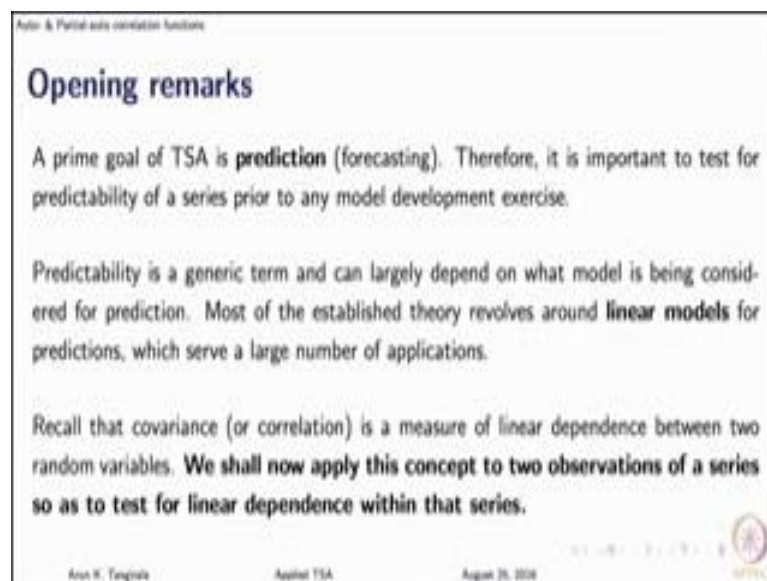


**Applied Time-Series Analysis**  
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**Lecture - 26**  
**Lecture 11C - Autocovariance & Autocorrelation Functions-1**

We enter the world of kind of time series modeling I can say and remember the goal of modeling is prediction right that means.

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Auto. & Partial auto correlation functions

### Opening remarks

A prime goal of TSA is **prediction** (forecasting). Therefore, it is important to test for predictability of a series prior to any model development exercise.

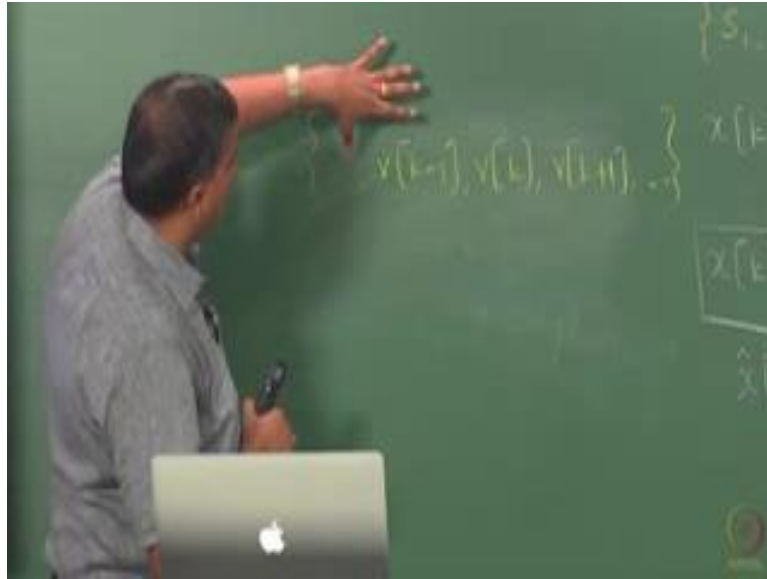
Predictability is a generic term and can largely depend on what model is being considered for prediction. Most of the established theory revolves around **linear models** for predictions, which serve a large number of applications.

Recall that covariance (or correlation) is a measure of linear dependence between two random variables. **We shall now apply this concept to two observations of a series so as to test for linear dependence within that series.**

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The series should have something in it for us to predict therefore, the first step in modeling is to determine if there is predictability in the series and that is what we are going to discuss and also will remind ourselves that we are working with linear models, we are not talking of general predictability here, what we want to test is whether the series has anything in it that allows us to will linear models, in other words we are going to work with covariance's, we have already seen that covariance is the measure of linear dependences.

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So, what we have is essentially a signal and infinitely a long signal right here you have and now will shift over to this  $v$  notation  $v_{k-1}$ ,  $v_k$ ,  $v_{k+1}$  and so on and I would like to see if the series is predictable, in other words in a linear sense. So, what I am going to use is measure of linear dependence which is covariance and I am going to examine the covariance between any two observations and then see if there is if any two observations are correlated. So, for this purpose we introduce auto covariance function.

