Applied Time-Series Analysis
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Lecture – 37
Lecture 16B - Models for Linear Stationary Processes-1

Let us get back now to the main discussion of today's lecture, where I will briefly introduce the notion of this model of a linear random process and then we will continue the discussion tomorrow. So, if you look at the history of time series modeling as I have always said the story began from prediction, but taking a slight deviation from the prediction discussion, how was a time series thought of and even today the this approach is used for many time series.

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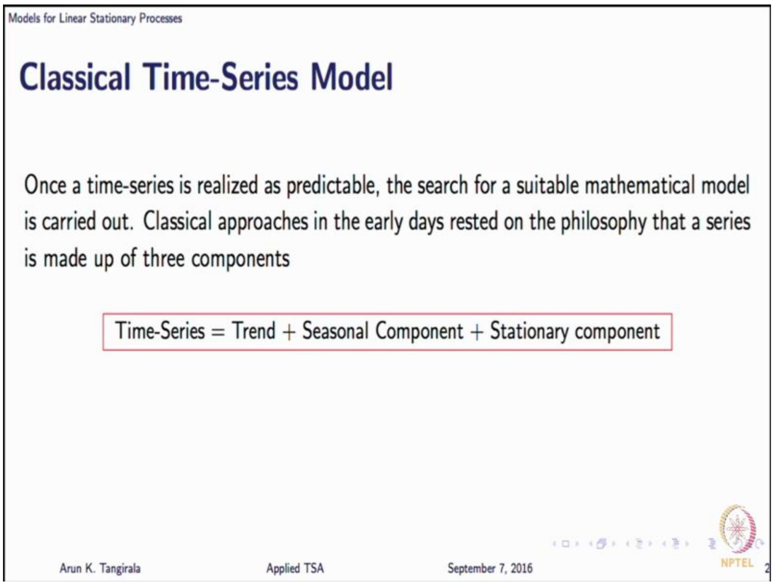

Classically a time series was thought of to be made up of a trend, we have seen what trends are right some linear trend, quadratic trends, some kind of polynomial trend plus a seasonal component; you should read that as a periodic component, we have seen some series which have some periodicities for example, there is a series.

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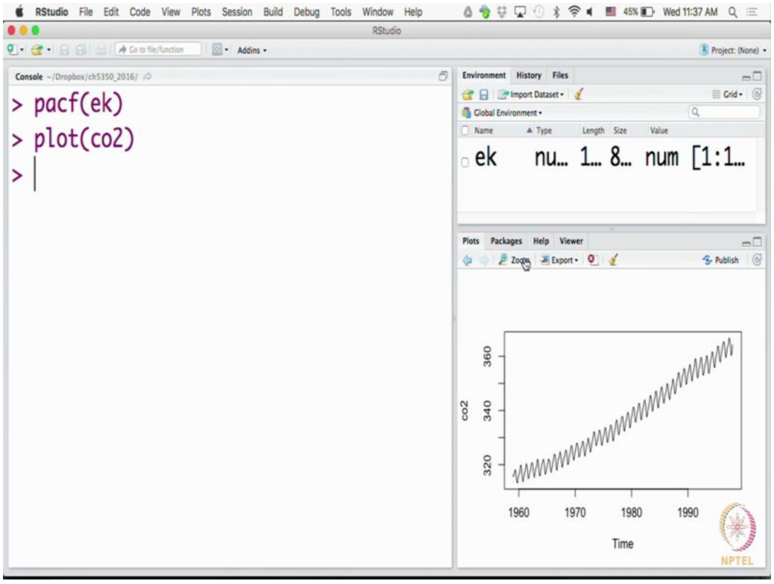

