Applied Time-Series Analysis
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Lecture – 24
Lecture 11A - Introduction to Random Processes-4

Continue with our discussion on non stationarities and then we will briefly talk on about ergodicity and kind of close or qualitative discussions and move on to more quantitative stuff where will start looking at auto correlation, auto covariance and auto correlation functions. So, what we were discussing yesterday is towards end of yesterday's lecture is the trend type non stationarities and today will talk about the integrating type non stationarity.

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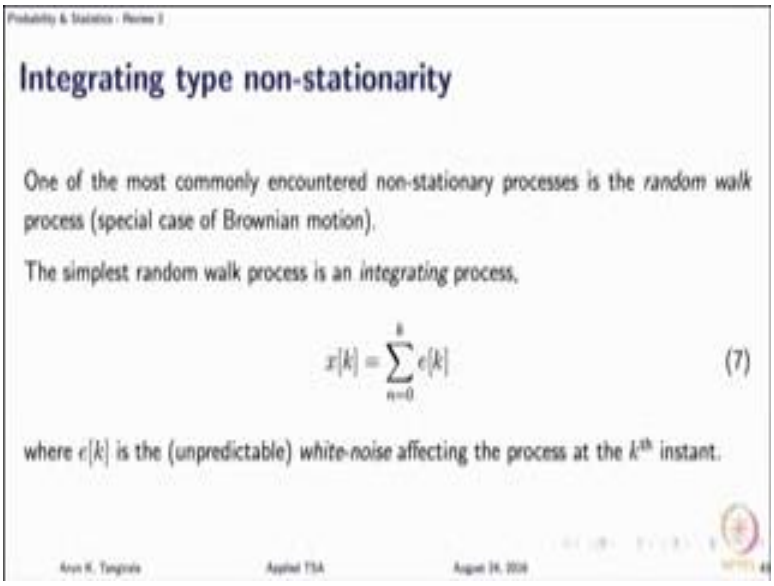

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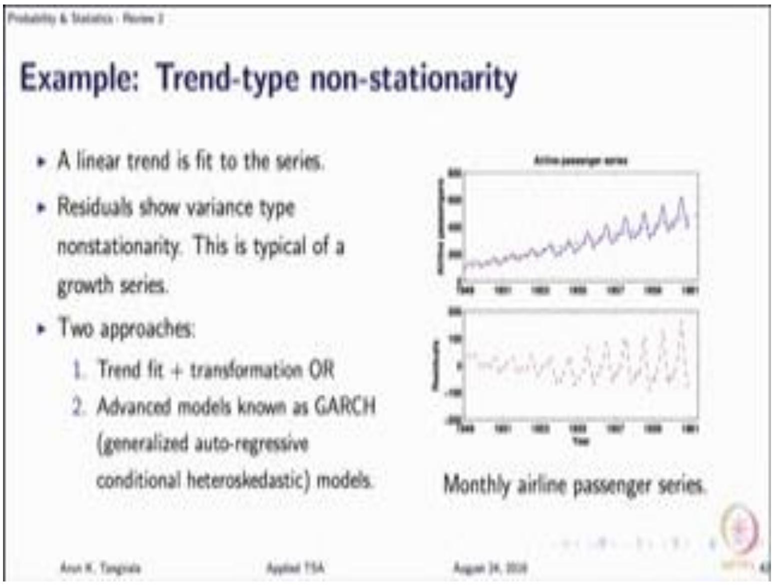

This is a stochastic kind of non stationarity, if you compare this with a trend type non stationarity as I mentioned yesterday, trends by definition is deterministic function of time, when nothing is mentioned and when your said that is a series contains a trend, it is kind of assume that you are looking at a deterministic function of time and of course, there is a stochastic component riding on top of it.

And as you can see and you have seen at least couple of a examples, there are many real life examples that fit into this frame work at every stage, you have to tell yourself that none of the models that we are building is actually true representative of the process I mean it is not a true reflection of what is happening, what we are seeking are not correct, models what we are seeking a working models, if your model is able to explain the phenomenon you would develop more faith in that model, but then you know you do not get up stress with that model and think that the model is a process itself. So, that is where you have to detach yourself. So, there is also a philosophy there, an attachment, detachment philosophy so, coming to stochastic kind of trends.

