Applied Time-Series Analysis
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Lecture – 45
Lecture 19C - Models for Linear Stationary Processes-9

Now, that is an alternative viewpoint that we have discussed in the last class that if I want to recover e k, I look upon the moving average model of order 1 and AR 1 for e k where v k is being driven, remember we this is our original schematic we assume that the process is actually being driven by e k, this is the process generating v k and we assume that this has being driven by white noise.

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But now if I have to make a stable forecast from v k, then I need something some device which will actually take in v k and generate this; do not think that they are in a loop and just saying that this is something that I implement in parallel and; obviously, qualitative speaking this as to be in some sense and inverse, do not think I am taking the physical system and actually dumpling it over or anything it is a system mathematical inverse.

In fact, you can look at this for those of you who work in probably speech processing or a or a field called source separation, which is a fantastic field in itself in speech processing in kinomatrix and so on, where you are given mixture measurements and the classic problem another classic problems that is presented in source separation is there are a few people sitting like you know you are all seated here and I have some 100 micro phones recording whatever we are speaking, and we are only given the recordings and we are suppose to figure out how many people are sitting in the room.

So, that is the problem of source separation; these 100 micro phones actually pick up this speech signals comings up from that many speakers in the room and we assume a certain mixing model that is the speech signals are actually mixing in a certain manner at the point where the micro phone is sensing the audio signal. So, you come up with the mixing model and you assume a certain mixing model and you use the mixture recordings readings and then you figure out what the speech signatures are, how many speakers are there the simpler problem is I am given how many speakers are there like m is given for example, and I figure out what the source signatures are.

In fact, if you look at the model carefully the moving average model carefully, it is nothing but a mixing model. So, you are mixing the white noise sources from the past and producing v k right. So, it is the mixing model, what you are doing now to recover e k is de-mixing. So, you can think of this inversion also as de-mixing; there are so many nice perspectives depending on what you are comfortable with.

So, coming back to the point, what is moving average model for v k is actually an auto regressive model for the recovery of e k and we know straight away that any auto regressive model is a will produce a stable in the sense stationary signal at least for AR 1 we know that the C 1 as to be less than 1 in magnitude, this is another way of arriving at a same result very good.

Now, there is now it is formal set up. So, we have looked at two different perspectives now we want to extend this idea; essentially what we are saying is you will have multiple models when you estimate an MA model, but you will have to pick that one which is so called invertible; invertible meaning that, which generates a stationary e k, that inverse which actually generates stationary.

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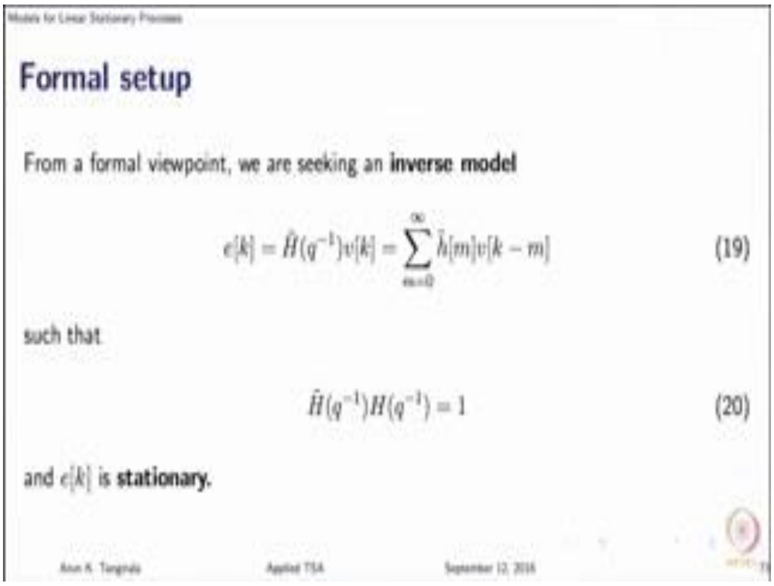

So, this process is what we have actually denoted by H of q inverse and now we are seeking some kind of inverse H tilde for now call this as H tilde of q inverse and this H tilde should be such that when it operates on H it should yield 41; that means, it should exactly recover e k.

