Applied Time-Series Analysis
Prof. Arun K. Tangirala
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture – 31
Lecture 13C - Autocovariance & Autocorrelation Functions-6

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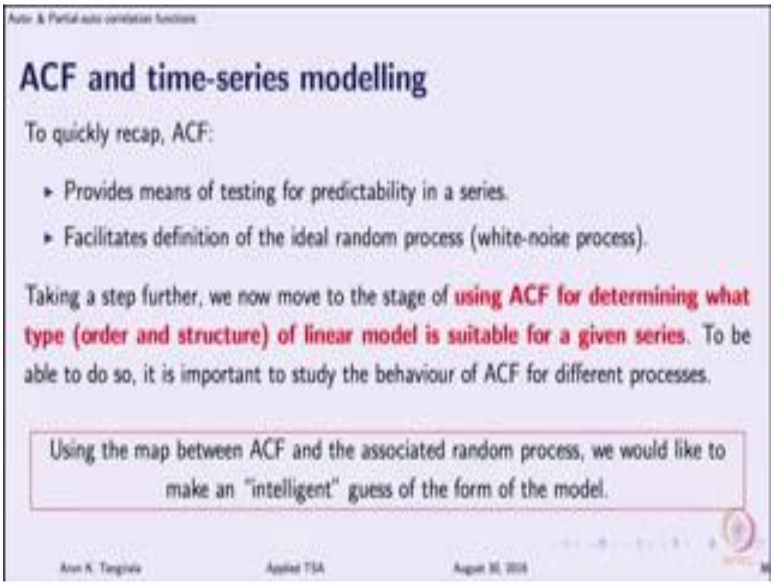

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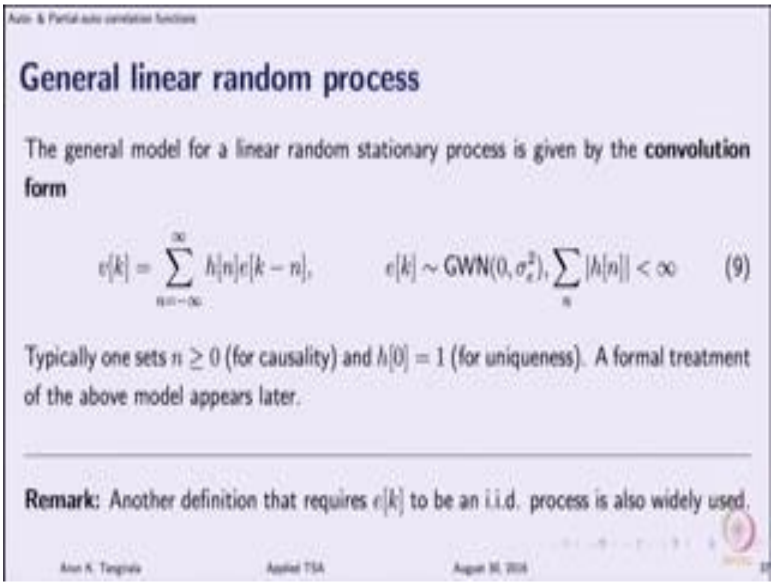

So, let us move on and study the ACF signatures for two different class of processes and the first class of process is a moving average process, but before I go to the moving average process just a quick preface on what a linear random process, how it is define; we will come back to this definition later on. But just to give you a feel some kind of curtain raiser of what are linear random processes. Because it is this special cases of this linear random process that for which we are going to study the ACF signatures. So, linear random process as you can see on the screen is defined as that process which can be generated by a linear combination of the past present and future shock waves; white noise, Gaussian white noise sequences.

Think of it this way; now that at least I have qualitatively kind of convinced you that we can bring random process in to this frame work; think of this as an input and this as the output and we know how linear systems are defined, some basic background in linear systems theory will tell you that the equation that you see on the screen is nothing, but the convolution equation; that is typically seen for all linear time in variant systems. It is like the mother of all equations for linear time in variant systems. So, you should see now that slowly we are able to draw parallels with a deterministic linear world. So, if you were to look at a deterministic LTI system linear time in variant system then naturally you will write this kind of a convolution equation.

But here it was very difficult to come up with the definition of linear random process, until somebody conceive this frame work. If there is no cause how do you define a linear random process it is very difficult whereas, in the deterministic world it is very easy, I can say y k should be a linear function of u k; that is what is linearity from mathematics I can borrow the definition, but it was not easy to do this random processes; it took lot of effort and formalization.

