Applied Time-Series Analysis
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Lecture – 05
Lecture 03A - Motivation and Overview 5

Yesterday we were talking about the challenges in time series analysis and we did mention the concept of realization and it is very important to realize this concept of realization because that is the concept that makes the analysis of random signals quite different from analysis of deterministic signals. When you are looking at a deterministic signal, whatever you have is the only possible thing that you could have obtained. Whereas with a random signal whatever you have observed is one of the many possible things that you could have observed and that makes a huge difference and at this point I want to make a very important point.

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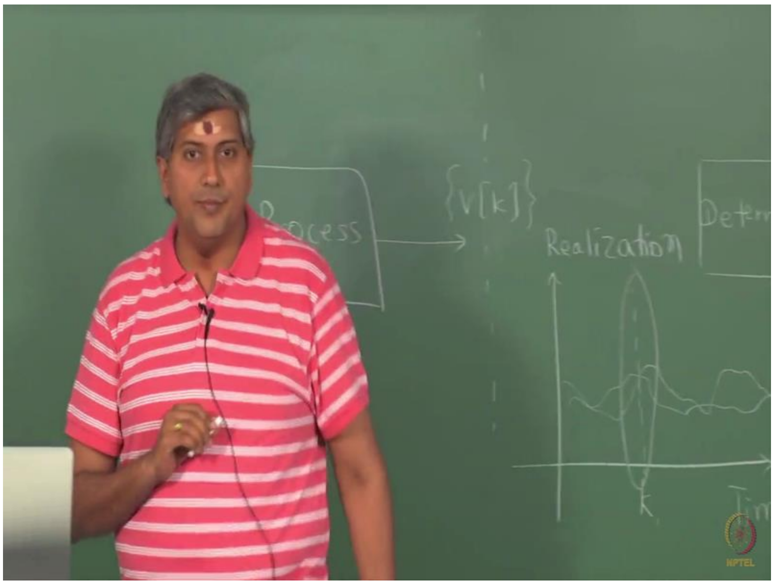

So, yesterday we had this schematic where we said we will represent the random process as a box for now and we observed this signal V K; it is a discrete time random signal and that is a reason why I use a square bracket set, if it was a continuous time random signal I would have used a parenthesis, but we will only deal with discrete and random signals therefore, you can keep the notion of continuous time random signal aside for the course.

So, here is the signal and what we mean by realization is as follows. At any instant K that is in discrete time, we imagine that there are many possible values; it is our imagination why do we imagine that yesterday we discussed this we said that is the natural framework for handling random signals or for handling signals that cannot be predicted accurately. Since I am unable to predict the signal accurately, I list out all possible values at any instant in time and then assign chances to each of those possible values or if it is a continuum then we talk of a density from probability distribution or a density function.

So, it is not that truly speaking that there are many possible values, may be there is only one solution and the process is giving me that solution. Many a times we think wish if that has happened then or maybe I would have gotten that job or I would have gone to this university or maybe that spouse or this movie and so on, we think there are multiple solutions; unfortunately there is only one solution, you are probably you are destined to get only that job, you are only get destined to get married to that person and you are destined to probably take this course only. So because we want a framework for prediction, we imagine that there are many possible values of which I have observed one and that is because of our inability to understand the process very well, we can as well say that it is because of our ignorance.

So, randomness is a very euphemistic term for our ignorance that is how you should actually look at, but we would not like to keep saying I am ignorant, I am ignorant, I am ignorant we rather we blame the process, we say the process is random and that is what leads to the concept of realizations. So at any instant K, there where many possible values of which I have observed only one and now you extend this over a period of time. So, at every instant there are many possible values I observed one of them and now when I take a record of data, I have a combination of many possibilities of which I have observed one and that is what is the notion of realizations.

