Applied Time-Series Analysis
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Lecture – 43
Lecture 19A - Models for Linear Stationary Processes-7

Let us begin. So, what we will do today is actually go over this moving average models bit more in detail and also get started an auto regressive models, but before we do that will go back to our linear random process the model and talk about the identification viewpoints; remember one of the things that we said is apart from many other restriction we said we will fix the leading coefficient to unity and let us talk about that first today and then move on to moving average models talk about invertibility a bit more in detail we did a very (Refer Time: 00:54) discussion on the day.

So, it will do a more discussion more at leisure today relaxed one. So, if you go back to the linear random process model that you must have written at least several times in your notes by now, as why as I said in the class last time said as far time series modeling is concerned, what I am given is an observe set of observations of v k and what I am suppose to find out are this coefficients here and sigma square e right.

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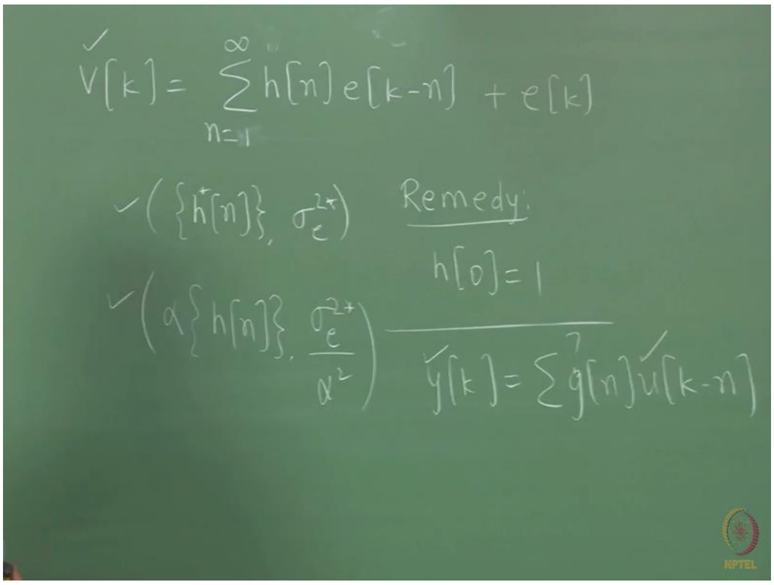

So, let us restrict ourselves to causal models here and as we discussed in the last class we have infinite unknowns and so, (Refer Time: 01:45) we will come to that shortly, but having said that let us say somehow whether it is a through moving average model or auto regressive model or through some procedure we have a way of estimating these unknowns h and sigma square e, clearly there is a problem of non uniqueness here. What we mean by that is suppose there is a solution which gives me; let us say h star and sigma square e star as one of the solution. If this is a solution then there is also another solution completing with this, which is equally it is stands an equal chance of being a solution and that is alpha times h of n and what would be sigma square e accordingly?

Student: (Refer Time: 02:47).

Sigma square e by.

Student: (Refer Time: 02:53).

Alpha or alpha square or square root of alpha, alpha to the power 3 by 2 non of these that is how we are used to, come on I gave it in a favorite format multiple choice.

Student: (Refer Time: 03:12).

By?

Student: (Refer Time: 03:17).

Sure, sigma square e by alpha square not by alpha good that is correct. So, sigma square e star by alpha square because if you scale the coefficients by alpha, it amounts to scaling this by 1 by alpha and therefore, sigma square e also proportionally gets scale by 1 over alpha square and this is true for any alpha non zero value of alpha. So, I do not know therefore, which is a solution and we do not want to be in such a situation where I have one time series model and there are infinite other possible candidate models; we want to have a unique solution and therefore, we have to do something apart from the fact that we are estimating h of n and sigma square e, we have to do something and therefore, one solution is to set h naught to 1.

So, this is only one remedy, but this is not the only remedy please keep that in mind; if I fix one of the coefficients then I get read of this situation here, if this is a solution then this is also solution in general, but now when I fix h of 0 to 1, I am excluding all the other possibilities and I am only considering that possibility where h of 0 is 1; why cannot it be 2? It can be it can be 1.05 also does not matter, but one makes a lot more sense because the moment I say h of 0 is 1. Then I can rewrite this here as summation running from 1 to infinite plus e k, which fetches us the same interpretation that we have been living with until now which is that e k is a unpredictable portion of v k. If I choose h of 0 to be 2, I would have 2 times e k, but I can always observe that and again come back to e k. So, it is better to set h of 0 to be 1, this is a somewhat semi rigorous way of explaining why h of 0 is 1; a more rigorous explanation can be given through spectral factorization and so on, but we will not go into that right now.

But this is the major issue that we have as compared to the estimation of deterministic processes. As I said if you look at the deterministic world, we have this convolution equation, but the difference is this is known and this is known, all I have to figure out is what are this impulse response coefficients and since the input is fixed there is no ambiguity about g, I cannot say if g is a solution alpha g n is also solution that is not possible because input is unknown, here I am running into this kind of a situation and we are fixing the situation by requiring that h of 0 to be unity.

In other words the linear coefficient of my time series model will hence for be always fixed to unity and as I said the nice interpretation as a consequence of this is, I can always see the white noise appearing as an integral and indispensable component of v k, which I will be never be able to predict given any amount of past. So, the white noise stress that we talked about earlier they tell us if I have a scope for including this term in the model for v k right; if there is predictability then; that means, there is something to do here I have to take care of this term, but if the white noise stress comes out and tells me know the series as white noise characteristics then that is it I mean I can go home and live happily ever after I do not have to do any modeling right, but we want to build models otherwise how we will be get a salaries when we get placed. So, we want to know now how to model this any questions on this now. So, I hope you are clear why we fix this coefficient to unity purely because otherwise we would have non uniqueness issues fine.