So, just for you to remember this definition; we will come back and talk about this definition more in detail. As long as you are able to express your process as a linear combination of the past, present and future shock waves such that two things; the driving force is a Gaussian white noise and the coefficients that you see in the combination there in the convolution equation are absolutely convergent; why the absolute convergence is required we will talk later on, but for now you can assume that is necessary and sufficient condition for stationarity of the series.

The absolute convergence guarantees stationarity of the series because generally writing some model is not enough the basic premise the basic assumption is v k s stationary. So, your model should also generate stationary series. Now there are some restrictions that we place on the summation so as to admit only causal processes; what do you mean by causal processes, future shock waves should not affect the present. Of course, you know most of the processes that we see are like that, but there are many other processes which are not like that, if you take you know human psychology, behavioral psychology and so on we look in to the future and get worried. Now that is a non causal behavior, so there are processes that are non causal, but we not worry about it. As a result we restrict or summation the lower limit to 0 alright and there is yet another restriction, we will not worry about that which is the first coefficient is to set 1, we will not talk about it now.

But now let us look at quickly the special case of this linear random process which is called the moving average process alright of finite order, this is called the moving average process of infinite order in the terminology and let us begin with that and ask what is ACF signature of such a process. I am going to erase some of the things on the board so that you do not get confused alright.

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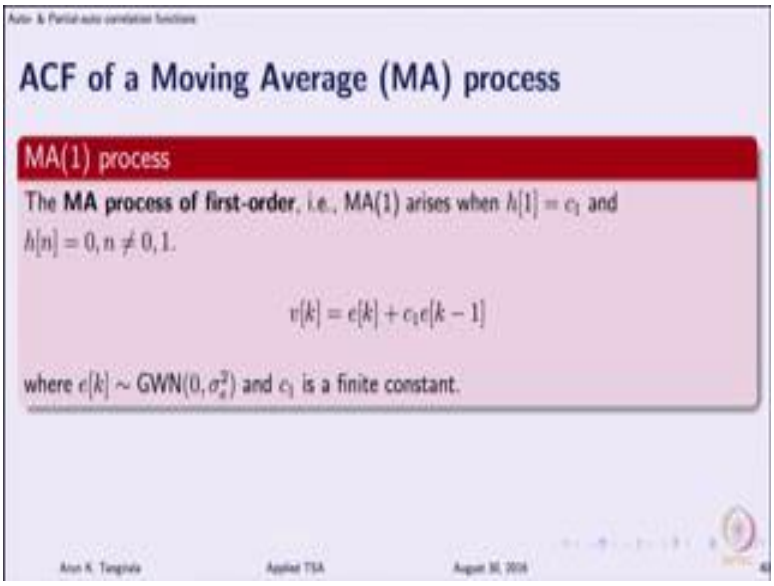

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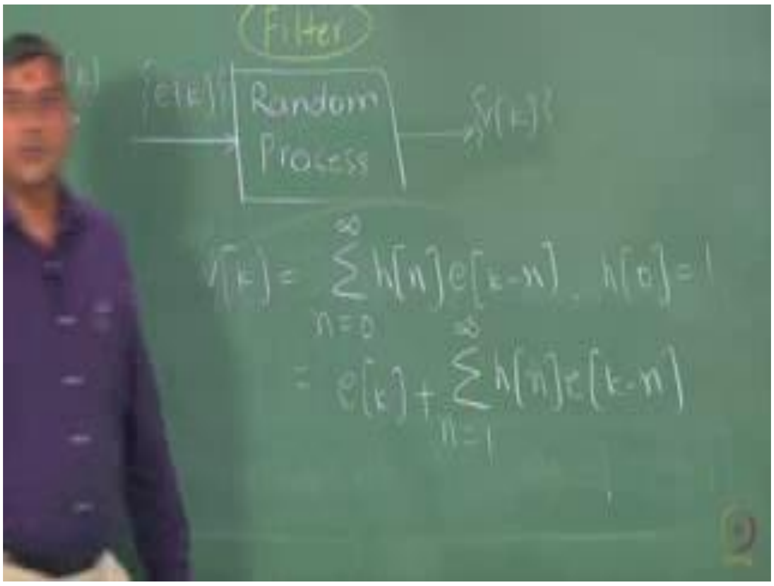

So as you can see the equation that I have written for the moving average process of order one; first of all the general equation is that v k is this infinitely non convolution equation n equals 0 to infinity and remember we said we will work this restriction that the first coefficient is 1, we will understand why that; the case later on. In other words it is generally you will find the equation written in this way; you said bring out the e k from the summation and write this one. So, that you clearly see that white noise is an integral component which is the unpredictable process, is an integral component of v k what I meant earlier is for a random signal when I said whether a predictable component exist or not the unpredictable part has to exist, you can understand here whether this term exist or not; this e k has to be there in your model.