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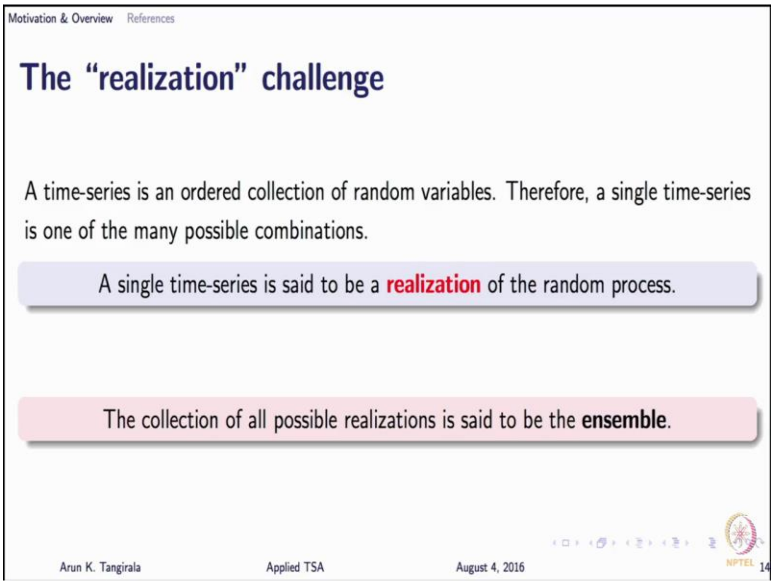

So, realization is just one possible record of data that you could have observed, it does not matter whether it is finite time or infinite time, the fact is when I keep observing a random process in time, I generate one realization and again you should ask why we call this as a single realization it is because if I had used another sensor for example, then I would have observed a different record.

So what we are doing is; we are trying to model that process, but unfortunately there is randomness in the observations that we are actually clubbing together with the process. So we are not exactly modeling the physical process of interest, we are also modeling the measurement process; you should remember that the process may be deterministic.

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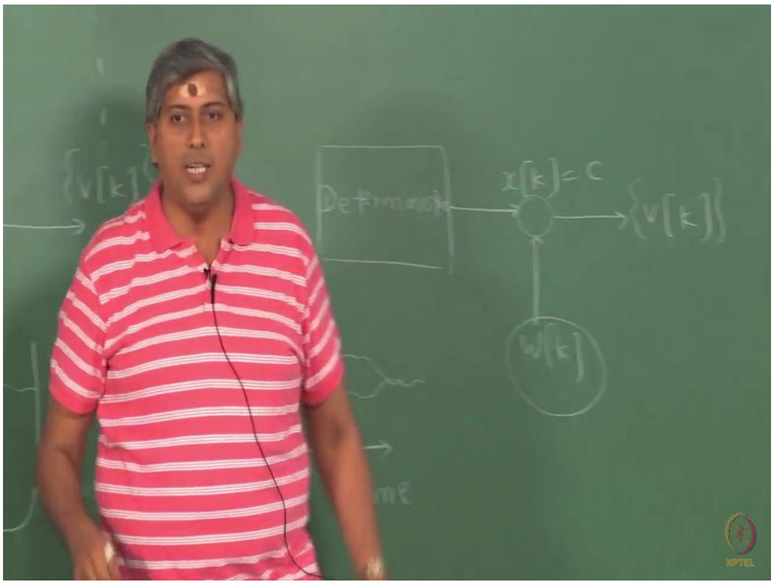

For example here you could have a nice deterministic process like we have been trained to imagine all the time. So, as far as the deterministic process is concerned, it will probably give me some x; k let us called this as x; k it is a deterministic sequence, but I am unable to observe this. The moment I start to observe this with a sensor then assuming that the sensors error or perception adds on to this deterministic signal.

Let us say that is w; k what I have with me is v; k, so I do not know what this is, I set out to observe these deterministic signal with a sensor, but the sensor adds its own perception, its own error; you can say measurement uncertainty and produces v; k. So, to me who is an observer what I have is v; k and it is made up of this deterministic component x and the measurement error w for now we do not really look upon v k as being made up of x and w, but I am just explaining to you the different scenarios that you can run into.

