Applied Time-Series Analysis
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Lecture – 49
Lecture 21B - Models for Linear Stationery Processes 13

Now, comes the important fact an important point which is the equivalence between that there is any equivalence between auto regressive and moving average models. And we have discussed it, we were already now can sensed it in many different context that an AR model can be re return as a moving average model and vice versa.

And in general if you have an invertible moving average process, you can give a stationary auto regressive representation and vice versa, but obviously not of the same orders. In fact there is now a different way of defining inevitability of moving average models. Moving average model is set to be invertible if and only if you can construct the stationary AR representation. Sometimes you will see this definition in certain text that an invertible moving average model is that which allows you to construct the stationary auto regressive representation.

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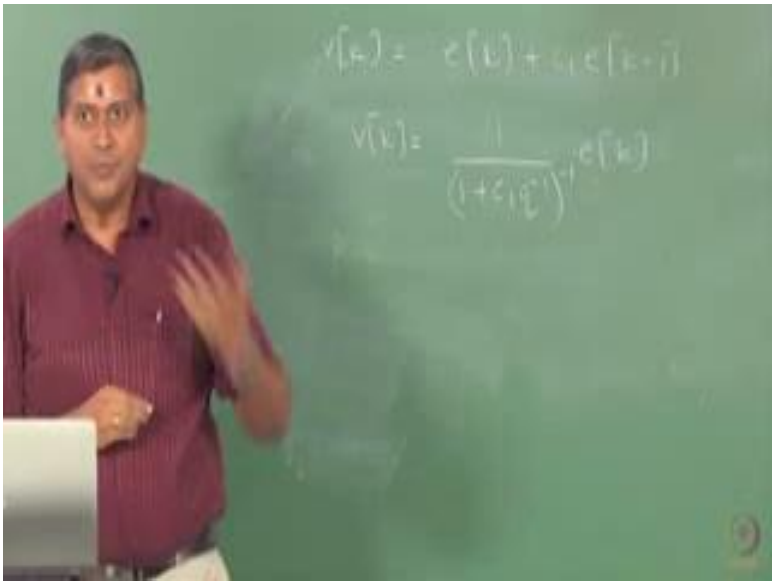

Because we know that the moving average model can be re-written as an AR representation, but stationery. So, if I have a MA one I have v k as e k plus c 1 e k minus 1. And I can rewrite this as an AR in an AR form that is here I have 1 plus c 1 q inverse

times e k on the right hand side I can rewrite this in this form. And straight away see that invertibility implies stationarity, but in the sense here if it is invertible the zeros here have to be outside the unit circle. Now when I look at this here this is not in an AR form right, an AR form would be let me put it this way impact so that you see it more clearly v k equals 1 over 1 plus c 1 q inverse inverse; you can see even at this way, either way is a same thing. If the process is invertible then you can construct the stationery AR representation and vice versa. But obviously, you can see straight way finite order moving average models would have an infinite order AR representation. And a finite order AR representation would manifest as infinite order MA.

You can even imagine this from the ACF view point when we looked at ACF of AR models let us say AR 1 it had an exponential d k, but suppose I did not know that there is there is a class of models at auto regressive models exist I did not know, I stop listening after moving average model let us say in the class. Then I view any process from a moving average view point from that view point place the AR process will manifest as the moving average of infinite order, because it does not really died on at finite lags theoretically. So, from that view point also you can say that a moving average model also an AR model manifest a finite order manifest as moving average of infinite order.

Either way now the question is which model is good; that is the most important question that keeps bugging us. But will come to that shortly, I am going to skip this you are already discuss. Now let us actually look at the mixed effects model first and then ask is come back to this question as to which model is suited. So, in general we need not have pure AR or pure moving average models, there be mixed effects. In the sense that, a process may be evolving as a function of its past and also evolving as a function of the past shockwaves, both. So, what I have today is partly because whatever I have done in the past, whatever my reactions have been in the past plus whatever I have been you know whatever uncontrollable things have been happening to me; e k is something that is unpredictable, I do not know I mean in the sense its shock way. So, there are these unpredictable things happening in my life these into have an impact plus my own reactions to those unpredictable things in the past. Yes, sorry.