So, what this means is in other words this is what is a predictable portion of v k, whatever is not contained in e k you can in principle predicted, if it were not the case I would have absorbed it in e k that is a point, but let us not worry about prediction at the moment.

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Let us ask now for the special class of processes where we assumed that we have of course, always we have h of 0 is 1 and let us assume h of 1 is some coefficient c 1 then the model becomes v k equals e k plus c 1 e k minus 1 which is the equation that you have on the screen, we call this as a moving average process order of 1; why is it moving average we somebody has use a term moving average earlier, but I kind of assume that you talking about moving average of the observations, but here the moving average is with respect to the input. We are averaging that you can see, we are averaging e k and e k minus 1; the present shock wave which is always going to be there and a past one; why is it moving, as you move with k the window that you are considering for e k is also moving and that is why it is called moving average; that is very simple.

Now, we want to ask what kind of autocorrelation function, whether first of all the series is co related; what do you think do you, think the series will have some correlation clearly because there is some predictable component as I already said, but what is that signature, how do we figure that out, we write the definition of ACF first of all ACVF.

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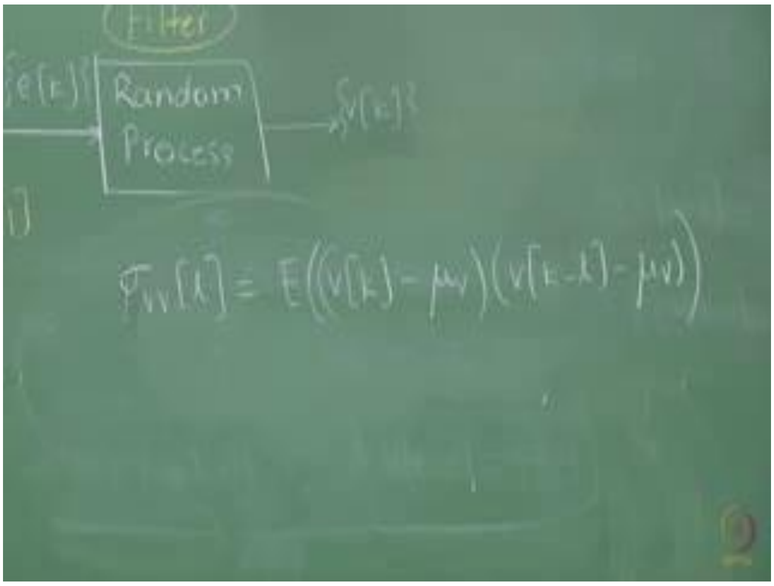

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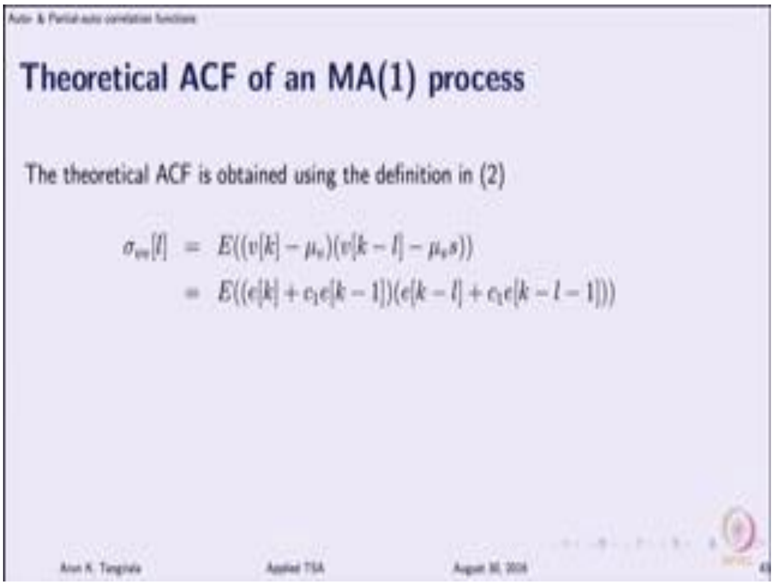

We say that we would like to determine the theoretical ACF and for this, what you do you. You invoke the definition; expectation of v k minus mu v times v k minus l minus mu v, now before we go ahead; you should ask me a question how on earth can a guarantee that the v k is stationary because in writing this I made the assumption that the mean is same, what in this model guarantee is that v k is stationary at least mean stationary; ek 0 mean what is the mean of v 0 expectation of v k is 0.