Let us look at this equation here we have, v as H e; H operating on e and I assume that e k is H tilde v k. So, when H operates on H tilde I should get this as unity. In fact, it should be the other. So, what I show on the screen is H tilde times H equals 1, but that is also because this operators are actually commutative; that is q inverse operating on q should get me 1, q operating on q inverse should get me 1. So, whether you write as H tilde operating on H or H operating on H tilde it does not matter, they should come out to be 1.

You will think why I have written this way I could have straight away written this as H inverse? I mean I could have written H tilde here as H inverse itself right why I have not written this? Because it is not so straight forward to assume that simply an inverse of H will get me 1, what one needs to do is you have to actually formally show you have to formally show that sorry that H tilde is H inverse and I am just avoiding that proof the way you show that it is formally H inverse is you go back to this series that you see in equation 19, which is in now you have written e k as an infinite combination of the present and the past signal.

But now the coefficients are H tilde; using this H tilde and using this summation here and then using the fact that H of q inverse is described in terms of h, I will have to do some formalization there, I do some multiplication and eventually show that H tilde are indeed the coefficients of the series expansion of H inverse right.

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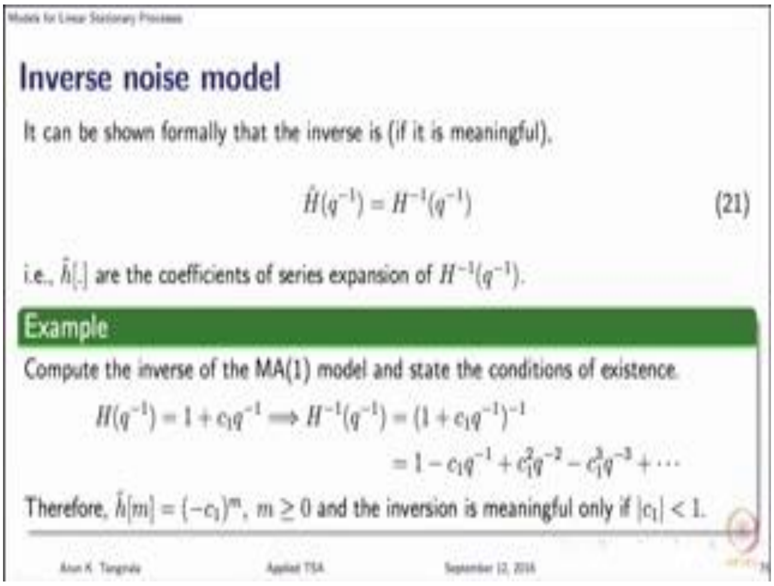

For example here when I take an MA 1, H of q inverse is 1 plus C 1 q inverse and H inverse that is a series expansion of H inverse is as you see on the screen, as you have done also previously in a class 1 plus C 1 q inverse and you have 1 minus no C 1 q inverse, plus C 1 square q inverse square and so on; one as to actually formally show that your H tilde that you have for e k is H tilde of 0 is 1, H tilde of 1 is minus C 1, H tilde of 2 is C 1 square and so on.

But we are avoiding that you can take it that the H tilde is nothing, but H inverse, provided it exists what we what we mean provide it exists is that when I use that H inverse, it should produce a stable e k, that is what we mean by provided it exists. So, the summary is essentially your H tilde is simply the inverse of H and assuming that it exists. So, this is how you figure out the inverse of any time series model and as you can see clearly even in the inverse, the leading coefficient is unity. It as to be because the leading coefficient in H is unity and if H is a finite order like your MAM, your H tilde is infinite order right suppose H was auto regressive, what would be the inverse?

Student: Finite.

It will be finite right it will be simply 1 plus d 1 q inverse, but if you cast the auto regressive in a moving average form which we have done before once you run into an MA infinity process right. So, when in general this is true; a finite order moving average process manifests itself in the inverse domain as infinite order AR process and vice versa, but we will talk about that a bit later right now let us talk about inversion. So, the formalization is that the inverse should exist and if it exists then we say that the model is stable.