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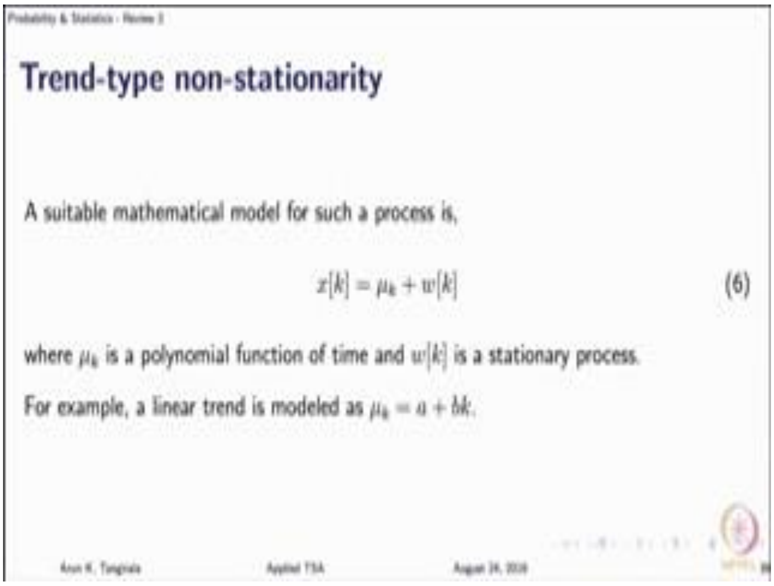

What do you mean by stochastic trend? Let me take you back here, remember we said when you have a trend type non stationarity x that is your random signal can be thought of as a mu plus some stochastic signal stochastic signal.

Now, this mu if it is a deterministic function of time, you have a trend type non stationarity, now your mu is not a deterministic function of time, it is a stochastic function I mean I do not say stochastic function, it is coming out of a stochastic process and there are; obviously, many possibilities, now we are interested in one particular type of stochastic trend and this is the integrating type non stationarity, there are different names to this and as we call is a integrating time in physics or in statistics also this known as random walk phenomenon, then you have Brownian motion and so on named after the bottom is Brown. So, if you look at the history of Brownian motion, you will find that Brown had actually discovered this phenomenon in his feel, he would he was observing the moment of pollen, but of course, he did not get into any math.

Because perhaps you did not have the time or he was not interested and so on maybe he was, but it was not meant to be that way it is nearly essentially later that Einstein stumbled upon this kind of phenomena and then gave a more quality quantitative statistical very regress explanation to this kind of phenomena and that is when the name of Brownian motion came along and in all of this, it was called random walk, I am sure you understand what is random walk, I am standing here, now the random of phenomena follows, I am standing here, now I have to decide whether to go to the left or to the right and if there is equal probability that is 1 kind of suppose I toss a coin, a head turns up, I move to my right and a tail turns up, I move to the left.

And that is in that it is a fair coin then it is going to be equal probability and I am going to do this at every step. So, I toss a coin; let us say tail shows up, I move to my left then again a toss a coin then the same story. So, this is an example of a random walk phenomenon and such kind of phenomena are not fictitious, in the sense it is not some kind of very abstract phenomenon as I said starting with the observation of Brown people have observed that several phenomena in nature and even kind of man made follow kind of a random walk phenomenon and we are interested in one particular class of random walk phenomena which is the integrating type.

Now, why is it called the integrating type? Of course, I have an equation for you here, but this equation would not make much sense at this moment to you until I defined what is white noise, the e k that you see in the equation is nothing, but the white noise signal. That would not make too much sense for you until we define auto covariance function and then define white noise we will see later on that this integrating type process is also an auto regressive process of order one, so at the moment leaving aside the definition of e k.