Student: (Refer Time: 05:08).

Correct.

Student: (Refer Time: 05:15).

Yes you do, but then what happens is now whether; it is a good question what you mean what you are asking is if I have an AR 1 model for example, v k minus 1 cumulatively contains the effects of all the shock ways. But, now if you look at the architecture of an ARMA model versus a architecture of AR model e k minus 1 let us say v if I have an AR 1 e k minus 2 e k minus 3 there affecting e k all of them through v k 1; that is architecture that you have. So, it is an indirect way of affecting. But, there is this are they if I look at the architecture of ARMA model what it is saying is the shock wave not only in the past whatever past shock waves or not only indirectly affecting your presence, but also directly coming and affecting.

They have another propagation path, and that is possible you can have multiple path ways. There is one path way which is indirectly affecting even now and probably for many instant to come, but then there is a direct path way as well. When I even if you take any physical process and I give let us say there is some earthquakes somewhere, there is an indirect path way, but there is also direct path way may be a delayed one; we do not know. It come back and affect and then when earthquake occurs the building starts to crumble and then may be effects of this path shockwaves. But then something else also the path shockwaves actually are coming back through some other path and affecting your structure, but we take sometime though.

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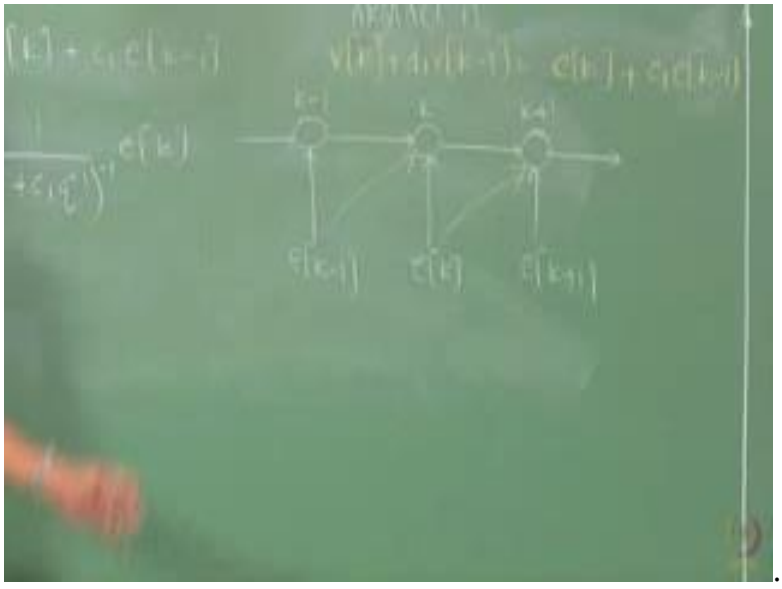

So, this ARMA architecture is different if you at to draw graphically may be that will help you. For an AR 1 graphical representation would be; suppose this is your k and here so let us say its k, k minus 1 and then you have standing at k plus 1. Here you have e k, e k minus 1 and e k plus 1 and so on. So, AR 1 has this kind of a graphical representation. This is a chain of connections or path way you can say. Whereas an ARMA suppose an ARMA 1 1 in interest of time I am not going to re draw suppose you have an ARMA 1 1, then what you would have is in additional path way directly affecting, so e k minus 1 directly affects k. What is an ARMA 1 1 the model would be v k is v k minus 1 or v k plus d 1 v k minus 1 equals e k c 1 e k minus 1; I reach the border. Now there is a direct path way; and so on e k would affect e k plus 1 directory also. So, there are two path ways right. That is a ARMA architecture for you. And MA architecture would not have these connections; it would not have these direct connections only this connection that is it. This will always we present although we have pulled is out again I should keep saying this all the way pulled is out this is internally a part of the process itself; just for the sake of modeling and representation we pulled them out. Is that clear now?

