Applied Time-Series Analysis
Prof. Arun K. Tangirala
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 40
Lecture 18A - Models for Linear Stationary Processes-4

Good morning; we will continue our discussion with linear random processes. So, what we shall do today is go over the definition of this linear random process a bit more in detail as you saw yesterday and also as you see on the screen on today, the definition as a certain restrictions right on the driving force as well as the coefficients; will discuss the restriction on the coefficient have bit more in detail and then move on to discuss a class of processes, which we have already seen earlier which are the moving average processes and again examine such models more in detail today and as I said we will talk about the issue of inevitability fine.

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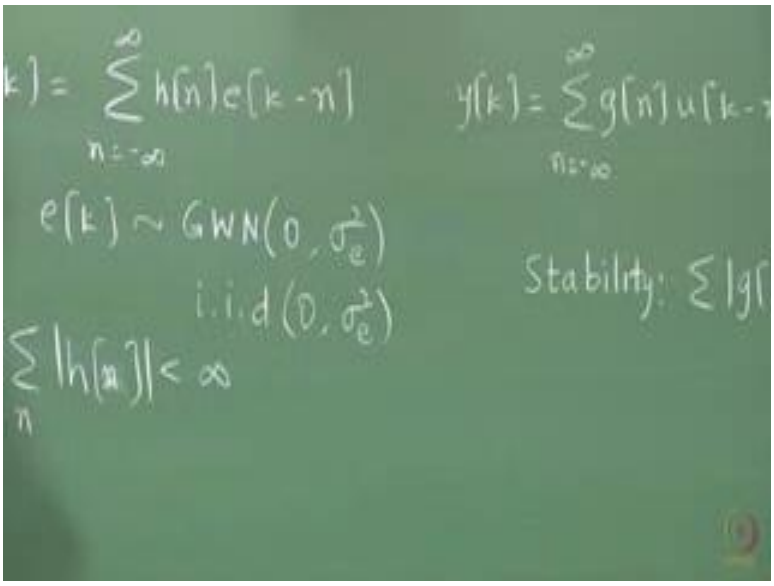

So, let us look at this definition that we have for a linear random process and yesterday we did talk about it is strong similarities with that of the convolution equation that you see in deterministic world, I will just write that by the side for your convenience and this is not surprising because we have already said that the idea of developing a time series model amounts to imagining v k to be driven by white noise, linear time in variant filter driven by white noise.

So obviously, the moment you say LTI systems, you should expect the convolution equation to come in and that is what has happened. But the big difference as I said yesterday is that this convolution equation is derived from the property of linearity and time in variance.

Whereas very often the definition of a linear random process itself is given in this form; so the definition of a linear random process is tied to this a bill this representation itself and of course, when you move to the non-linear world, then you have to worry more about what are the other representations and so on, but since you are on the linear world we will work only with this definition and as we have noticed in general we also allow feature shock waves to effect v k, but very soon we will restrict the summation to such that n equals 0, in other words we will only allow causal models.

So, you see this restriction here that of course, we have also said that the driving force should be white noise, but let us add the Gaussian to it and as I mentioned earlier in many text particularly on non-linear time series analysis and a few on linear time series analysis, you would see this e k being represented sorry as a restricted to i. i. d sequences, but we know that the Gaussian write now this is also an i. i. d, so we are all not really violating this particular requirement, any way if you look at the history of time series modelling a one of the major contributors was Bartley and if you go to Bartlett's definition of a linear random process, you would see that the restriction of e k is on i. i. d is not on the white noise as such.

But gradually people replace is with white noise, now it is because anyway we are only worried about a second order stationarity right and white noise is anyway a second order stationary uncorrelated process. Now the more serious part course I mean I am not saying this is not such serious, but the more serious part is on the restrictions of the coefficients; here you may choose to switch between Gaussian white noise and i. i. d, but if you look at this sequence here n is just a dummy variable, we have required that this sequence here is absolutely convergent.

Now, let us talk a bit on this requirement of absolute convergence of the coefficients, we have already seen this kind of a requirement here we have set for stability, one requires the impulse response coefficients to be absolutely convergent. Here there is no notion of stability, but we have this issue of stationarity and so on. So, if you read stability loosely, essentially what it amounts to saying is any bounded input should produce a bounded output that is the kind of stability we are discussing here. Here one of the requirements of v k for v k to be stationary is that v k should be bounded for example, magnitude of v k should always be less than some finite value, but since v k is a random variable, we do not talk about magnitude of v k we talk of expectation of magnitude of v k. So, that is one of the requirements, all though that is not the only requirement for stationarity.

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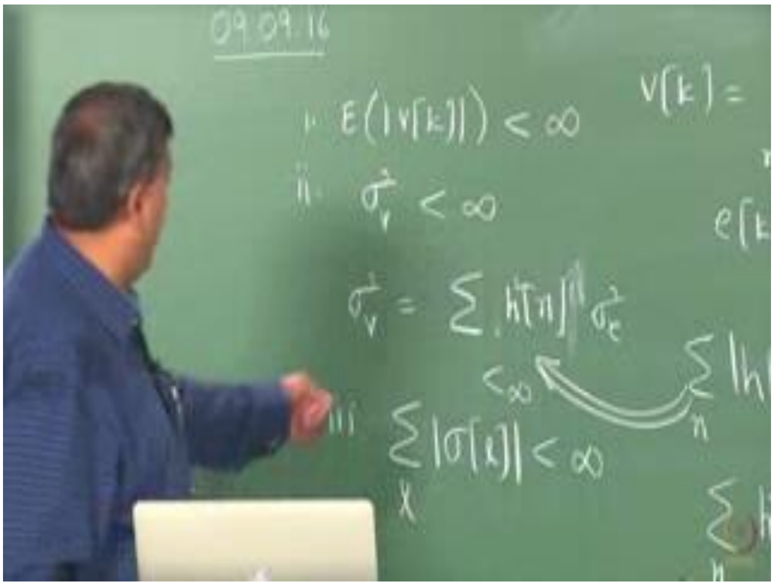

So, if you look at it from that view point then this condition here guarantees that is fairly is to prove; we actually take the magnitude on both sides apply expectations and then use the (Refer Time: 06:02) shorts of the triangular inequality to show that if this is satisfied then this is guaranteed.