Many a times you perform experiments, even though you may have understood the process very well, but you still collect data; maybe to estimate some parameters that is a usual scenario and all such situations can be explained by this schematic here. You are very well understood this process and you are collecting data to estimate some parameters, what you have with you is v; which is one possible record, have you used another sensor or the same sensor again maintaining everything else the same, that is the operating conditions fixed you obtain another record of data.

So, what is the big deal about this realization challenge I mean where does it pose difficulties is what we should understand. At least briefly for now and the entire course will actually tell you what is the challenge that one faces, but before we discuss that let us also understand the notion of an ensemble, these ensemble is nothing, but a collection of all realizations, all possible things that records that you could have generated. We will never be able to see that, in general it is not possible to actually generate all possible realizations, you may be able to do it on a computer and maybe you know for academic purposes, but in reality it is not possible to have this ensemble with us; nevertheless it is a very useful concept for theoretical analysis and this is something to keep in mind whatever I am going to tell. When we talk of statistical properties of a random signal, we are actually referring to this ensemble; not the realization.

What I mean by this is normally, if you are given a random signal and I ask you what is the mean, what would your answer be. Suppose I give you a random signal and I ask you what is the mean or the average of the signal what would your answer be

Student: (Refer Time: 08:07).

So, that is the golden trap that everyone falls into. It is not the average in time, when we talk of averages of random signals; we are not talking of averages in time. When we talk of averages for a random signal, it is actually with respect to the ensemble. What I mean by this is; suppose I am, I want to define the average or I am looking at the average of v; k, this random signal, I do not walk in time to find the average that is not the theoretical definition at all. The correct way of looking at averages for a signal is to freeze time; that means you stand at any instant in time; t or here in discrete time instants k and walk across all possibilities. So, if you take a random signal as you must have understood now; there are two dimensions one is of course, time or probably space or frequency whatever; is your independent domain in which you are observing and the other dimension is the realization dimension.

One record may give a signal of these sorts and then another one could give me this one and so on. So, when I am looking at any statistical property beat average, variance or third order moment and so on, I should position myself at some instant k and then look at the average of the signal along this direction, that is the correct definition of the any statistical property of a signal.

Now then why are we trained to think of averages in time, the moment we talk of average, we always think of average in time correct. The reason why we are trained to do that is that the sample mean that is so called mean in time is an estimator, it is an estimate of the theoretical average which is in this direction. So, the theoretical averages or theoretical statistical properties are always defined in the ensemble space; in all in the sample space or you can say it is ensemble whereas, estimates are obtained using the observations in time; why because it is not possible for me standing at a single instant in time to generate all possibilities that is not possible if I were to sit down to collect all possible values of the temperature readings at any instant in time that would be not possible.

So the only records that we have is to rely on this record that we obtained in time, that is a practicality and therefore, the question is when I have a single realization in time which is what I will typically have or maybe about three or four in many industries or in certain applications, if the variable is of interest; if it is a critical variable that I am measuring then I would have multiple sensors measuring that variable maybe about 6 or 10 and so on; maybe if you go visit a nuclear reactor, you will see temperature is one of the critical variables, you will see about 6 sensors measuring the reactor temperature, so that if one sensor fails the other sensor recording can be used and so on.

So, in such cases maybe you can obtain 6 realizations or 10 realizations, but that is also nowhere close to the ensemble. So for all practical purposes, we should be asking the question. If I use the realization that is the record of the signal in time to estimate the averages that are defined in this direction, is it valid, when can I do that, so let me actually give you an example.

