Applied Time-Series Analysis
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Lecture – 42
Lecture 18C - Models for Linear Stationary Processes-6

Now, very quickly let us talk about moving average models and I just want to talk about inevitability, because we have already seen moving average models. Now this is your general definition of linear random process and at this point we branch of into two special classes of models: as we have seen already moving average and auto regressive. Now before we do that, why are we doing this, when should ask and again this is not so widely discussed in any of the text, but look at it from a modelling view point.

From a modelling view point what is given to us? So, assume that the process satisfies all the requirements of a linear random process and now I am set out to modelling, we have got and out of this theoretical glory details ok and now let us ask from an estimation view point, what is it that I am I am going to skip this slides here we have already discussed let us go on to the moving average of presentation, but before I move into that let me give you a quick a quick preface on why we are moving to moving average models. Yes it may turn out that somebody in the past has come up with this proposition and so on, but in Hines side perspectives always help.

So, if I look at it from a time series modelling view point, what is known to me? V k correct that is what is given to me and I decide to build a model of this kind what is it that I have to estimate that is all.

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We have to estimate sigma square e right that is the big difference between estimating the impulse response model for the deterministic systems and for stochastic processes. In deterministic systems I already know I already know the input. So, I do not have to worry about the input at all my focus is only on estimating the g's, here I have an additional responsibility which is that of estimating sigma square e as well. So, always when you report a time series model you have to report the coefficients and sigma square e; if you do not report the value of sigma square e, then you are giving me incomplete information about the model that is a first point that one should remember.

Now, how many h's do I have to estimate if I were to go by this model, so how many do we have? Countable.

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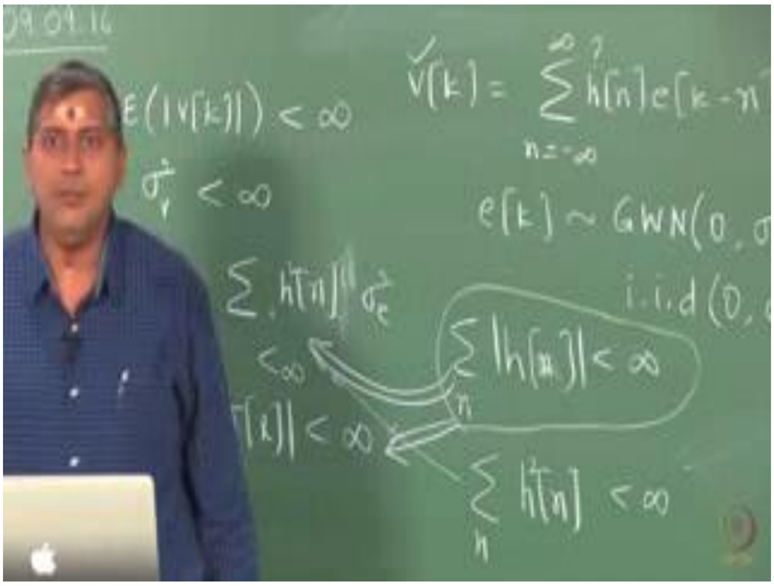

We have an infinite number of unknowns all I know is v right, what I do not know is this h am I right?

So, I have to estimate this infinite number of unknowns is a practical and given finite v right and given a realization, but I am supposed to estimate infinite. So, what do we do now? We can say that we truncate this to we restrict ourselves to causal processes no problem. So, what do we do know, what are the options that we have? Any suggestions? These are the same suggestions root approaches that I have taken even in the deterministic world when we want to identify the g's sorry.

Now what is the one of the options I am not saying the solution, one of the options is to assume that the process is described only by a finite number of h, this is exactly what is known as the finite impulse response model in a system series; to begin with we are saying a linear random process is described in terms of infinite impulse response coefficients or h.

But from a practical view point I cannot estimate this infinite impulse response coefficients, I have to somehow do something to handle this in infinite number of unknowns business and one of the ways to do that is to assume it is only an assumption that the process can be described by finite number of impulse response coefficients and that is what leads us to MA models.

How do we know that the process has an MA description whether this assumption is correct? ACF comes and tells us your assumption is fair enough, that is why we studied the ACF signature, you see now things are starting to connect; from an estimation view point I cannot actually guarantee that I cannot estimate this infinite number of unknowns