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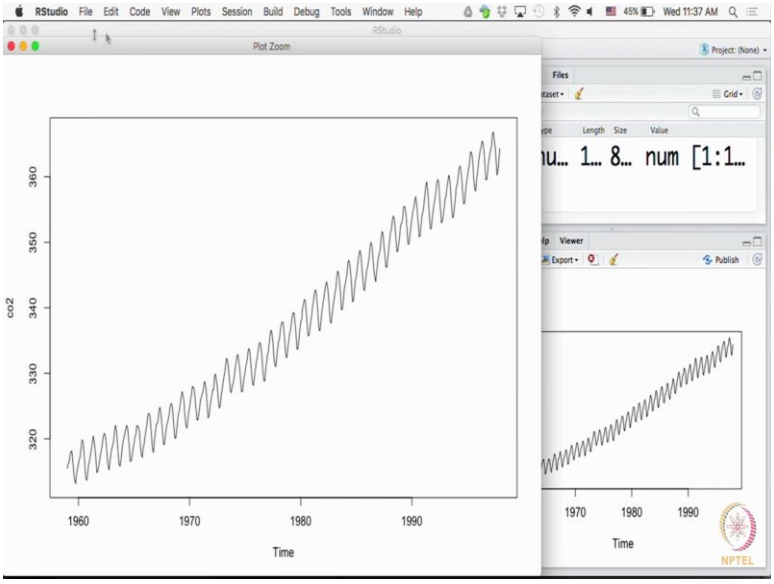

I will show you let me show you in art of the carbon dioxide levels let me zoom out here. So, you can see this figure right here on the screen, you can see that there is a trend and then there is a periodic component, these are carbon dioxide levels and under oth and atmosphere in some region over some duration, let us not worry about that. So, you can see there is a trend there is a kind of a linear trend at least visually and then there is an oscillatory component then there may be we do not know after we have removed the trend and seasonal component, we may be left with an irregular component in other words kind of a random series. So, there are many ways of imagining these series; the classical imagination is that all of these superimposed on each other right, but that is not the only way to imagine times there is something else that I will talk about very soon.

Now, then what people did as far as this model is concerned is, they would first figure out what the trend is by let us say linear regression. So, for example, if I give you this series, if you want to extract the trend then you would fit a model such as this.

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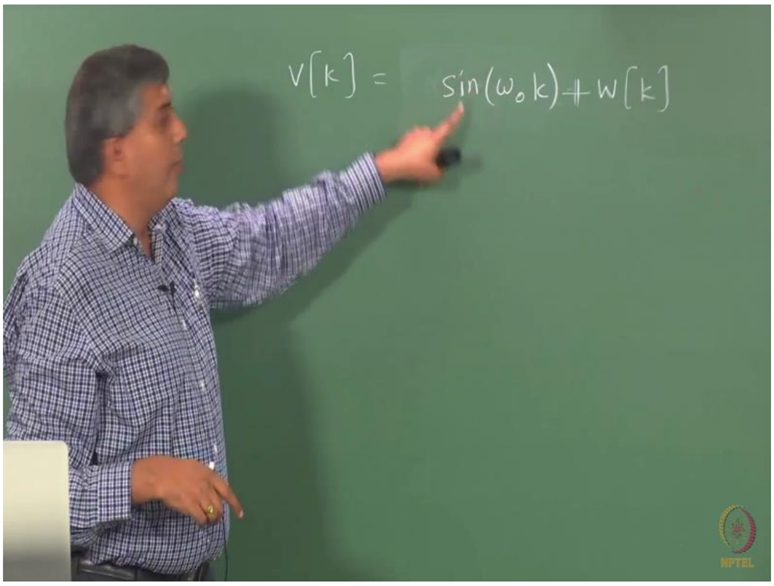

So, your series let us say is V K, if you want to extract the linear trend then you would fit a model of this kind right and focus on estimating alpha naught and alpha 1 if the linear trend is right how do you know if the linear trend is right? Well visually there is some feeling that there is a linear trend so you go ahead and fit and once you have obtained optimal estimates of alpha naught and alpha 1. Typically, it is cast as a linear leastwise problem and then you work with the residual. So, the residual will be representative of this w k, once again you see if w k has a trend. formally there are tests for trends, which more or less are based on what you learn as test for regression, whatever tests are used in linear regression those are kind of used in this statistical test, at this moment we do not get into that we will talk about that a bit later because we are focused right now on modeling stationery processes, but I am giving you a full picture right now and then we will quickly converse with the stationery part.