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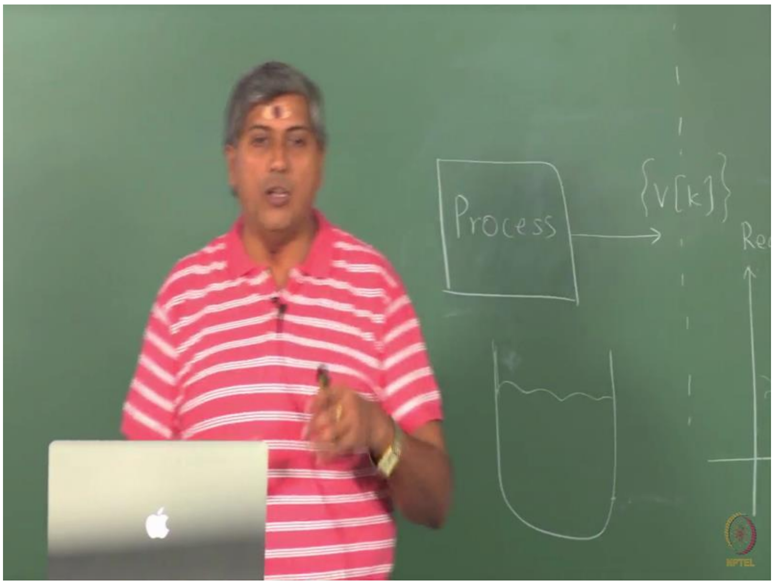

So all of us are familiar with a liquid level system, how does a liquid level system look like, we have seen many a times in our homes everywhere, in the toilets everywhere you see this liquid level system gives us for example, or a classic example of liquid level systems. These are very commonly used in industries which as a flow in and flow out, nothing else I mean it is a very simple innocuous system.

Let us assume that there is no flow out and flow in, it is a very imagine that you are measuring the level of water in a beaker or in a glass and so on. Now what you see on this slide are 3 of the million possible records that you could have obtained for the level reading. We do not have a flow in, we do not have a flow out; assume that the table on which your apparatus is set up is fixed, there is no vibrations, there are no earthquakes at the time of your experiments and neglect evaporation losses; that means, essentially the level reading is fixed. If everything was ideal, then the reading that you should have obtained is a flat line in time, so you have here three plots from three different sensors.

Now, the reality is that is when you look at the measurement unfortunately there are going to be fluctuations and that is because of this w k, x k is constant; in this case we know that it has to be constant c, but I do not see that constant line; what I see is in fact, fluctuating measurement; I know very well that I should not be obtaining this, but I cannot do anything about it, it is the sensors artifact that is producing these fluctuations.

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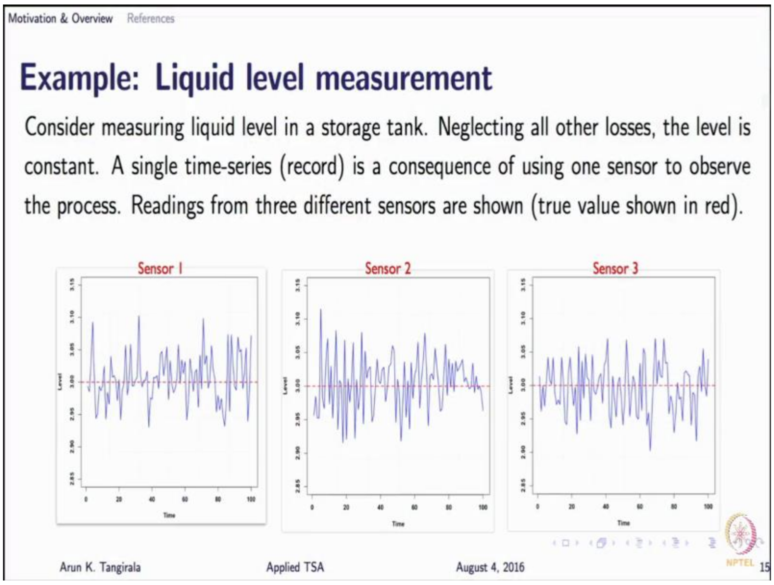

Of course in the simple example, I am able to say clearly that whatever fluctuations I am seeing in the reading are slowly due to the sensor. In a more practical or general scenario, it is not possible to do that; it may not be possible to attribute the fluctuations to sensor or to inherent process disturbances and so on, but this is a simple experiment, helping us understand the notion of the realization.

