Applied Time-Series Analysis
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Lecture – 56
Lecture 24B - Models for Linear Non-stationary Processes 4

Now let us move on, we have spoken enough of tendency seasonality now its season for integrating effects. One thing that you should remember as I said there are parametric and non parametric methods and among the non parametric methods essentially you use filtering ideas. And as you have seen the trends are low frequency characteristics and you use some filter either to extract or eliminate depending on what you want to do.

And as I pointed out differencing is one way of handling trends, we are not including seasonality here and this discussion. You can handle seasonalitys also through differencing. For example, if I know that there is a periodic component here of some period; again it assumes that you know the period a priory. In the c o 2 data we had cycle of 12, cycle city of 12 months; and you can use the fact that the signal has a periodicity of 12 and construct a differenced series.

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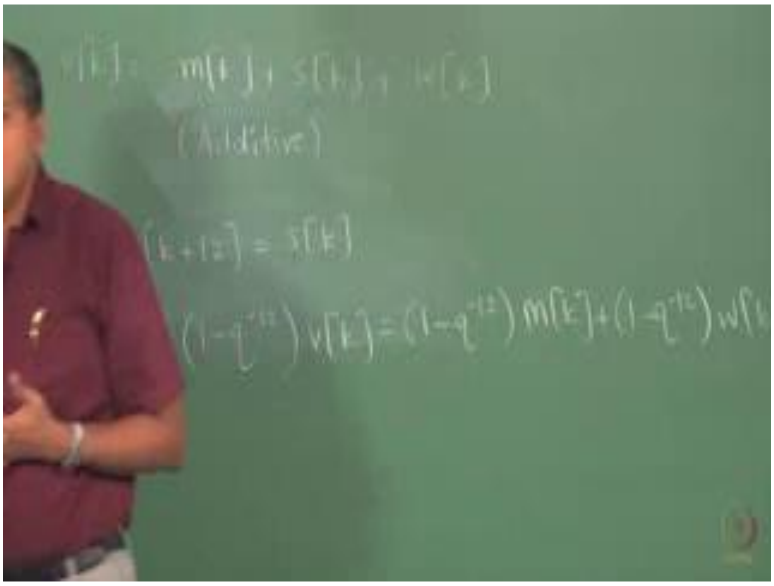

The regular differencing takes care of linear trends as I show you on the screen. And the other differencing takes care of the; so suppose I have a pure periodic signal of period 12 then I know that s k plus 12 is s k for the c o 2 cycle for example, c o 2 data. Then, I can actually construct here delta 12 which is different 1 minus q to the minus 12 I can perform this kind of an operation on v k; what does that do v k minus v k minus 12.

If this holds, this is correct for the seasonality then I would have 1 minus q to the minus 12 m k plus again 1 minus q to the minus 12 w k. But this is not a great way of doing things because as we have seen yesterday whether you use this approach to handle trend or seasonality you will be introducing artificial zeros on the unit circle, in the stationary part whatever you will be left with and here also be your modifying the trend. So, you have to be careful when you use differencing approach. By all means actually you should avoid this differencing approach when it comes to trend and stationarity. On paper it may look very nice very good, but in practice it can present a lot of difficulties.

So, will skip this part, and move on to integrating processes now.