So, have been fix this, now we move on to the estimation of h and sigma square e and as we discussed in the last class we have infinite number of unknown to estimate, even after fixing this of course, and this is the prospective that you will not probably find in any time series test because that was not the prospective with which people proposed moving average or auto regressive models, the approach was completely different for example, we have seen how an auto regressive model takes birth right, we started off with prediction equation we said I build a linear predictor using some p past observation and then straight away I get an auto regressive model and that is the standard presentation that you will see in every textbook.

But this is a completely different and a fresh prospective on time series modeling, which is purely from a practical view point I have to estimate infinite unknowns and of course, goes without saying I have to estimate sigma square e; that is only one unknowns does not matter always in a time series model I have to estimate. So, the issue is this of infinite unknowns and this problem arises even in the deterministic world, where I look at the convolution equation there are infinite number of impulse response coefficients those g is that I have to estimate and it turns out that whatever strategies one adopts in the deterministic world is pretty much same strategy in fact exactly the same strategy, everything actually really carries forward here as well, that is no difference at all but you do not see this connections generally in time series takes because most of the authors who write time series text are only concerned about stochastic processes to begin with and therefore, efforts are not made to actually connect the deterministic worlds even though they may be aware of it.

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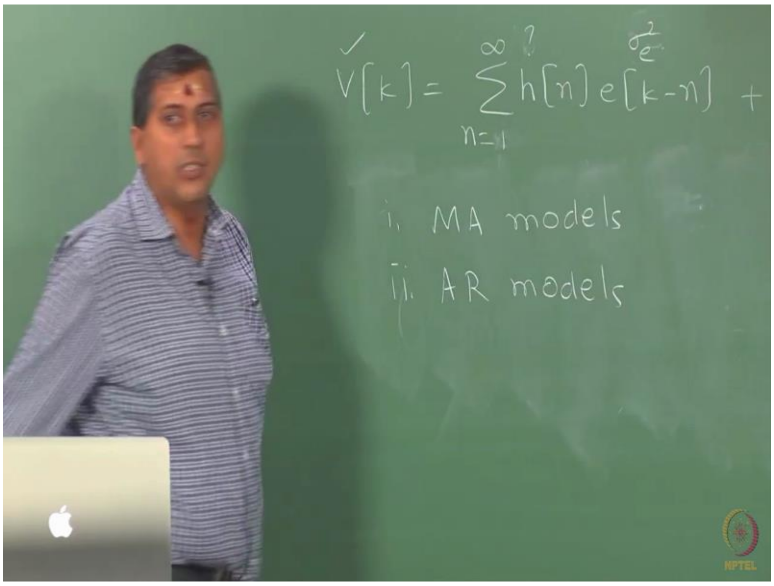

So, there are two approaches for us here; what are the two approaches? One approach is to assume that the process is characterized only by a finite number of h. So, somehow I have to get rid over or overcome this problem of infinite unknowns, the worst thing is a very straight forward one, I assume that no the model may say there are infinite unknowns, but there are processes how there where I do not require this infinite unknowns, it is such that only finite unknowns are present because this is a very generic model not all processes need to described using infinite impulse response coefficients.

Now, when we adopt this strategy or this assumption that a process is describe by finite number of impulse response coefficients, we are led to the moving average model; and in the deterministic world we are let to what is known as a finite impulse response model that is fir model. So, whatever is an fir modeling in the deterministic world is the moving average model in the stochastic world; the approach is same the assumptions are the same and so on, but keep in mind that we are dealing with stochastic processes.

So, as you see on the screen, the assumption that we are made is h of n is some constant at each n, up to some lag m and then zero there after abruptly it goes to zero. Before we proceed we should ask is this possible, is this possible that a process can be like this, it is possible there are many processes where the correlations only last may be 2 or 3 or may be 10 observations depending on your you know what is you sampling interval, but beyond that there is no effect. In fact the effect becomes so negligible that you can as well treat it as a model like this.

And of course, you have this other class of processes, where the effect is not that is a correlation is not dying down abruptly, but rather exponential like many of our behavior may be if you look at the human behavior, there are quite a few things that continue to effect from our childhood, not as much like I got some you know in my seventh standard I got this many marks do they effective right now? If you look at that then it kind of a moving average phenomenon, but they may be else, some other performance it keeps struggling you for a long time, it will die slowly it will died on right until some other event actually takes over and drowns that other one that is an auto regressive effect.

So, if you see out own lives there are both moving average and this auto regressive effects and that is why we will be eventually led to call the ARMA models, where there are some effects that have died their natural death, they do not really bother me any more right, they say I have come out of it that is the phrase that we use, but then there are certain things that keep troubling some are personal, some are professional and so on. So, now, to be have on one hand the assumptions leading as to moving average models and the second one we will perceive which is which will lead us to auto regressive model, but we will not jump the gun right now, we will see what kind of assumption on h will lead us to auto regressive model. So, we will already seen how auto regressive models take birth, which is by which is from the prediction view point like we wrote a predictor as a linear function on the past that is all.

But now we want to ask from an impulse response estimation view point, what assumption on h leads me to auto regressive models? It may not be the way that auto regressive models where conceived in the literature of time series analysis, but whether it was conceived or not the implication remains and that is what we would like to know, when I assuming a move when I assume a moving average model the implication is the impulse response coefficient die of after a finite time, whereas as we will see with the auto regressive models, the implication is that the impulse response functions do not die in a finite time, but rather take infinitely long time they died on asymptotically, but in what way we will look at that a bit later, let us focus on the moving average models (Refer Time: 14:59) any questions? Somehow my class does not ask me too many questions; I would like a lot of questions to be asked so that I can learn.