So, traditionally what people have done and continue to do so for several processes is to first extract the trend and then look at the periodicity, but if there is periodicity, sometimes it is advisable to first get rid of the periodicity and then fit a trend and then work with a stationery component and for the stationery part, always one tries and fits and ARMA model; that part is the same whether it is classical approach or modern approach, it is only the way you handle the trend and seasonal components that makes the difference between the classical approach and the that is one of the prime differences the other difference I will point it out point out shortly and the. So, let us actually move on.

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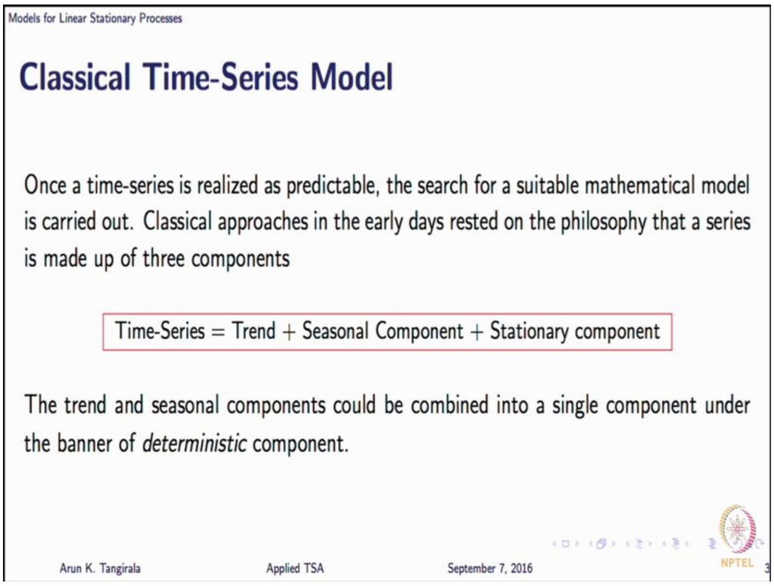

Now, you could combine the trend and seasonal component into a single component known as a deterministic component, both are actually deterministic. When I say seasonal, I do not know how many of you actually understand what is seasonal, but many examples can be given for example, if you look at sales of cloths or you know sales of sweets, they will probably shoot up every Diwali or some other festival time. So, there is a certain periodicity there yes people do consume sweets on a daily basis, but that is one phenomenon happening on a daily scale, then there is another phenomenon happening on a probably yearly scale if assuming that Diwali is only time that people real this sales shoot up, but there are other festivals too, but taking that only into account you have a yearly phenomenon that is sup that is riding on top of this daily phenomenon and you and then if you think that no there are going to be two occasions on which the sales will shoot up then you have a half yearly phenomenon and then you have a yearly phenomenon and so on.

So, these are called seasonal phenomenon, essentially periodic phenomenon, but the basic premise in the classical model is that this trend and the seasonal component and the stationery component are adding up, they need not be that is just a premise that has been made and people have worked with these kinds of models several methods have been developed to extract the trend, the seasonal component, and then finally to estimate the ARMA model

So, we will talk of these approaches at least some not all, there are non parametric approaches and some kind of semi parametric approaches to extract the seasonality and trend and then one works with are residual to fit an ARMA model and these methods use the ideas of regression.