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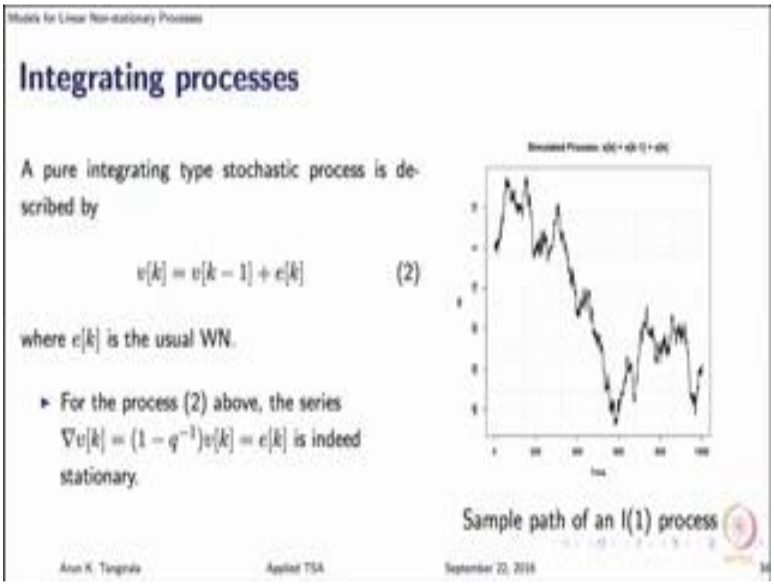

Integrating processes are of a different nature. I am showing you for example on the screen snap shot of an integrating process that I have simulated, it is just one single realization. And you can see that, that if you look at the long series that is if you look at a longer realization you will see that the mean is not really changing does not have it trend deterministic trends that we have seen earlier. But, the truth is the mean does change with time, in a stochastic way not in a deterministic way.

And that is the feature of integrating process. We have already seen what an integrating series is.

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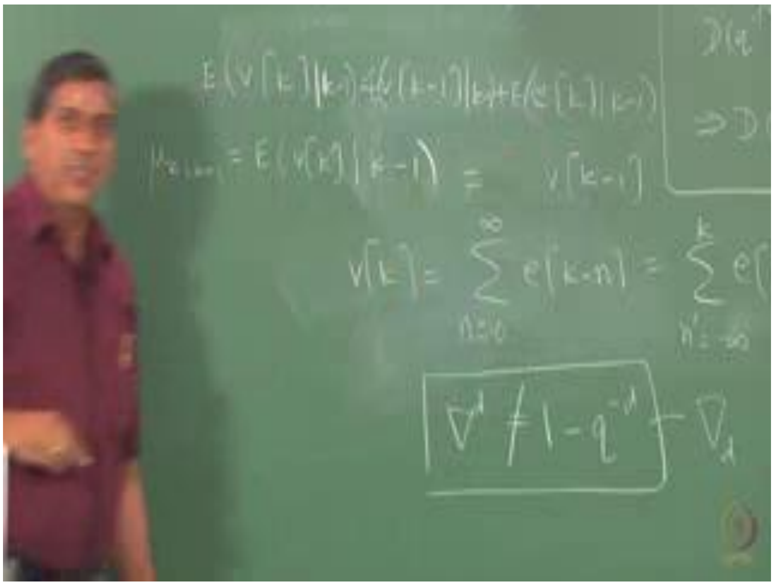

So, we have seen that v k is for example and integrating process of first order pure integrating process is of this type. So, that the expectation of v k given v k minus 1 is what is the conditional expectation. What is it?

Student: (Refer Time: 04:10).

Absolutely sure, good, correct; so v k minus 1, but this is nothing but your mu at k given all the series up to k minus 1 this is a conditional mean. So, given that the process has evolved up to this instant in this fashion the mean at the next instant is the observation at the previous instant. So, which means the mean is in some sense changing with time, but in a peculiar way. This is not mu k remember, this is not mu k because first of all to write mu k we will have to write it in a different way, in fact you can say the overall mu k is 0. How can you show that? Is it possible to show that the overall mean is 0?

What is in alternative way of writing this integrating process? So, has a summation assuming there it begins from 0 initial conditions then you could you can write v k as sigma e k minus n k n running from 0 to infinity right, sorry it is an accumulation of all the parts shock waves until this instant. Is it right? What I have written is it right or not? Is it correct? Or one could write e of n and then n running from k to minus infinity to k and alternative way of writing this is sigma e n prime; where n prime runs from minus infinity to k. Maybe this will give you better picture. This clearly shows that it is accumulation or integration of all shock waves from time shmemorial until now.