So, that is the difference now between. And how do I know now, how a now do I know how the process is wide inside. Nobody knows and that is it beauty of it, that is as I always says good thing that nobody knows. I means that how you have may have hundreds thousands of papers and methods and so on. So keep guessing; that is what God says keep guessing does not matter. It will and you lively wood and keep moving on. Even at the end of our life we would not know. May be in the soul form we would know not in the physical form. Anyway, so what kind of ARMA models do we admit that have both invertible and stationary.

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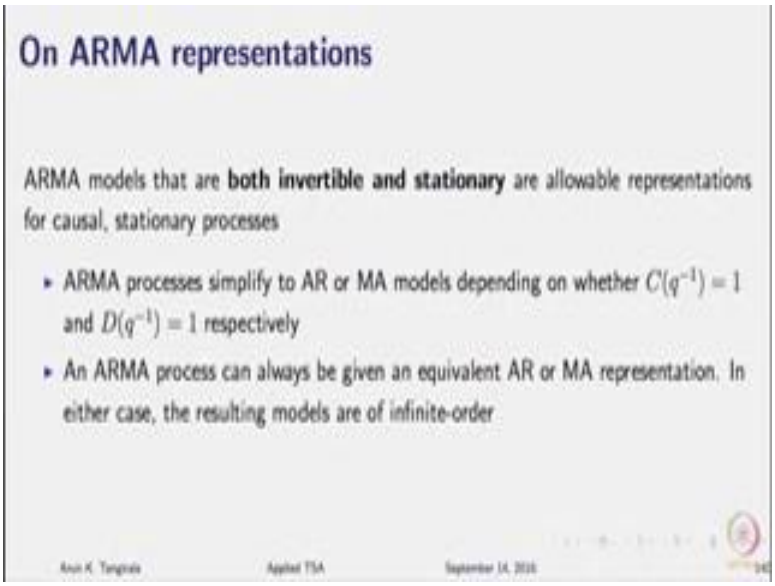

You should understand now these two requirements; stationary obviously is very obvious requirement, invertiblity you can understand is a requirement because there is moving average component and when you have to write the forecast equation for example, for an ARMA 1 1 here. The forecast would in involve e k minus 1. And again I go through the same ordeal of recovering e k or e k minus 1 from the past observation. And once again the requirement of invertibility will crop out.

So, we only admit ARMA models that are invertible and stationary. In other words even though the process must have been generated by noninvertible one I will identify invertible and stationary ARMA model; which means now the zeros and the poles have to be in some region. If I say zeros and poles in a classical sense that is in the sense of z then they have to inside the unit circle. So, when you are given an ARMA model you should check upfront if it is invertible and stationary by examining the poles and zeros.

Now what about the ACF of an ARMA process? Again we are asking this so that in practice I can guess if an ARMA model is suited. We have learnt already how to figure out if an AR model and MA model is suited. Is there a way I can determine whether the ARMA model is suited? Unfortunately you cannot, even I mean I am giving you the procedure for calculating ACF but this is practically of no use, only it is useful in calculating the ACF theoretically like your ARMA ACF for example. Use as this procedure for compute the ACF if you at intuitively ask how the ACF would look like for

an ARMA what is your answer, how would it look like? Remember there are super positions of effect.

Students: (Refer Time: 11:55).

So, by moving average model; the moving average component will stop contributing after the certain lag and then the AR component takes over. And that always results in an exponential value. So, if I have an ARMA 1 1 up to lag 1 you will have both moving average and auto regressive effects contributing, beyond lag 1 its only the AR component. And now you can extend this to an ARMA p comma m model, but what is it I mean when I look at the ACF it is not going to help me any ways; I cannot say- yeah after this lag I see an exponential d k forget it you no way. This behavior itself is complicated and an on top of it you will have estimation errors keep creeping in.

Student: (Refer Time: 12:45).