At the heart of the auto covariance function we have covariance as the measure you should not forget; that there is nothing more complicated about this, I am given a series the first that I want to make is to determine whether this predictability in the series and we use covariance has a measure of predictability or dependency, if there is dependence then; that means there is scope of predictability.

So, this auto covariance has you can see even yesterday we have introduces, is covariance between any two observations of random process  $v_k$ ;  $k=1$  and  $k=2$ .

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Auto & Partial auto correlation functions

## Auto-covariance function (ACVF)

The **auto-covariance function (ACVF)** is defined as the covariance between two observations of a series,  $v[k_1]$  and  $v[k_2]$

$$\sigma_{vv}[k_1, k_2] = E\{(v[k_1] - \mu_{k_1})(v[k_2] - \mu_{k_2})\} \quad (1)$$

where  $\mu_{k_i}$  is the mean of the process at  $k_i$  instant.

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Again we have slightly we have uses slightly diluted notation here, its expectation of  $v[k_1] - \mu_{k_1}$ , times  $v[k_2] - \mu_{k_2}$ , in other simply covariance between  $v[k_1]$  and  $v[k_2]$ , this is a very generic definition assuming the process to be non stationary; when the process is stationary at least second order stationary, then in distribution then we know that the second order stationary processes are only dependent on the time difference, that is a joint distribution of a pair of observation is only dependent on the time difference not at the time instance and that the mean remains invariant with time therefore, we satisfy ourselves we kind of work with slightly relaxed version of this definition, where we drop the subscripts on  $\mu$  and essentially we say now  $\sigma$  is the function of the lag  $l$ , rather than  $k_1$  and  $k_2$ .

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Auto & Partial auto correlation functions

## ACVF of stationary processes

For stationary processes, recall that the mean remains invariant and the distribution is only a function of the time difference or lag,  $l = k_1 - k_2$ . Consequently,

### ACVF of a stationary process

The auto-covariance function of a stationary process is only a function of the lag  $l$  between two observations,

$$\sigma_{vv}[l] = E\{(v[k] - \mu_v)(v[k-l] - \mu_v)\} \quad (2)$$

where  $\mu_v = E\{v[k]\}$  is the mean of the stationary process

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But if you have a non stationary process, you should follow back to the original definition, which we want at this moment and for many lectures to come.

So, now the auto covariance function  $\sigma_{vv}$  at  $l$  is nothing, but expectation of  $v[k] - \mu_v$  times  $v[k-l] - \mu_v$ . Now I should caution give you couple of caution, but I will talk about it tomorrow let me conclude today's lecture in the interest of time, that this auto covariance function is not a mistreace function, if you have understood the covariance very well then you have a understood auto covariance function fairly well what we mean by this is remember covariance is an unbounded measure; it is a symmetric measure; it is a measure of linear dependence and it does not tell me whether one causes the other, all of this properties hold for the covariance. Auto covariance function inherits all those properties, the auto covariance function therefore, is symmetric with respect to lag  $l$ , it does not tell me whether  $v[k-l]$  is influencing  $v[k]$  or  $v[k]$  is influencing  $v[k-l]$ , but since we assume that the already that there is a flow of time here the causation is kind of taken care of that is not an issue right we do not assume  $v[k]$  is influencing  $v[k-l]$ . So, that causation is taken care of, but it is unbounded because it is sensitive the units of  $v$  this could be a temperature series I could use any units or temperature and change the value of the auto covariance function, what did we do in the case of covariance to fix all those issues? We introduce correlation likewise we will introduce auto correlation which is nothing, but the standardize version of this auto covariance.

So, noting so mysteries about it, what is important is to understand how from this auto covariance function, I can discover what model is suited for the random process and that is what we will spend a lot of time on time on understanding the signatures of different processes, what you mean by signatures is, what kind of auto covariance function a particular random process has and then do a reverse engineering, in practice I am given series I can estimate auto covariance function, by looking at auto covariance function I will try to guess a suitable model and then estimate the parameters that is the sequence that we will follow.

Tomorrow will discuss a lot on auto covariance and auto correlation.