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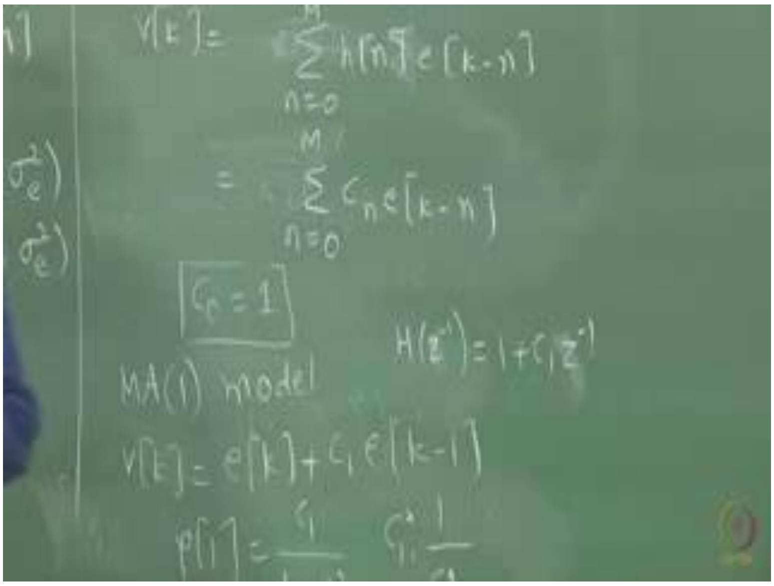

So, one solution is to assume that I have to begin with we say here h n e k minus n and n runs from let us say 0 to M and how do I know what is the value of M, whether this is a plausible model by looking at the ACF and ACF also fortunately tells me gives me a good guess for M; theoretically it gives me straight away the value of M, but when it comes to practice it gives me a good guess of M, because we are not working with a full with the ensemble we are working only with the realization.

So, whatever I am going to work with is a estimates of ACF. So, I will get a good guess of M, keep that in mind now we will make us a small change to the notation here whenever we talk of MA models of finite order, we will introduce this notation C n instead of h n for a reason I will tell you later on why.

So, we have moved from h to e we will reserve this h generally for the linear random process infinite impulse response model and also use this h of q inverse and so on, we will continuity use h of q inverse only that may now change the notation. Now there is another point that one should keep in mind that you restrict ourselves to C naught equals 1, this is an added restriction that we impose, again this is from an identify will be constraint that I will explain more in the next class, but let me just take a couple of minutes and explain to you this concept of inevitability. So, let us consider a simple MA 1 model and I will explain to you what is inevitability we will deal with it more formally later on.

So, consider an MA 1 model and we know what is an MA 1 model; with this restriction as I said I will explain to you why that restriction occurs later on. So, here I have e k plus C1 e k minus 1 and from a modelling view point what I am given is v k and I am suppose to estimate C 1 and sigma square e we have already fix c naught to 1. Now in practice we do not estimates C 1 and sigma square e from v, but rather from ACF for some other statistical property. If you were to use the ACF which is generally the case then we know that the equation that help us estimating the C 1 from ACF is this relation in the mapping between the ACF and the parameters, straight away we know that there are two solutions to this.

In practice I know rho. In fact, I am going to replace rho with it is estimate and I hope that the model satisfies estimate also as I said this is the method of movements. So, there are two solutions one is C 1, if you say C 1 star is one solution, what is the other solution? 1 over C 1 star.

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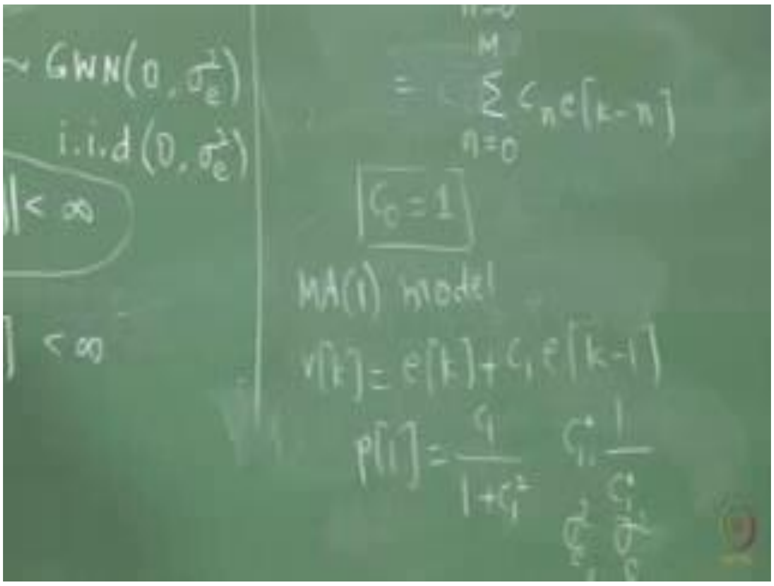

Now both satisfy this equation here which one should I pick; remember associated with C 1 star is also an estimate of sigma square e, let us call this as sigma square e and the other one as sigma square e tilde. So, associated each coefficient there is a sigma square e that is why it is an important to report both.

Now, a question is which one is the one that I should select? Theoretically both give rise to the same ACF, so using the ACF alone I will not be able to figure out which model to work with. Now at this stage we turn to the an important aspect of time series modelling which is forecasting; when I fit this model eventually what I am going to do with a model? I am going to make a forecast right that is what I am going to do. Now when I make a forecast and I will just explain this very quickly just a minute, when I actually do this forecast for the moving average model.