It is all theoretical, everything is ideal; idealism is good for dreaming. You are right, at some point you may see suddenly exponential d k setting in. But how good is the transition theoretically itself depends on the models of MA, AR and values of the coefficients and so on. And to top this, like in your pizza you have toppings you have an estimation errors which really dominates and that depends on the sample size that you have. So, there is no way you going to observed this with any distinction it is not worth pending that effects, it is not at all worth.

So, all these theoretical things that we are study at least for ac AR and MA it is pretty clear, I can get a very good guess. But when it comes to ARMA there is no way my ACF or PACF is going to be give me any good guesses, because it is so difficult to dissent angle the contributions of deposition couple the contributions of moving average and auto regressive. So, what do we do in practice? Trial and error, but in a more systematic manner this course is not to teach you modeling on trial and error bases, but in a systematic manner. And we will go through an example later on I will show you how to fit a model time series model systematically, but let us ask this question.

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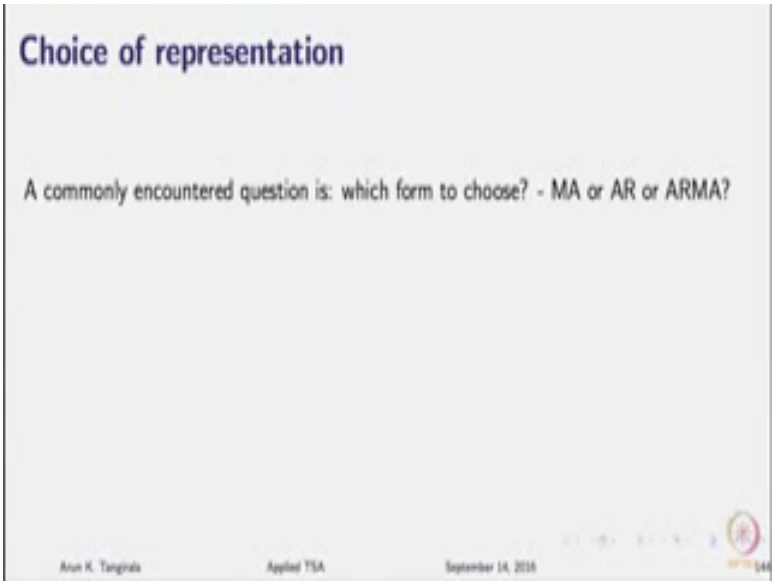

If already talked about the order; let us ask is question on which model do I choose: AR, MA or ARMA for a given series, I give you a series you pick any of this series that you are seen before which you being to be stationary which or is a stationary. We are not talked about stationarity test let us say that you are given at a stationary, which model would you fit?

Student: (Refer Time: 14:41).

Both are infinite; let me tell you in practice nothing like that you want to see. Only for few select processes you may see all that nice things. Somebody else are some other answer; you will pick ARMA.

Student: (Refer Time: 15:05).

Zero; you hoping that will happened, so you start away straight away with ARMA over parameterize and see if it works. Do you understand how painful it is to estimate an ARMA? The moment you have a moving (Refer Time: 15:26), painful in the sense that today completed as does it we are not going break a sweat, in fact if you are sitting AC not given drop or sweat. But, you will not get unique optima and there are issues still there are efficient algorithms for estimating ARMA models, but then why you know why study all of this, if you are going to start off with ARMA why study any of this. So, there are some guidelines we cannot give theoretical answers has to- yeah this is a metric or

this is indicator nothing like.

As you says you look at ACF and PCF, that is a first step always. Now you see if you were to fit a MA model what would be the order? If you were to fit a AR model what would be the order? That mean and the order will tell you how many co efficient you are going to be estimate. If it turns out and if there is no preference for a particular type if you turn out that an AR model of order one or two can explain your series go ahead with it.

That is it an MA which may take a few more coefficients. And we know why we prefer AR models because the predictors are linear and we learn later on that with least square approaches even the Yule Walker method you seen very easy to estimate. Even recursively I can estimate AR models over a sweeping; you know I can just sweep through a set of orders to figure out which AR model is good, in other words I can look at PACF. With MA that is not the case I cannot estimate order just like that there is no recursive algorithm.

