together the model, let us say you put together you have explained v d using an ARMA model and now you together this differencing operation and the obtain model to obtain a model for v k. So, you say that the model for v k is 1 over 1 minus q inverse h h of q inverse e tilde of k.

This is the model that you have now in all of this what you have done is yes you have by way of differencing you have forced a a pole on the unit circle, but in this process you have also place to 0 at unity and you cannot expect that to cancel out here because you are not going to fit an m a model here which will guarantee that the 0 will set on the unit circle for you, we have going to work with v d. So, I will identify some ma model where a 0 is going to be very close to unity, but not exactly unity has a result it does not really cancel out the effects here. So, do not expect the fettered m a model for v d to cancel out the pole at the unit circle.

So, the point is now you we have created problems when there are none more over this 0 here at unity unfortunately places 0 at the non invertible location, we only build ARMA models for series that come out of an invertible and stationary process, but unfortunately v d is coming out of non invertible process. Therefore one has to exercise caution when it comes to over difference.