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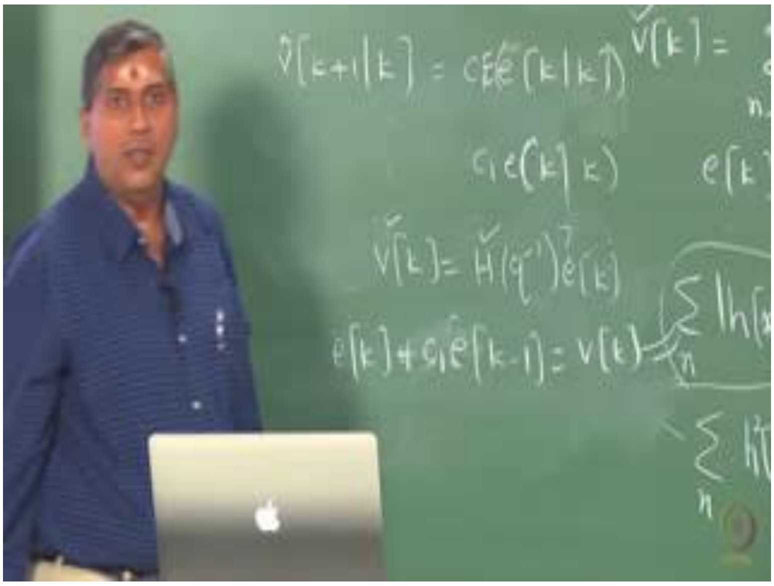

Let us say I am given information up to k and I am predicting what happens at k plus 1. What would be v hat of k plus 1 given k? K plus 1 given k the it is a conditional expectation right, the expectation of e k plus one given all the past is zero by definition of white noise process and we are left with you can say hat or you can say the conditional expectation or just for the sake of discussion we would write C 1 e k right. In fact, it is not exactly e k. e k given all information up to k.

How do I recover e k is e k is given to me? No, how do I recover e k? E k is the shockwave that has generated what I see as v that shock was unpredictable, when nothing was given to me when only the past of e k is given to me the shock is unpredictable, but now I am not given the past of e k what I mean by given k is past of v; v contains a effects of e k right v k contains the effects of e k, therefore I can use that to recover.

As a simple example I would not be able to predict an earth quake, it is like a shockwave, it causes some damage, but when I visit the site of damage I would be able to estimate the extend of earthquake right it is not a prediction, it is only an estimation. E k given k is not a prediction it is estimation, by looking at the damage I can indeed access the extent of shockwave not exactly, but to a certain extend that is exactly our e k given k, how do I get this e k? Remember v is H of q inverse e k and I have to now recover given v k and given H I am suppose to recover e.

So, I will pick one of those models there because I have identified C 1 star and 1 over C 1 star and I ask which one now gives me a stable estimate of e k? It turns out that when you use one of those models, only one of those models gives you stable estimate; that means, an estimate of e k that satisfy stationarity, the other model will give you an unbounded estimate of e k and to see this all you have to do is rewrite r inverse this equation invert this equation right when you invert this equation, what happens e k is v e k plus C 1 e k minus 1 is v k correct.

Now, my goal is to recover this e given v; when I write the time series model, I want to forecast v given e, but to forecast I need the estimate of e. So, therefore, I rewrite this equation imagine v k to be driven e this is not the case, but I want to estimate this. Now what is this kind of a model, when you think of v as the input and e as the output? It is an AR model; it is an what is an m a model for v becomes an AR model for e correct assume both e k and v k are stationary that anyway is guaranteed.

Now, we have already seen that this AR model is stationary; if and only if what is the restriction? Naught C 1 is less than 1. So, which one do I pick now? I have two models here, I pick the one such that modes even star is whichever is the solution that is less than one in magnitude; in general when you extend this to higher order moving average models, do not think that the coefficients individually have to be less than one in magnitude, in general you can relate this to what are known as the zeros of H of q inverse.

So, here the zero of h of q inverse is minus C 1, what is H of q inverse here? 1 plus C 1 q inverse; so, the zero of h if you think of in terms of q or z, it is actually minus C 1. So, in general we require that the zeros of when we say when we say zero you just equate that nu H of q inverse to 0, it is a roots of that they should lie within the unit circle for the MA 1 model, the zero restriction and the coefficient restriction are one and the same because there is only one root.

So, that is what is the story of inevitability; when h of q inverse as zeros inside the unit circle when we say zeros inside the unit circle, it is in terms of what you do is you replace q inverse with z or you can still replay work with q inverse, we say the zeros in terms of z, many text books would give you another conditions the zero should lie outside the unit circle, but that is in terms of roots of z inverse; when you write roots in terms of z inverse, the condition of invertibility is that the root should lie outside the unit circle, but here it is roots inside the unit circle when H of q inverse satisfies this condition we say it is invertible.