So, when I take the expectations on both side assuming that here of course I have assume that the process start from 0 initial conditions which is ok. The mean turns out to be 0, but what we are talking about is local means. The locally the mean is changing with time, and of course that is also true for other kinds of stationary processes, but this is here peculiar in the sense that the mean is exactly the previous observation, whereas for stationary processes also if it was an AR 1 process it will be true as well. For example, instead of v k minus 1 I had 0.8 v k minus 1, then expectation of v k given v k minus 1 would be 0.8 v k minus 1 locally the mean will change. But, this is a peculiar thing where this observation determines the mean of the next one the conditional mean.

The other way of looking at it and if you look at the history of this integrating processes which we are again observed by Bugs; Bugs made in all much contributions to time series if full name is George Bugs. What he and his researches colleagues observed that there are series which look very similar regardless of the shifts in the series. If you look at the entire series locally there may be different in terms of mean, but over all they look very similar. And of course, you can think of it from this random work view point as well.

So, the way the integrating process was born is that you say even if I shift the series by a certain amount the overall process remains the same. In other words, it should be insensitive to the shifts in mean for example here, if D of q inverse is the polynomial your regressive auto regressive polynomial you would say for some constant c if I shift v k it should be the same as this. So, local shifts in the series should not cause problem, in the sense there should look very similar. And from this you can derive a conditions straight away that D of 1 is 0; you can I leave it as a simple exercise to show that this demands that when I evaluate the polynomial with q replace by 1 it should turn out to be 0.

That is fairly straight forward to see because, you have D of q inverse operating on c to be 0 and this should be true for all non 0 constants, so the only way this can happen is when D of 1 is 0. Because D of q inverse operating on c should be 0, c is a constant.

What this means is invariably the D of q inverse should have a factor of 1 minus q inverse necessarily. This straight away implies that D of q inverse should be of this form 1 minus q inverse time some d prime of q inverse so that D of 1 is necessarily 0. Where, d prime is such the d prime of 1 is not 0. You can have 1 minus q inverse rise to many term powers also that is not an issue, but at least there should be 1 minus q inverse and that is how the integrating process can be thought of as well. So obviously, how do you handle the integrating process?

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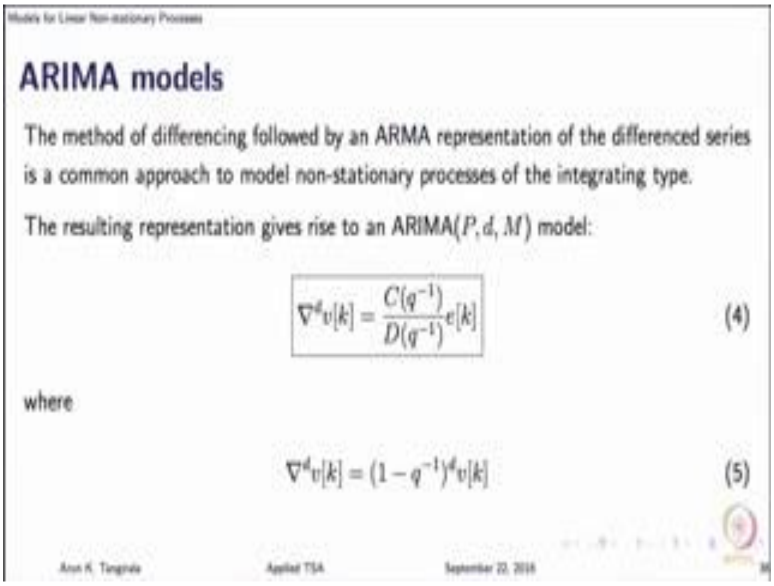

Well, as I said through the differencing operation and together when you put together the integrating effect along with the stationary one a multiplicative model is 1. Then an ARIMA model is 1, where we say an ARIMA model of order p, d, m is an ARMA model on a series that is differenced d times. That is another way of looking at an ARIMA model. That is if I difference series d times then I end up with a stationary and invertible series for which I can fit and ARMA model of orders p under; that that is very easy way of understanding it.