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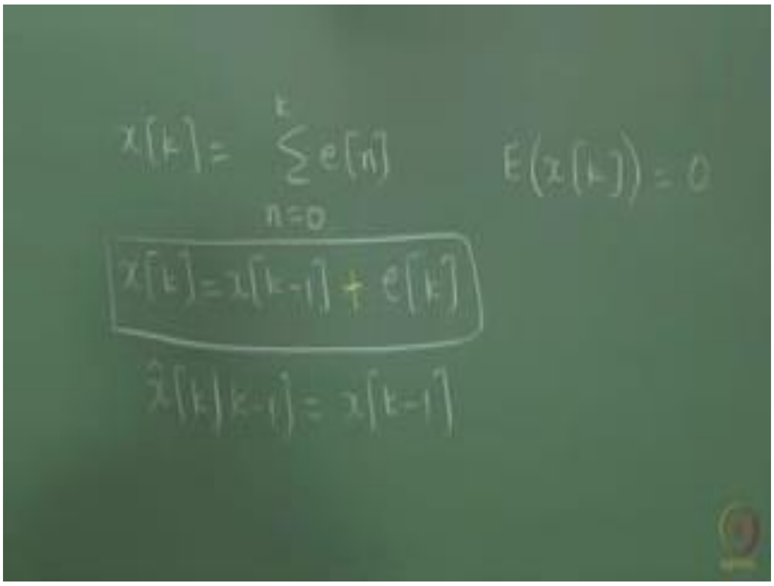

Think of just accept that to be white noise, if you look at the definition say n runs from 0 to k, now that is the definition of white noise here, there is a mistake, there in the equational, correct, it should the index dummy index should be n, that is fine is a minor 1, will correct it.

What is happening is think of this white noise as of now, a qualitatively as a shock wave that is unpredictable and you can see that the current state of the process is an accumulation of all the shock waves right from the beginning. In fact, strictly speaking I should be writing here minus infinity, but for now will assume that the beginning is indexed at 0. So, because of the accumulation of the shock waves at any instant in time what your essential doing is integrating them and that is why the name integrating effects of course, it is not so obvious why this is a random walk phenomenon the random walk nature comes about much more clearly when we write what is known as a difference form of this equation and slowly you should start getting use to this a difference form equation at convolution equation and so on.

This is said to be in some out convolution form how would you write the difference version of this x k, let us write a backward difference minus x k minus 1 equals e k very good, all right, this we call as a difference version and this version of that is this equation representation for the integrating type process is a lot more useful, we work with this equation much more often, then this sometimes we use this for theoretical analysis. In fact, now we can rewrite this as x k equals x k minus 1 plus e k. Now you can see some kind of a random work phenomenon coming out at if you are standing at k minus 1, the next state is the previous 1 plus some random component which is an unpredictable 1 adding out and that is why the name random work comes about to this process. Of course, there are different types of random work phenomena, but this is simplest and as I told you this is also called the integrating type process.

Now, this is an auto regressive process also because the process is regressing on to itself you should get use to this term auto regressive and of course, will talk about that more in detail, later on. X k if you look at the process x k minus 1 is actually regressing on to x k, regression is the term that comes from hypnotherapy and so on where you looking at effects of passed regressing on to the represent now there are where if you move is based on that right will not get into that now anyway. So, you can see that essentially a random work phenomenon and another way of looking at this integrating type process is from a prediction view point given all the information up to k minus 1 the best prediction of an integrating type process is the previous value itself.

By the way, write this way, by the way you should slowly get use this notation x hat of k given k minus 1 implies that I am predicting x at k given all the information up to k minus 1 not just at k minus 1 and that happens to be x k minus 1 why is that. So, qualitatively sorry, predict the correct because e k is unpredictable by definition what we mean by unpredictable given any amount of history of e k the it the prediction of e k is the mean itself typically these e k is are assumed to be 0 mean. So, the best prediction is the previous value itself. Now what is the non stationary part of this we have not seen why it is called non stationary will prove this a bit later on, when we learn how to analyze the auto regressive processes, we can prove it one way.