In fact, that is one that is also the way that one proves boundedness of y k in the deterministic world, the only difference is in the deterministic world we do not take expectations. So, we will not prove that this proof is available in many texts including the book by Brockville and Devis and so on.

Now, the other requirement remember we said v k should be stationary of course, we can straight away see that the expectation of v k is 0 given that e k’s are zero mean variables, they it is likely that the process that you are looking at does not have a zero mean, in that case what do you do right, all you have to do is replace v k with v k minus mu assuming it to be stationary already, so the model would be for v k minus mu in place of v k that is a only difference.

In other words you go to add a constant to the right hand side that is other way of looking at it. So, we will not worry about that it does not spoil any of the results that we are discussing. So, coming back to the discussion, we know that if v k were to be stationary which is actually guaranteed by this then expectation of v k is 0. So, that is the first condition of second order stationarity satisfied or the white sense stationarity satisfied.

What is the second condition? Variance should be finite. Now we want sigma square v to be less than infinity, it turns out that again this condition guarantees this I am not going to prove it, but it is fairly easy to see that see first of all what would be variance of v? What is the expression for sigma square v?

Student: (Refer Time: 08:17).

Student: (Refer Time: 08:18) summation over n.

Right this is what even times.

Student: (Refer Time: 08:32).

Right correct. So, sigma square e is anyway given to be finite. So, the finiteness of sigma square v slowly depends on this, now we know from the theory of sequences right properties of sequences that if a sequence is absolutely convergent then is the square is also convergent not the other way around right, you should remain recall this properties of sequences that absolute summability does guarantee the summability of the squared one. In fact, I do not even need to use the magnitude here because it is going to be squared and we are only anyway dealing with real valued sequences here.

So, this implies the convergence of this sequence, but not the other way around right. In fact, in many text books you will see slightly diluted version of this requirement for a linear random processes that is instead of placing this restriction you will find this to be the requirement, it does not take away too much from the from the definition of the linear random process but this is a stronger requirement and will tell you why it is stronger require requirement.

So, you understand now that the variance is; obviously, going to be finite whether this holds or this holds. Now what is the third requirement for stationarity? The auto covariance should be independent of lag l and it also should be bounded right and in the sense auto covariance cannot run away at any other like since we are proved variance to be bounded the auto covariance also will be bounded; remember the maximum value of auto covariance is at lag 0.

So, you can prove that the auto covariance in fact, in many text books instead of proving variance to be bounded, you would actually require that the auto covariance we bounded, but there is a more important requirement that we are forgetting anyway. So, we will talk about that soon, first is that the conditional expectation sorry the expectation of the absolute value should be bounded and you can prove that again invoking this requirement, it is very hard to use this requirement there and you can proved that the variance is bounded using one of these requirements as a result auto covariance is also bounded.

But there is another important and even more important requirement we will first talk about another important one: remember we said we can write the definition, we can think of this representation only for processes; with what property?

Student: (Refer Time: 11:32).

That is that is going to give me guarantee optimality. So, the jointly Gaussian requirement is for optimality. So, that the conditional expectation and this linear predictor or linear representation will give me same predictions, but my ability that to form such a representation for v k rests on some condition on the auto covariance absolutely convergent right; the we have said that we have said the spectral density should satisfies certain conditions, but firstly a spectral density should exist and for the spectral density to exist like we said yesterday the auto covariance function should be absolutely convergent.

So, you need also to show that sigma mod again sigma l should be bounded of course, I have dropped this subscript, but this is the requirement that we need to guarantee and it turns out that this guarantees absolute convergence as well and you again a proof of this is available for example, you can see the text in Hamilton, I will see if I can upload the proof on the slides, but we will not spend time in the class proving these things, this is for your information in the sense to understand why this there is so much why this requirement is necessary and in how many different ways this is essential.

Now, there is an even more important requirement which is that this summation here that we are looking at first of all should converge to a random variable, generally when we talk of summations we talk of convergence to a real number for example, when we talked about spectral density, the spectral densities a discrete time Fourier transform of the auto covariance function that is at least relation wise and there we straight away said for that summation to converge the auto covariance should be absolutely convergent, but what we meant by convergence there is, convergence to a real values number.

Here when we are talking of convergence here we expect the summation to converge to a random variable, random variable in the same space as the space on which you are looking at that is the same sample space on which your examining the process and the convergence of sequences of random variables is lot more involved then the convergence of sequences of real numbers or in a deterministic variables.

We will not go over the theory of convergence of random variables at this stage; it becomes highly important in estimation theory when we talk of consistency that is convergence of estimates to the truth and so on. So, I will postpone the discussion of the theory of convergence, but you should know that this summation here just straight away does not necessarily guarantee that the net result is going to be a random variable defined on a certain sample space.