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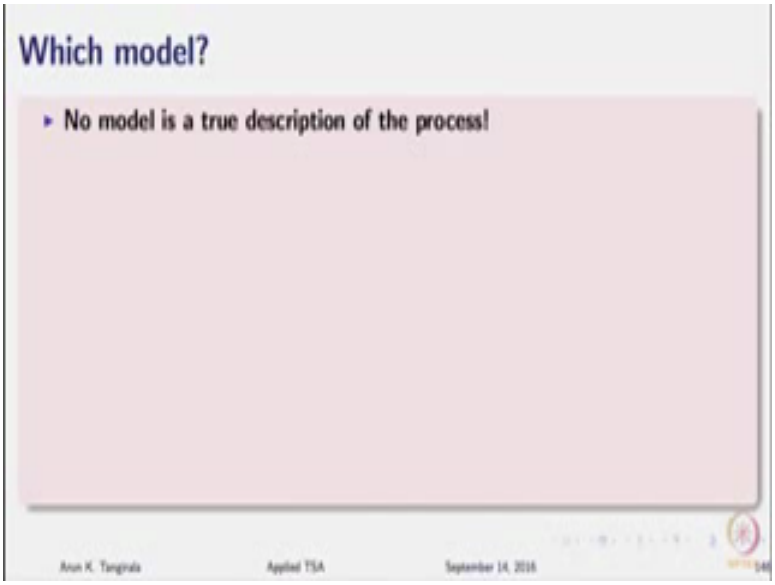

So, there is definitive answer. And the most important thing is there is no model that will give you a true description of the process that is all a myth. There is nothing like a correct model, we are only seeking or working model. And what are the principles which we want to build over model? One of the most important principle is parsimony be stingy; if this is one place where you want to be really miserly. You want to keep the

model as simple as possible. So, this basically says that in modeling keep things simple, since no model is going to be actually correct, why are you actually running after complicated models when a simple model will the job for you, but you cannot keep it simple then simpler, I mean you cannot simplifying and I will not model anything do you want a make to keep things simple.

So, what it says is there is always a trade of involved as you keep increase in model complexity, yes you will get better and better fits that at some point you will start over training the model, will start remembering all the numbers in assignment questions also. You do not want that, you only want the concept to be understood, leave aside the numbers for the specifics. So, if an AR model of a low order manages to explain then so be it. In fact, generally when there is no preference for a particular model one would fit an AR model because it is easy to estimate.

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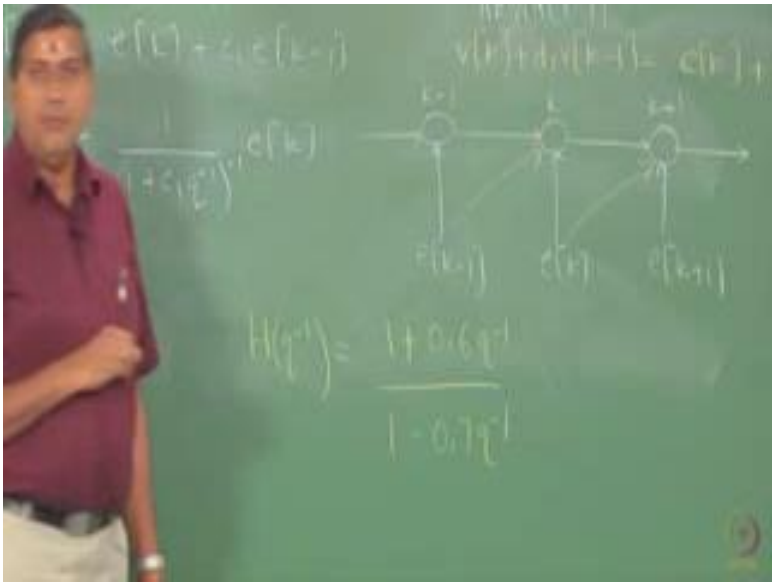