The other way that you can see right know instead of getting into the auto regressive world there is a way to prove that this is non stationary, how do you prove that it is non stationary typically what is the procedure that we adopt? We will try to see the mean is changing with time, what happens to mean of this process? So, if this is the equation, we can use this equation to arrive at the mean, what would be the expectation? 0, because expectation operate as a linear operator on the right hand side the expectation yields value of 0. So, that is not an issue it is stationary in the mean what about the variance how do you calculate the variance?

Student: (Refer Time: 11:11).

Why where is (Refer Time: 11:13).

Student: It is a sum of (Refer Time: 11:16).

Central limit theory is only use to arrive at the distribution; I mean a part of also that tells you what is the variance but the focus of the central limit theorem is to give you results on the distribution part. Simple x k is made up of k random variables right some of k random variables, how do you compute a variance of some of random variables?

Student: (Refer Time: 11:46).

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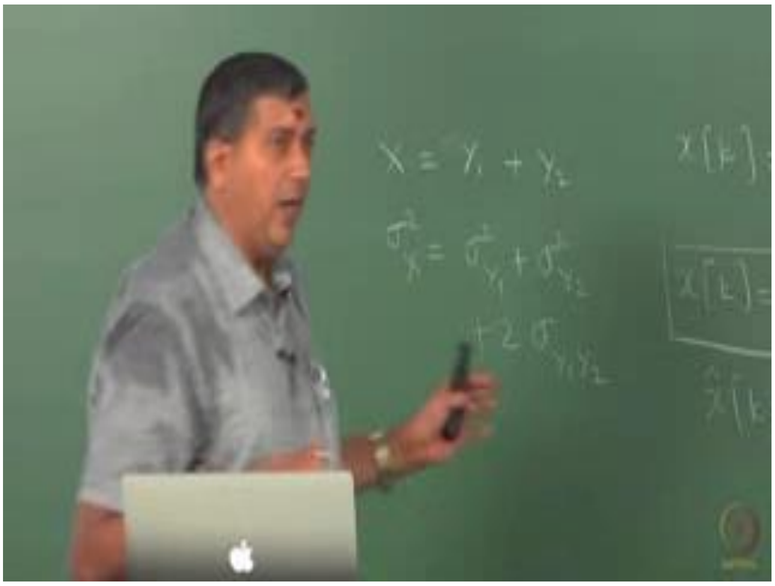

Sorry, suppose I have let us take a simple example, suppose I have a 2 variables y1 and y2 and x is made up of; x is essentially y1 plus y2, 2 variables y1 and y2 and add up to produce x, what is the variance of x?

Student: Variance of y plus variance of x minus co variance (Refer Time: 12:11).

Minus, why minus? So, sigma square y1 plus sigma square y2.

Student: (Refer Time: 12:18).

2 times, very good, 2 times covariance between y1 and y2, now you can extend this equation to n variables or k variables. Apply this to find out the variance of x k, sorry, k times sigma square e, why, what happen to the covariance?

Student: (Refer Time: 12:45).

Did I say independent? At the movement it is uncorrelated, there is of course, will talk about to 2 different processes white noise and IID, the covariance terms vanish by definition of the white noise remember or white noise sequences such that it is unpredictable. What do we mean by that unpredictable? In a linear sense, let me make that complete statement and because it is unpredictable in a linear sense, there is no linear dependency between any 2 observations or in at any 2 instance the signal is uncorrelated.

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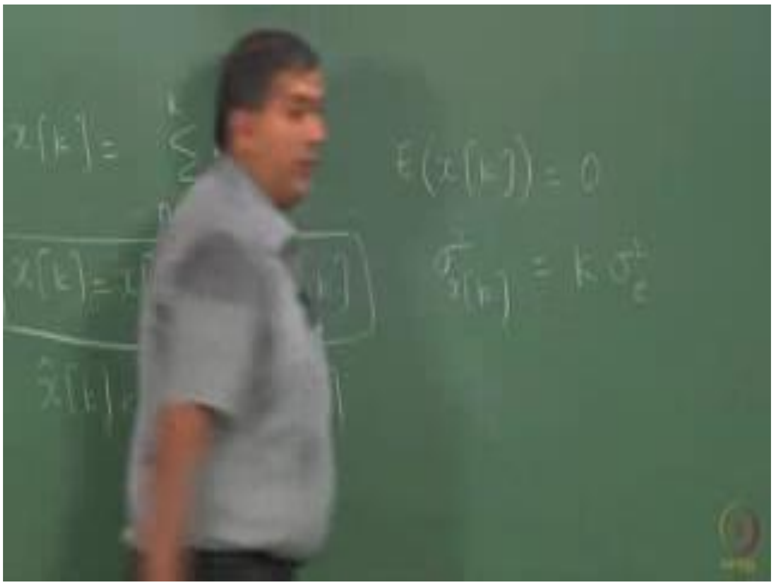