Now, before we state the formal result what we mean by inverse existing and the inevitability? Let us also take some effort in rewriting the prediction equation in a formal manner in terms of the inverse. So, here earlier if you will go back to the previous slide, we wrote this prediction equations specifically for MA 1, but now if I am given a general H how do I run the forecast equation and the way you do that is using this a notion of inverse.

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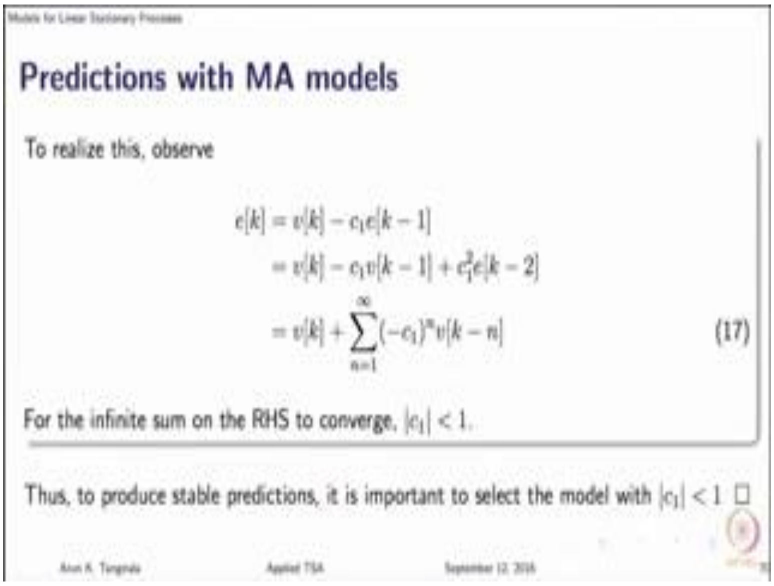

So, what you do is, you say v hat of k given k minus 1, it does not matter whether you have v at k plus 1 given k or v at v hat I mean whether you are predicting k given k minus 1 or k plus 1 given k does not matter.

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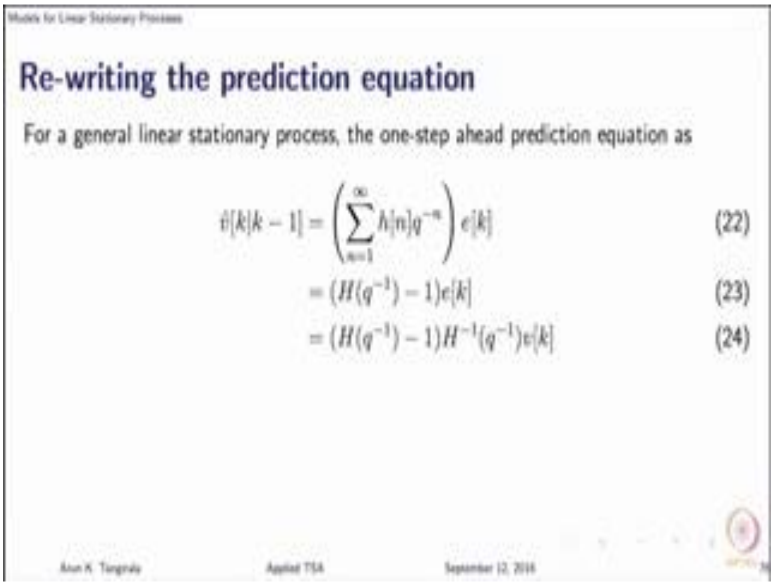

So, what I have a done here in writing the prediction equation for v k given k minus 41;if you look at equation 22 what I have done is, I have thrown away, what I have thrown away? The e k term right because that is unpredictable given any past; what is left with me is this sigma H n q to the minus n, n running from 1 to infinity instead of 0 to infinity operating on e k, but that part is nothing, but the H that I have minus one remember that my H is actually in a general for a linear random process, H of q inverse is 1 plus sigma H n q to the minus n, where n runs from 1 to infinity; this is my H for a for a linear random process.