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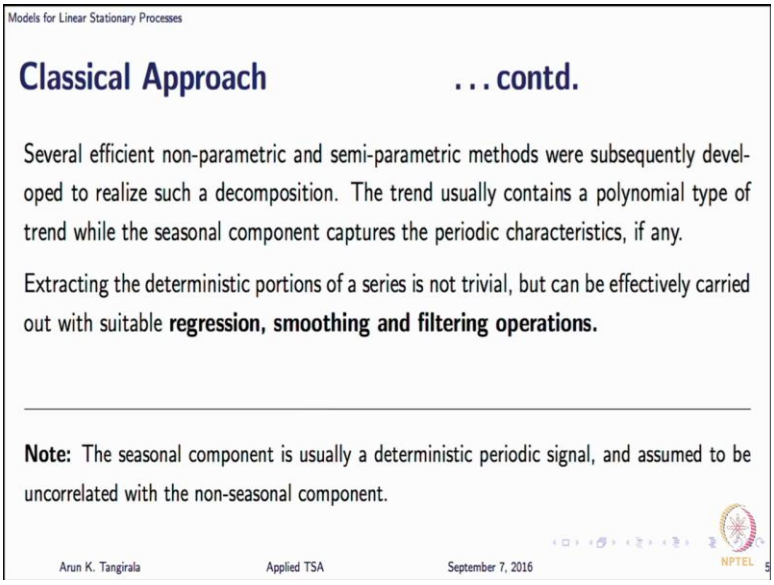

For example to fit the trend or sometimes even to get your periodicity; smoothing and filtering ideas to extract the seasonal component, as a simple example suppose there was only a seasonal component and the stationery component there was no trend let us say that is how the series was right. Now we know that now there is no uncertainty randomness about the sign here, it is a deterministic sinusoid here it could be sinusoidal cosine does not matter.

We know that this sinusoid has an average of 0 over 1 period right. So, if I know the period now that is one of the assumptions, if I know the period then I can average the series over 1 period or many such periods and then the seasonal component is gone. So, I construct a new series by performing some kind of averaging of the series, so that the seasonal component vanishes and I am only left with w k of course, averaged of w k and then I can extract the sinusoidal component that is one way it assumes that the period is known.

How does one determine the period? Suppose this is a series, by looking at the series it will be it will not be possible to figure out omega naught right, but when you look at the auto correlation function for some of you have already done and some and the rest few are going to do it, you will see that the ACF is way more cleaner than the series in terms of giving you the periodicity, that is the basic advantage. Why do you think that that is the case? One of the things that you should question here is first of all is this is a stationery signal or a non stationery signal?

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Why?

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Where is the time dependence time dependence of what? Mean correct. So, the expected value of course, e k is zero mean, expected value is sin omega naught k. So, strictly speaking we cannot use the definition of ACF that we have learnt, because the definition of ACF that we have learnt is only for stationery processes we cannot use that; however, what I have asked you to do in assignment is to work with the sample ACF, the time averaged ACF. The time averaged ACF can always be used, whether it has to do something with the theoretical ACF or not is a separate question, but if I look at the time averaged ACF then you can show by assuming ergodicity on e k, that your ACF comes out to be of the same period as the signal that you are searching for which is omega naught.

The big the big advantage of working with ACF or the sample ACF is that the effects of noise are gone, in the sense gone in the sense they are condensed to a single point at lag 0, because theoretically if you look at e k just now we saw the ACF sample ACF of e k right what happened the only significant lag the lag at which only where you see ACF being significant is lag 0, almost at all other lags it is pretty much in insignificant. So, what ACF is doing is it is allowing you to go from the time domain space to the lag space and in doing so it is actually collecting all the randomness that is present in at every point in time and condensing it to a single lag at lag 0.

So, it is collect I mean it is a crude interpretation, but it helps what is happening is if you are in the time domain, at each instant the randomness is affecting your signal this is a very classical problem of detecting a sign embedded in noise has been studied for decades and what you are learning is some of the solutions to this, later on we will learn a much better solution through spectral analysis, but ACF is also frequently used to detect periodicities.

So, the point that I was making earlier is at every point in time here the signal is affected by the randomness, coming from the white noise therefore, it makes it very difficult for me to detect the periodicity am I right. On the other hand if I look at the sample ACF of v, we cannot talk of theoretical ACF v, ACF, but if I look at the sample ACF of v you can and as a n goes to infinity; that means, as the sample size becomes very large more or less it will converge to the ACF of a deterministic signal plus the ACF of this stochastic component.