Which means all the cross terms would vanish and therefore, we would be left with so, sigma square x k remember strictly speaking your suppose to use the random variable, but as I said will slowly dilute that notation and work with sigma square, here x k will be used for both observation as well as the random variable k times sigma square e if sigma square e is the variance. We will give a formal definition of white noise shortly will e k is assume to be stationary by definition.

Now, can we conclude that x k is not stationary because the variance is changing with time that is good enough to tell us that it is non stationary, we will arrive at another I mean we will look at, arrive at this same conclusion by looking at the poles so called poles of the auto regressive model, but will reserve that for a later discussion good. So at any signal that any instance or is the signal is accumulation of all the shock waves and therefore it is called integrating effects.

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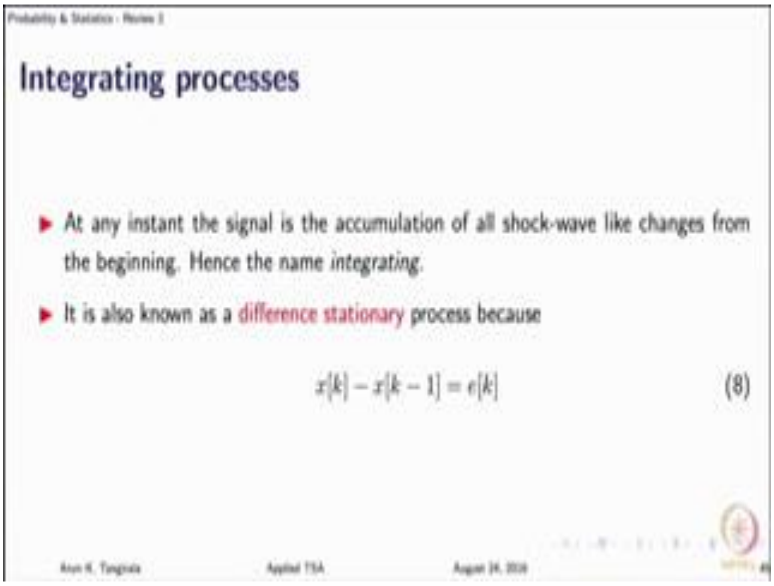

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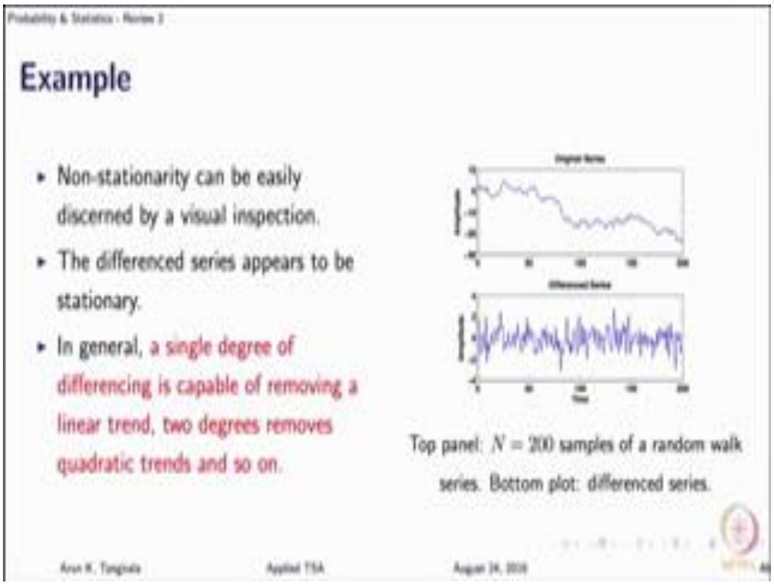

This is something that we have already discussed, let us get a feel of how this, such a process looks like I am showing you just one realization of an integrating process you can easily simulate integrating processes even and are you can generate this realizations. I will show you later on how to simulate integrating processes or even in general arma and arima processes may be couple of lectures down the line.