Now, suppose I want to estimate this see; that means, I want to know the true level which is shown in the red line; I do not know how well some of you are able to see it, but maybe if I zoom in, you will be able to see the red line; let me actually see if I can do that. Are you able to see the red line now clearly? There is a dashed red line for each sensor and that is nothing, but the truth. Let us say I have calculated it, I know it exactly based on a simple observation or whatever.

Now, the goal is to estimate that value corresponding to the red line from the readings that I have given you alright. How you do that, well an intuitive way of doing that is to take the time average of the record that I have and use it as an estimate; it is only going to give me an estimate. So, what we have learnt is a very important aspect and hopefully clear the misconception which is that the averages of random signals are not defined, not only average any other statistical property it is not defined across time, it is defined across realizations. Later on we will come across this notion of an expectation and introduce an expectation operator, I am sure you are now familiar with that assuming that you have sat through the MOOC that I have recommended for this course on statistical hypothesis testing.

So, what we obtained through the sample mean as we call or the average in time is only an estimate of the red line. Now if I use sensor 1; I would obtain 1 estimate, if I use sensor 2, you will agree that you will get another estimate and likewise for sensor 3 and so on.

Now which one is correct, which one is the truth; none of this is the truth or each of this is only going to give me an estimate. Now the reality is that I will have record either from sensor 1 or sensor 2 or sensor 3. How reliable is now you are estimate, ideally what I should be having is all the possible records at any instant in time and then I would be able to get the average at that time. The true average here is invariant with time alright, but I do not have that, now I have to rely on a single realization, the question is how reliable are the estimates that I am going to calculate from a single realization, when can I rely on a single realize estimates computed from a single realization and if I can rely how reliable meaning, what is the kind of error that I am going to incur in using the single realization, how confidently I can draw inferences about the true process, here it is a very simple; even in a simple process like this, it is hard to answer just like that; one needs to have a formal theory.

Later on in the course, we will answer all of these questions in a very precise manner, we will quantify the error that we will incur an estimate, we will learn how to estimate this red line value using many different estimators sample mean or the time average is only one of the many possible estimators. So, the summary is truth is one which is independent of time is a Kallah theorem; that means, it is independent of time, but estimators are many, we will learn in estimation theory what are the different possible ways of estimating a single truth which has a certain definition and this definition is across the realization space or in the ensemble.

Now you look at a deterministic signal on the contrary, there is no such issue at all because simply this dimension is missing. In the case of a deterministic signal whatever you observe in time is the truth, but remember the moment I say whatever I observe these thing comes in. Therefore, any science or any analysis that you draw based on observations is going to be contaminated with some errors. So, all external observations whatever externally that you are observing that is observer is different from what is being observed we will suffer from this limitation and time series analysis or I would say uncertainty analysis is all about telling you or giving you the tools for taking into account such uncertainties in your records and then also going to telling you how this uncertainty in data is going to propagate to the uncertainty in your inferences and that is what I meant by earlier when I said engineer should be well versed with handling uncertainties.

You should know how to quantify uncertainties in data; in your observations and you should also be in a position to determine how those uncertainties in data propagate to uncertainties in estimates that is a very very important thing to keep in mind and estimation theory is all about; it is such a beautiful theory which tells you how uncertainties in your data will propagate to uncertainties in your inferences.

No where you can get rid of uncertainties, but you should be in a position to draw error bounds, to draw uncertainty bounds and so on. Whether it is your modal parameter estimate or whether it is your statistical property estimate, whatever may be the case you should be able to do that and estimation theory gives you all the paraphernalia to do that alright, so let us get back to the presentation mode; yes.

Student: (Refer Time: 20:39).