So, straight away this identity comes up. So, all I am doing is I am instead of that summation I am substituting H of q inverse minus 1, this is plane algebra there is nothing magic there. Now what I do is in place of e k. So, ultimately I what do I want? I want a forecasting equation in terms of the model and in terms of the data that I have, this e k is only an intermediary. So, I have to get rid of that e k and e k is nothing, but H inverse v if it exists right. So, now, I make use of the properties of H and H inverse and ultimately what is expression that I get?

Student: (Refer Time: 12:30).

Sorry what do I get for v hat of k given k minus1? Since I am asking to complete that; what do I get time something then we will look at the width of the bag parenthesis and that is how it should be standard coaching staff sorry?

Student: V k 1 is H inverse.

V k minus H inverse that is in

Student: On v k.

So, correct. So, how do we write is, 1 minus; does it look I know it is v k, but this is ok. So, is that equation I am predicting v at k.

Student: (Refer Time: 13:42).

And on the right hand side I am saying I want I need v k, something strange is not it, some trap, somebody needs a phone a friend.

Student: (Refer Time: 14:00).

Then I can see somebody is sending sms and finding out what is the answer.

Student: Right side on is the

Right side one is the no left hand side is the predicting one, right hand side is your available signal.

Student: (Refer Time: 14:18).

V k as?

Student: (Refer Time: 14:22).

Then what happens?

Student: Strictly cancel.

Sorry.

Student: So, the one inside v k was v k cancels (Refer Time: 14:40).

Yes.

Student: The leading coefficient, the leading coefficient of fetching is also once. So, the right hand side actually (Refer Time: 14:52).

Very good; this is a independently arrived answer yes very good thank you. So, that is the main point here the leading coefficient in H inverse that is why I emphasized earlier, just as the leading coefficient in H is unity the leading coefficient in H inverse is also unity. So, although it appears a bit awkward the equation looks a bit awkward, it is end; it is just a compact way of writing your forecast expression if I were to write in an expanded form then I have to bring in depending on what H is H inverse may be finite or infinite I mean in terms of summation.

So, instead of breaking my head on that, we should generally we leave it at that depending on the scenario either this 1 minus H inverse would lead to an infinite summation or a finite summation, but regardless v k does not participate on the right hand side; my the first term that participates in the equation is if at all v k minus 1 then that should be v k minus 1; maybe that is also not there it depends on the model, but that is the first term that you can get at most. So, that is something that you should keep in mind and you I expect you to remember this is like one of the golden equations for forecast, given a times series model you should straight away remember this as an expression for one step ahead forecast.

When we talk a forecast later on we will talk a p step ahead forecast and so on say sometimes you want to make forecast p steps ahead; like in a game of chess I want to predict the next 3 moves not just say 1 move, this is for a one step ahead prediction.

Now, given any time series model that is invertible, you can use this expression for calculating your forecast theoretical forecast right. I mean later in practice what you would do is, you would replace this H which it is estimate and you would replace v k with it is realization keep that in mind and now we come to the concept of inevitability and formalizer: any stationary that is the model for any stationary linear stationary process, is set to be invertible if all the roots of H of z inverse and we say at this moment we look at H of z inverse as a numerator kind of model, but when we move later on we will see when it comes to ARIMA models, H of q inverse can have a numerator and a denominator, in that case we will we will replace this roots with the zeros.

So, the zeros of H of z inverse should all be outside the unit circle if you are looking at in terms of z inverse or inside the unit circle if you are looking at in terms of z. Normally when we use the word zeros, it is usually calculated in terms of z that is a system theory convention, in which case this condition translates to the root zeros being inside the unit circle, but if you say roots of H of z inverse there is a choice that you can make whether you are expressing the roots in terms of z inverse or z. If you take for example, the book by (Refer Time: 18:07) they do not talk of z inverse of z, they use this backward operator b I find that convention a bit backward.