So, fine am safe, at least it should be means stationary for me to begin with this expression. In general we already know what is a condition for stationarity, for any linear random process the coefficient should be absolutely conversion, so what are the coefficient here 1 and c 1. So, all we are saying is 1 plus mod c 1 should be less than infinity; this is satisfied as long as c 1 is a finite and that is anyway stated in the model description c 1 is a finite valued constant, so I am guaranteed stationarity also, you will see that it is not easy to come to our conclusion for auto regressive processes when we look at them tomorrow.

Anyway, so let us quickly now work through this and write the expression; what is the expression that you obtained; can you work it out I give a minute. So, mu v is anyway 0, so I can throw it out of this expression and write a simpler one. Well the trick is to actually replace v k in this case at least with the governing equation that you as you see on the screen and then proceed further make use of the properties of white noise.

What you want me to explain that why mean of v 0, very simple when you are in doubt always go back to the basics; what is mu v; write down the expression mu is expectation. So, expectation of v k here expectation of e k plus c 1; e k minus 1 and expectation is a linear operator. So, expectation of e k is given to be 0 mean therefore, this would be 0.

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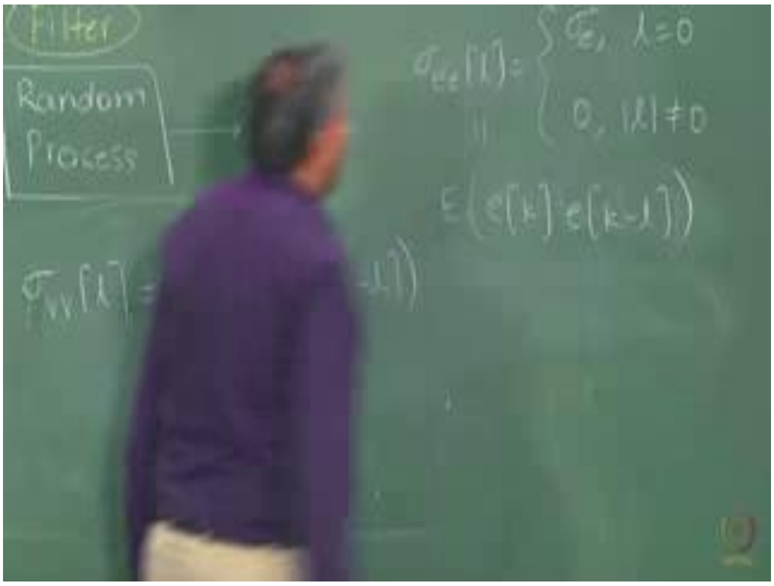

Remember that white noise, the auto covariance of white noise has an impulse like shape, it is sigma square e at lag 0 and 0 otherwise. So, you would get four terms in the product and apply expectation to each of those four terms. Remember sigma e e of l is nothing, but expectation of e k of course, minus mean, but it has 0 mean times e k mines l, any of you any difficulty? So let me know, these are the things that you should very comfortable with given that also you have a quiz coming up.

Anybody in the other hall has the answer, what you get, can you answer.

Student: (Refer Time: 12:37).

Ok, but you make use of this property right look at this white noise, you can substitute and then you can simplify it further, Sudhakar anybody has the answer there?

Student: No Sir.

Here also it is the same story as long as doing it is and trying to solve it.

Student: Sir c 1 sigma (Refer Time: 13:37).

c 1 sigma e square at what lag c.

Student: (Refer Time: 13:40).

At l equals 0; let us get the answer there what is that c 1 sigma square e at what lag?

Student: (Refer Time: 13:49).

Lag 0 that is not correct.

Student: (Refer Time: 13:52).