On the other hand it can so happen for example, if you have a process like this the two process is an ARMA is of this types 1 plus let us say 0.6 q inverse over 1 minus 0.7 q inverse. If we look at and you can try it out ARMA ACF allows you to compute the theoretical ACF and PACF, you will see that ACF if you were to look at fitting in a MA model it will be a high order; high in the sense related to word the 2 1 is. And so for the AR model as well. But there you have to know as you said you know if the ACF d k slowly PACF d k slowly, so maybe this PACF for this will actually die of only after lag 4

or 5 when you may want to consider and ARMA point, and that point you want to consider an ARMA 1 1, because an ARMA 1 1 has only two parameter. For this process and ARMA 1 1 will do the job for you (Refer Time: 19:33) v and AR 5 which will involve 5 parameters. From an estimation theory view point and this you will learn later on, more the number of parameter larger the error in the parameter estimates. And that is another reason also why parsimony is preferred.

So, always choose that model description which contains few parameters without compromising significantly on the quality of the predictions. And that is the basis several information theoretical criteria like (Refer Time: 20:05) information criteria and so on. Many a times what can happen is an AR 2 an ARMA 1 1 can actually do the job for you or the same number of parameters may prevail. So, what do I do now? If the number of parameter is only the guiding principle what do I do, and that is where the information theoretic criteria comes and tells you although you are fit same parameters; you can look at the estimation errors in these parameters, because in estimating ARMA model we may have use these non-linear least squares problem you may get a local optima. Whereas, in estimating in AR model of the same number of parameters right or does not matter how many parameter you will always get a unique optimum because it is a linear estimation problem. The parameter can be estimated using a linear least squares or Yule Walker equations.

You prefer in AR model in that case. So, there are some guidelines and that is what we learn through different case studies. And I will go through or a few case where in all of these over parameterization has to be washed out for. That way according to your approach actually I can go head and fit an ARMA 20 20. And then go back track but then you would not know how many you have to turn out to turn to be zero, because sometimes a situation can be tricky. When you come back then some may be non zero some may be zero and significant and so on. So it becomes very difficult always in modeling you want the bottoms up approach, not the top down. You do not want to the highest order and come back. You want actually, that shows your understanding of the process and that shows that you are being careful when it comes to modeling.

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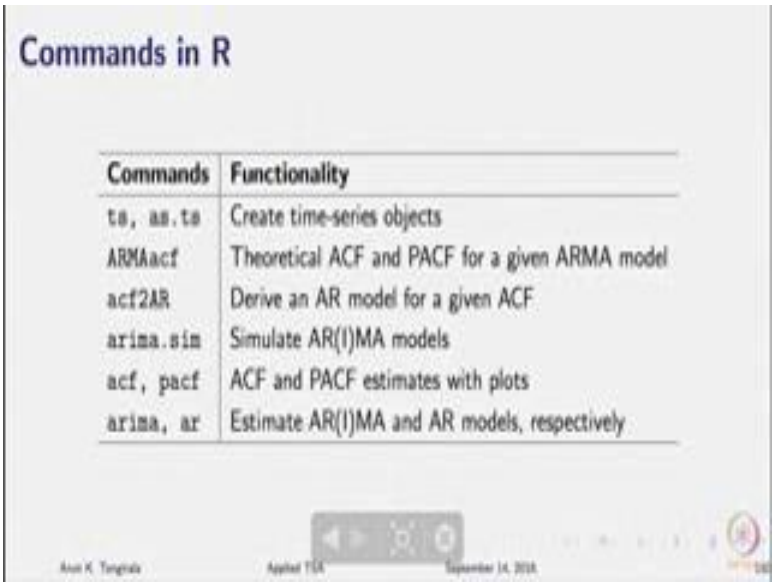

So, keep that in mind. These are some of the relevant comments R, whatever practical guidelines I talked about their best understood by way of implementation.