I told you long ago that we are defining ACFs only for stochastic signals, there is an ACF for deterministic signals as well, we have not gone through the definitions we will go through those definitions later on, but you can take it from me that ACF of a deterministic periodic signal is period, is also periodic with the same period and you can show that when you work out the expressions you will see the sample ACF. The sample ACF that you are using more or less works out to be the ACF definition for a periodic signal more or less.

So, the summary is because of the randomness I am unable to detect the periodicity in the time domain, but by moving into the lag space the what ACF has done for me is, it has gotten rid of the randomness in the lags; at each lag ACF is a deterministic function and the effect of randomness is only felt at lag 0 and at no other lag therefore, when I want to detect the periodicity I can look at the periodicity of ACF at non zero lags and come up with a much cleaner way of estimating the period. The spectral analysis is even more cleaner in terms of detecting the periodicity that we will learn later on. So, this is one of the ways for example of detecting the period.

Now, when I do that, when I detect the periodicity then I go back and use these approaches such as smoothing and filtering operations to a get rid of that component and then work with the here of course, we have e k, but earlier we had w k and work with the processed w and fit a model for it.

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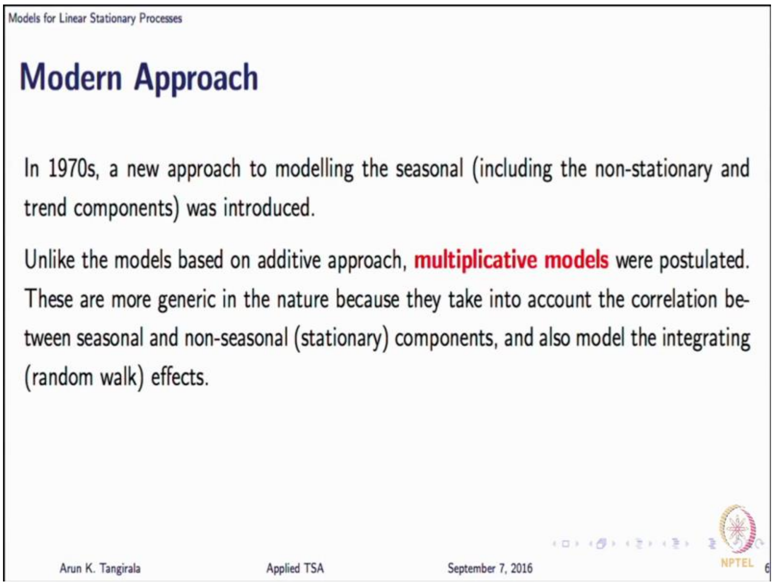

We will talk more about this later on, but let me quickly move on and tell you what modern approach has to do, somewhere in the 1970s this proposition was made by box Jenkins and today they are very popular, where they postulated and proposed that you could handle these kinds of signals which have trends, seasonality and stationarity all in one by the use of what are known as multiplicative models. Earlier we have seen additive model, but the models that box Jenkins have proposed are multiplicative models in the sense that the seasonal with let me say there is a transfer function for this seasonal component and then there is a transfer function that you have seen already for the stationery component they kind of multiply with each other they are in series.

Why have they become so popular? Because these multiplicative models they can handle of larger class of processes where the seasonality and the non stationery component are talking to each other, in the additive model the seasonal component is on it is own it is a deterministic function and the stationery component is on it is own it is a stochastic function, but why should the seasonal component be deterministic, a seasonal component can also be stochastic.