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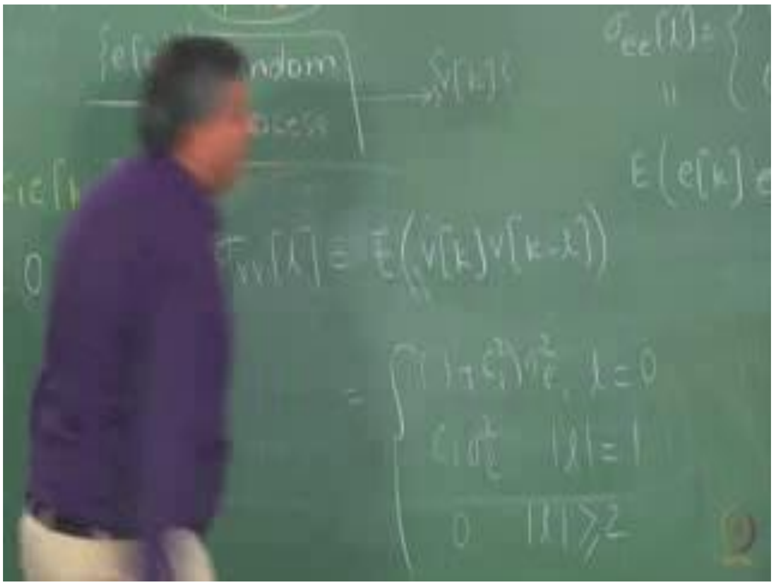

1 plus c 1 into sigma square e; that is not correct.

Student: 1 plus (Refer Time: 14:03).

At what lag?

Student: 0.

Lag 0 and then.

Student: (Refer Time: 14:12).

At lag c 1 correct, so the student that gave the answer from the other hall; the answer was correct, but for the wrong lag. So, we should remember our auto covariance is s symmetric function. So, you do not have to compute at negative lags after that what is the story; 0 very good. So, 0 at all lags greater than or equal 2; it is a very nice signature, the movement it is MA 1 of an MA 1 type; it says that the auto covariance goes to 0 soon after lag 1, it just goes to sleep and that is the answer that we have.

Student: (Refer Time: 15:15).

I am sorry.

Student: (Refer Time: 15:18).

Where no that was an example, but that was here the model is not in terms of its past, see it is different the model in terms of the past uncertainties past shock ways and no no that example was just an example. Here it is only MA 1 and it says that the ACF actually dyes exactly after lag 1. So, before I show you a schematic and I will also conclude this class with how to generate this theoretical ACF in r.

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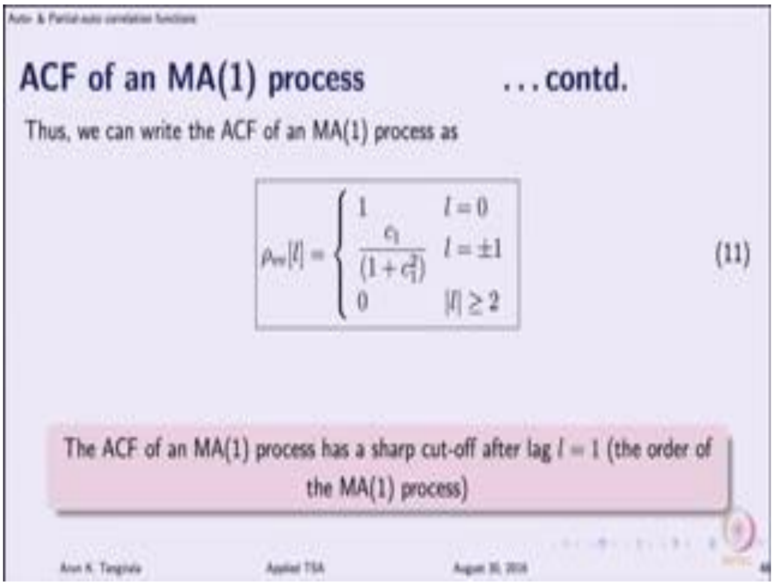

Let us look at the ACF; this is ACVF that you have and we notice on the screen, if you look at ACF is very simple, you divide the auto covariance with the auto covariance at lag 0 and when you that by definition auto correlation is 1 at lag 0 and 1 or c 1 or 1 plus c 1 square at lags 1; I mean I do not need to say minus 1 as well and 0 otherwise. Notice something interesting; the auto covariance depends on both c 1 and sigma square e whereas, the auto correlation only depends on c 1.