So, we I used q inverse, but whatever I mean some other textbooks use l for a lag operator, straight away they specify in terms of the b or the l which would amount to saying in terms of z inverse and therefore, the conditions would read that the roots have to be outside the unit circle; this unit circle comes about because the roots can be complex value you should understand that.

Only when it comes to MA 1 model, there is only one zero and therefore, that straight away translates to a restriction on the coefficient, do not for heaven sake ever assume that inevitability is actually a restriction on the coefficient, it is say restriction on the zeros or the roots which are a function of the coefficients, in other words do not assume that if I give you an MA 2 model, the condition of inevitability is that mod C 1 is less than 1 and mod C 2 should be less than 1 this is wrong; you have to set up the equation for calculating the zeros and then look at the roots and if you calculate the roots in terms of z inverse, those root should be greater in 1 in magnitude, if you are calculating root in roots in terms of z inverse.

Now having said this there are at least going to be a few students who will make this wonder in the final exam, which ever exam because you will see a similar condition for stationarity of auto regressive models also, there we look at the denominator of h, but that is for stationarity; here this is for inevitability, the conditions of inevitability are different from the conditions of stationarity. So, this is a story of inevitability and all the formalization surrounding it and in that process we have learnt this one step ahead prediction expression.

So, to summarize which means you know that the class is coming to a close that is your prediction; let us see if your prediction works out correctly what are the inputs that you can take whatever I have said and the time there are inputs that you can take to make a prediction.

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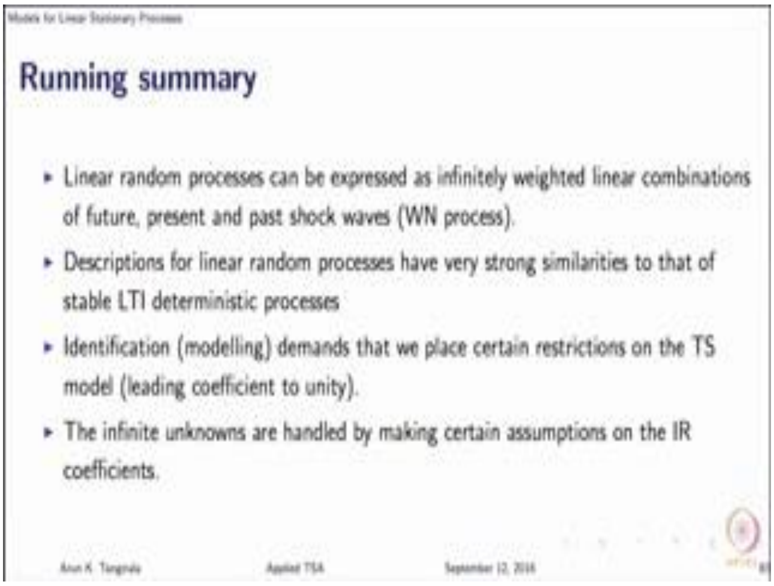

So, the to summarize linear random processes are can be expressed as a linear combination of the past, future and a present and future shock waves which are nothing, but your white noises, in some schools of thought it is an IID process and the description for this linear random processes have very strong similarities with that of the LTI deterministic processes and when it comes to identifying this models we are forced to play some restrictions; and one of the restrictions that we placed is H naught should be 1 right which is not the case in the deterministic world and the infinite unknown problem is over come at least in one way by assuming that the impulse response coefficients actually died on after a finite number of lags or whatever finite number of instance and that leads us to the moving average models and the MAM processes are equivalent to fir models, in the deterministic world in the stochastic world we figure out if an MAM model is suited by looking at the ACF, which also tells us what is a good guess for m in practice and among the and other fact is that when we sit down to mod fit an MAM model to a given process or a series, we usually run into generally we will run into multiple models and we pick the one that is invertible.

So, when you use commands like ARIMA in r which estimate the moving average models for you, there is this score within that routine which picks the invertible part and returns to you, it does not put that burden on the user it straight away gives you an invertible part.