That means, we look at random phenomenon at some observation scale let us say on a daily basis I look at sales of some item from a shop, there can also be another phenomenon as we discussed for example, the sweets purchased during Diwali, why should it be periodic in the sense why should the sales be exactly equal the next year? How they need not be, but whatever was whatever happened in the previous Diwali can have an influence the next Diwali not they need not be identical in value, but they have an influence that is one level of stochasticity, the other level of stochasticity is between the seasonal and the daily, for example, as soon as Diwali is over within the vicinity of Diwali the daily sales may be different. Not exactly on the Diwali day or on the festival day, within that vicinity there can be some effect. So, that is the interaction that you see between the seasonal and the daily or the regular stationary component that we talk of.

In a general sense today all these phenomena that we are talking of the seasonal component and the stationery component to a certain extent the trend, this class of signals or processes that we look at today are called as multi scale processes; that means, there are phenomena happening within a subsystem, there are several phenomena happening at different time scales and they are very common not just an econometrics, in many natural and manmade processes you will see multi scale phenomena. So, the sarima models are actually useful there, this sarima stands for seasonal arima models again we will talk about this later on.

Let me actually take one more minute and then we will adjourn. So, the focus for now is on looking at models for stationery processes alone. Let us learn how to model a stationery process and then we will worry about the trend component or the seasonal component, additive models and multiplicative models and so on later on.

Now, this is not something that we have there is something that we have not discussed before it is none nothing like that, we have already discussed. But let me now formally state a part of the story and then the other part of the story will be clear after we discuss spectral analysis.

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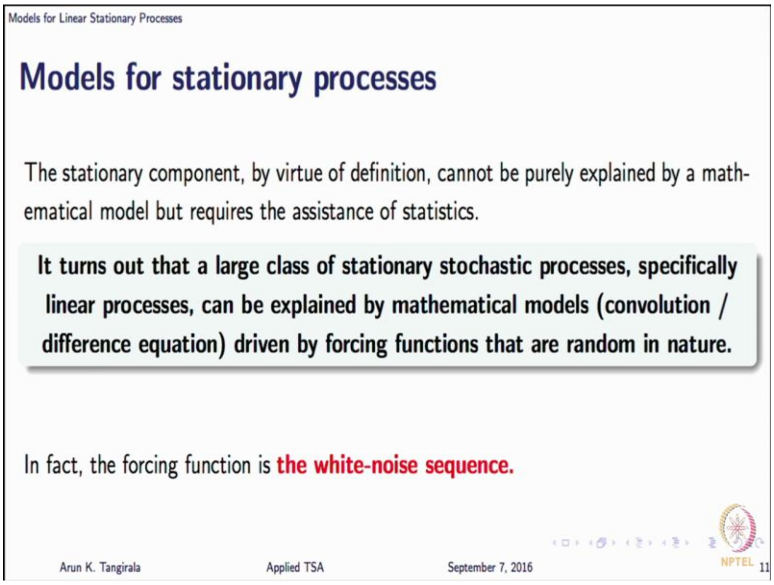

So, when I look at stationery processes when you look at the literature, it turns out that a large class of stationery processes as I have said before can be explained by a math mathematical model. What we mean by mathematical model is? A convolution type or a difference equation form driven by some random signal and this random signal happens to be the white noise sequence, we have already seen that interpretation before and the schematic representation is shown for you just so that you get better picture.

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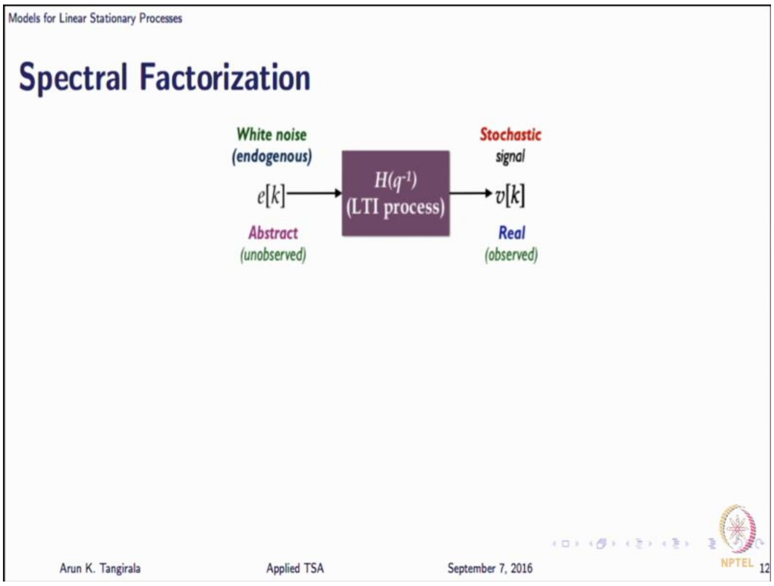