So, in practice suppose I find out for a series that the ACF is indicating an MA 1 kind of behavior; that means, I find that the ACF falls of a (Refer Time: 16:53) after lag 1; it goes to 0 and does not go to 0 because you will be dealing with estimates like you ask, but the ACF becomes insignificant, very small and I hypothesize or a postulate an MA 1 model and I want to estimate the coefficient, in practice I am only given the series in reality; how do I do that, I estimate the ACF that is one method, it is not the only method but is a very popular way of doing it although it is not a best way. What you do is you estimate the ACF at lag 1 and then you know theoretically for an MA 1 process ACF at lag 1 is c 1 over c plus c 1 square; you can estimate c 1; can you or can you not.

This is the basic idea behind what is known as a method of moments estimate us that you see in time series literature or in general time series modeling literature; why it is called method of moments, remember your ACF is a moment; it is a second order moment of joint pdf and we are assuming that the estimated ACF will also have satisfy the same equation as a theoretical ACF which is not necessarily true. But we are force fitting in such a way that the estimated ACF also satisfy the same equation as a theoretical one, is that the best way to do it not necessarily it works in some situations and it does not work in some other situations of course, when we go to estimation theory; we will figure that out.

So the ACF of a MA 1 process has a sharp cut off at lag 1; after lag one it just goes to 0 is this true for all other moving average processes of other orders, what you mean by other orders is suppose I had here is plus c 2; e k minus 2. Suppose this was a process that is m a 2, what you expect the ACF to look like no by just intuitively if you were to repeat the procedure.

Student: (Refer Time: 19:04).

Right after lag 2, it would go to 0 and now you can extend this argument, we will prove tomorrow that the ACF and of any MA process of order m goes to 0 exactly after lag m; it is a very beautiful signature, for these reasons this moving average processes of order m are called m correlated processes, but we will talk about that later on.

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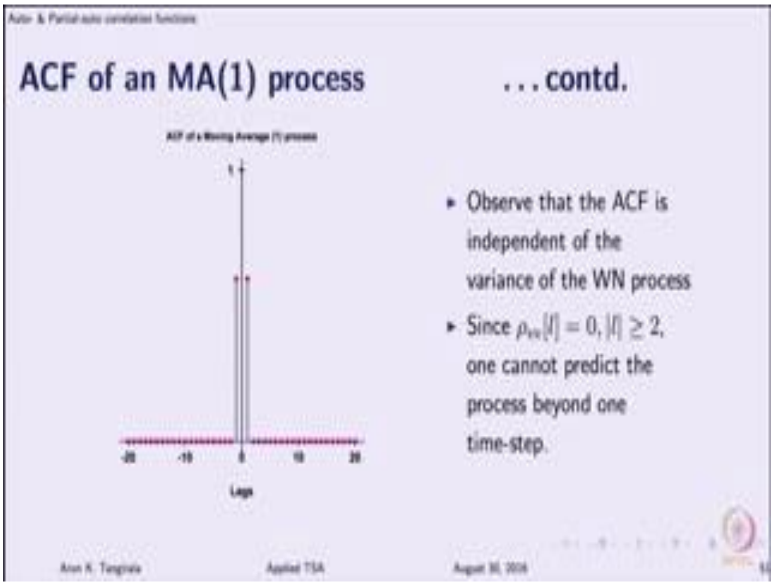

Let me just conclude the class with a schematic of this and quickly showing you how to do, how to generate this in your r; the theoretical ACF. So, this is the ACF of an ma 1 process that you see on the screen when the coefficient is chosen to some value and you can see that the auto correlation has gone to 0 after lag 1.

Now, the other point is because the ACF has gone to 0 after lag 1; that means, from lag 2 onwards, you will not be able to predict this process beyond one step; that directly tells you what is ability to predict, what do you mean by predict beyond one step. If I want to make a two step ahead forecast, I am standing now 6:26 right now the time is; I want to predict what happened said that is say your sampling interval 1 minute, I want to predict what happens as 6:28. If it is an MA 1 process, the prediction is 0 because there is nothing in this observation which is influencing or which has the information of the capability to tell you what is the happening two steps ahead, it can only tell you what happens a next because a process is such, it is not true for all processes. In general for an MA m process, you will not able to predict beyond m steps, so we will talk about that as well later on.