And you should not confuse this nabla d that I have used nabla rise to d with the other operator that we had seen earlier, do not think that 1 minus q inverse rise to d is same as 1 minus q to the minus d. There are obviously two different operators so be careful. Very often this 1 minus q to the minus d is denoted as nabla subscript d, that again that depends on the author. This is general convention that is used.

So, these are your ARIMA models. Now, the question is how do I know integrating effects are present, right? But we will talk about it very very I am showing you the transfer function operator; full transfer function operator form.

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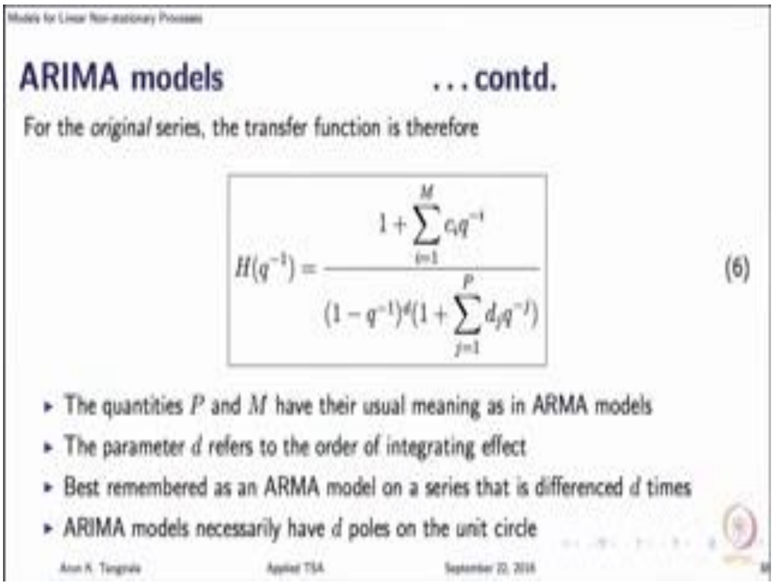

Standard thing there is nothing I have just replace the c and d with the respective polynomials nothing much. You have to determine d first by suitable differencing. Now that is where one has to exercise caution. Let us say you have detected the presence of an integrating effect. There are tests that are available, these are called unit root tests and I am going to very briefly talk about it a bit later. But let say you have figured out there is an integrating effect now the remedy for handling integrating effects is differencing; there is no that is by default the remedy that is used. How do you determine you difference once and then you again subject the series to test and so on? But one has to be careful in not to over difference.

Student: how much make a expectation of v k given v k minus 1 (Refer Time: 12:37).

How else it cannot be? I mean that is not the answer to the question, but the compute the expectation. So, v k is v k minus 1 plus e k I mean we are, this is only true for a purely integrating process. So, you have expectation even k minus 1 again here expectation of v k minus 1 given information up to k minus 1. So, now you have the answer you have the given information up to k minus 1 what you expect. So, Saturday for example if I give you quiz marks I will ask you what do you expect there, whatever what you expect, right.

So, I mean you will change your expectation. For expectation of v k minus 1 given k minus 1 is v k minus 1. The second term by definition of white noise is 0, because it does not depend on its past the best prediction is the mean.

Student: (Refer Time: 13:46) it is given v k minus 1.

That is different.

Student: that is like we know (Refer Time: 13:53).

Correct, so when we write like this the notation means you are given up to k minus 1. If you are given only v k minus 1 that would be explicitly indicated.

Student: your written second statement is (Refer Time: 14:09).

Where? I am so sorry, correct. Even then it is correct there is nothing wrong. Sorry, thank you, but even then the result does not change even if your given v k minus 1 the answer is correct, because expectation of e k given v k minus 1 is 0 anyways. Anyway, but thanks, correct. You can tell you friends that you we went through some fascinating stuff.

Anything else, you have some time one more minute before we urgent. We will continue on the discussion tomorrow. Certainly tomorrow we have going to start off on the Fourier part.