So, essentially what we can do is for a large class of stationery processes and when I say large class; that means, there is a class of stationery processes, which cannot be brought into this framework, but how do we know? Do we encounter such process in practice, we do not know. Theoretically we know that there exists some stationery processes which cannot be modeled this way, but in reality when I am looking at a process and I figured out it is stationery, do I know whether this that stationery process can be modeled this way or not it is very hard.

In such cases what happens is we will be building approximate models, but in any case we will be building an approximate model. So, for all practical purposes we do not have to so much worry about whether a stationery process can be brought into this framework or not. If the model that I built works for the process than I am happy that is the practical scenario. So, always learn to distinguish between theoretical conditions and practical implementations this LTI that you see in the schematics stands for linear time invariant or linear time invariance depends on how we use it.

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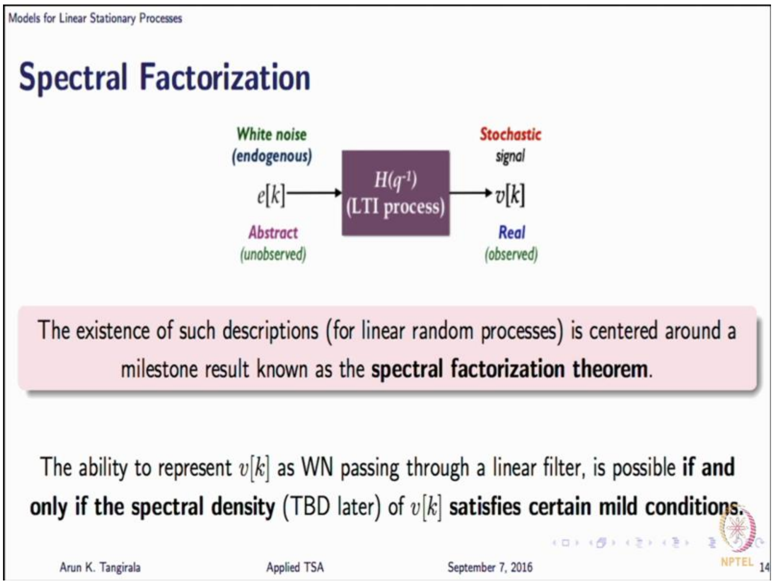

So, the idea is that this stationery process if it satisfies some conditions as stated by this spectral factorization result, then I can think of it as white noise passing through a filter as if it has been generated this way and what does what are these conditions? These conditions are some mild conditions on what is known as a spectral density, which we have not learnt until now, but we will learn this concept later on at this moment I can tell you that the spectral density is very closely related to the auto correlation function. In fact, the auto correlation function and the spectral density as I have mentioned I think on one or two instances are related to the furrier transform and that is the familiar (Refer Time: 20:24) theorem, we talked about it mention it briefly when we are talking of non negative definiteness.

So, if the spectral density of a process satisfies certain conditions which I am not stating here, then for that stochastic process I can give this kind of a nice interpretation that it has been generated by white noise passing through a filter that is the story. Now what are these conditions? We will learn later on, but what we will do tomorrow is when we come back to the class and hopefully we will begin on time is begin to begin with what are the general linear models that are available assuming that this stationery process satisfies this conditions on spectral density and then move on to the special models such as moving average and auto regressive models which we have already seen.

Thanks.