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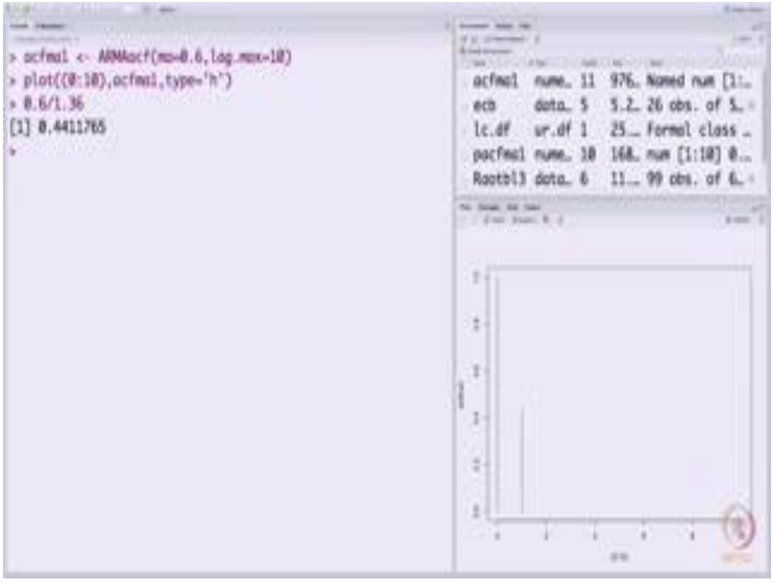

So in r, I do not know how well you can see that may be have to change here; fine I guess. So, the routine that calculates the theoretical ACF is the ARMA ACF, if this does not use data; you have to supply the model and then it compute the theoretical ACF. Essentially what you have done by pen and paper it has formulate to compute exactly the theoretical ones. So, you cannot use this data for data you have to use ACF fine, so suppose I use this let me actually store this here in ACF MA 1; I am asking it to compute the ACF for MA 1 process with coefficient c 1 set to 0.6 and I want to compute up to lag 10; we know any way that no point in going beyond lag 1, but we will just compute up to lag 10 and then now we can plot this.

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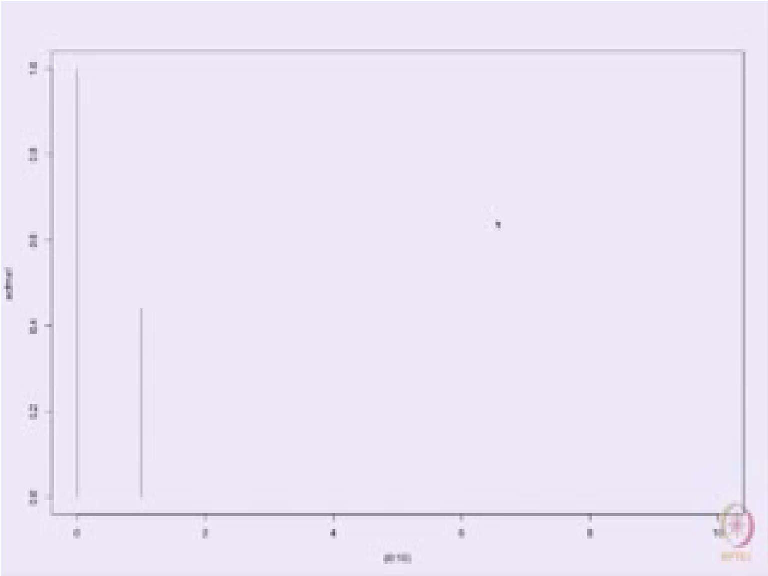

So, remember the ACF is computed from lag 0 onwards and I am just asking it to produce a plot so you can see this; I think I can just zoom this out. So, you can see on the screen that the ACF, this is the theoretical ACF; no data was used, no time series has been used. Can you check if it gives you the correct answer for ACF, it should be c 1 or 1 plus c 1 square. So, 0.6 by 1.36; roughly engineering calculations can you make; from rough calculations.

Student: (Refer Time: 22:59).

All kinds of numbers are being thrown on; less than right; is it that what is the value that you see roughly 0.4 plus; somewhere between point 4 closer to 0.45; we can actually make the exact calculation and then we can adjourn, so 0.6 by 1.36; like nearly 0.45; very good. So, we will meet in tomorrow’s class at 10:45; 11 o’clock, fine, see you tomorrow.