No it is not, let me reveal the truth it is actually this is simulated data; this is not experimental data, but it is representative what you will see in an experiment. So, this red line is something that I have chosen; that means, I have chosen a value for c and then superimposed some randomness on c. So, each realization that you see whether it is a sensor 1 for example, I have added 1 realization of a random signal and then sensor 2, I have added another realization; the red line is the same for all, it is me who has fixed it but.

Student: (Refer Time: 21:20).

Blue to a red.

Student: (Refer Time: 21:23).

That is what exactly I was saying that is wall estimation theory is about; estimation theory tells you how to get to the red line, it will tell you from finite data unfortunately you can never get to the red line, you will stay very close but you will never get to the truth from finite data. If your estimator is good which technically the term is consistency that is the term that we will come across in estimation theory.

If your estimator is consistent then given infinite data or let us be practical very large data then you will be very close to the truth, you will be able to get to the red line, but that depends on your estimator what I mean by estimator is crudely said the formula that you will use to arrive at the estimate of c. Sample mean for example, is a consistent estimator under some conditions; that means, if I were to look at the time average and I keep including more and more observations in time then I will get closer and closer to the truth, that is a very important property an estimator should have alright, anyway here it does it answer your question.

Student: Also you mentioned it unless we have something around 5, 6 sensor; we cannot measure multiple (Refer Time: 22:38).

Student: We have only one sensor then the same as the reading (Refer Time: 22:44).

Which average?

Student: If you have only one sensor.

Student: If you measure some (Refer Time: 22:52).

Student: Then we will two average (Refer Time: 22:55).

At any instant in time you mean there is no notion of average, you have only one point correct.

Student: So, does not may be that in case we should think of a thickness also (Refer Time: 23:07).

So, that is what estimation theory tells you as to if you use a set of observations in time then you may be able to get a good estimate of this average at any instant in time; you are right. If I had a single sensor, I would have had only one observation and there is nothing to average right that can also be used as an estimate of see for example, here I would freeze time, I would probably standing; now at this point in time and whatever observation I have for v, I would call that as an estimate of c, it will only be an estimate.

But as we collect more and more data from the single sensor, we would like to believe that if I average in time, I will be able to actually get to the average in the ensemble direction, but it is not always guaranteed that I will do with that, there are two things that we will guarantee time averages being suitable representatives of the ensemble average and one property is (Refer Time: 02:32), the other property is the consistency of the estimator.

So, your point is that; suppose I understand the question right, you are saying that if I can actually use some observations in time that is exactly what I was referring to earlier that you would take the time average and hope that is going to give me a good estimate of the ensemble average, but clearly just finite average of finite data, finite length signals; it is not going to give me the truth, it is only going to give me an estimate alright.

So, let us move on of course, unless there are any questions in that hall; are there any questions in that hall.

Student: Sir we are talking about the constant (Refer Time: 25:07).

Yeah.

Student: But that is if there is a (Refer Time: 25:09) then (Refer Time: 25:11) on time. So, if we do not know the result (Refer Time: 25:16) what you will be able to find the average of a single sensor.

It is a good question; we will talk about that in the sense when the truth itself is changing with time. We call that is suppose this average itself is changing with time right then we call such random signals that is the resulting v as non stationary signals. We can start off with non stationary signals if straight away if you want to restrict the audience to a single room, but we will talk about non stationary signals. Gradually it is good to begin with stationary signals, I have taken this very simple example, but it is not too restrictive, you will encounter these kinds of situations quite often. Once we understand how to deal with these situations then we will figure out how to handle non stationary signals.

So, one of the solutions would be for example, to assume that this time varying average is some deterministic function of time, but if you want more complicated stuff; this time varying average itself could be a random signal and that is where things can reach at their peak, but we will not talk about it at the moment yes.

Student: (Refer Time: 26:35).

Yes absolutely you are right.

Student: (Refer Time: 26:40).

Yes, we will answer that question shortly; any other question from that hall?

OK.