So, the main point here is going back to this difference equation form we have written it this way to see that it is a random work process, now will go back to the previous equation that we had to see that a single degree of differencing can produce a stationary series - that is x k is non stationary, e is stationary and lot of the theory that we are going to work with is applicable to stationary random processes. Now what this equation tells me is by differencing x once I am able to produce a stationary series, this is the pure integrating process therefore, 1 degree of differencing has actually produce a white noise and on the plot you see this on the top I show the realization of the integrating process and on the bottom, in the bottom panel you see the differenced series.

And at least visually when you look at it, it appears to be stationary whereas, the top 1 appears to be non stationary. Although it should be careful when you draw conclusions visually its lot more easier to detect trend type non stationarities visually then the stochastic once for theses stochastic once in particular the integrating type and so on, one has to formally conduct a statistical test and will talk about this so called unit root test. What we mean by unit root here is when you go back to the difference equation form or the recursive equation form you see that the difference equation form has a root at one at unity, yes.

Student: (Refer Time: 16:55).

Right, e has?

Student: Constant variance or (Refer Time: 17:01).

Yeah, it is stationary; the moment is station; that means variance has to be constant. So, by definition, white noise is a stationary uncorrelated process and we have already made use of the uncorrelated property in deriving the variance of x k and when I wrote this expression that point I said e we have made invoked the stationarity property of e k. Somewhere we have to fix things, so we assume that this white noise is stationary. You can have non stationary white noises also, but for now will work with stationary white noise and that is what gives us this expression. So, there are these unit root tests which will allow us to determine whether the process has random work behaviour I mean integrating type behaviour or not.

Now, many a times, 2 things can happen that is here it is a very simple case where I difference once and I recover the white noise, in a more general sense you may have to difference many times to make the process stationary, we do not know, typically you will not encounter the case where you have to difference more than 3 or 4 times and so on, but one may run into such a situation and this degree of differencing has to be determined on a case by case basis here, 1 degree of single degree of differencing as produce a white noise that is point number 1.

Point number 2, it is you should not thing that always this when is integrating type non stationarities are present, the moment I difference or how many I have a times a difference I will get a white noise process, it is not necessarily the case. This statement says that once you difference or may b the times you difference, you will recover a stationary process. In other words instead of e k you could have here some w k where w k is the stationary process, in which case w k itself has some kind of a model to be built and put together we called this as an arima process and, but the terminology will become clearer later on.

When single degree of differencing produces white noise we call is as a pure integrating process that is it, it is called an integrating process, when a single degree of differencing or any d degree of differencing produces as a stationary process which is not necessarily white noise then we run into what are known as arima kind of processes which we will talk about later on. Now are there processes out there which have this kind of behaviour? Numerous, you will find them predominately in econometrics, a lot of stock prices, stock market prices, financial variables and so on, have this kind of behaviour and a lot of disturbances that you seen engineering processes also have a random walk kind of behaviour.

We will shortly understand through the help of auto covariance function a different aspect of this random walk phenomena in terms of the correlation and so on, but we will restrict ourselves to this integrating type processes when it comes to stochastic, non stationary I mean stochastic trends. So, there are 2 kinds of non stationarities that will think of in fact 3 I should say including the periodic one, one is the deterministic one which subsumes the polynomial trends as well as periodic ones, in that sense we are only looking at 2, but if you separate the periodic one because we will give a separate treatment to the periodic one then we have this stochastic type trends or non stationarities in which we will considered particular the integrating type only, these 2 are enough to trouble us in this course.

To begin with we will restrict, will confine our discussions to stationary processes once we are in a position to model stationary processes then will bring up the non stationary part for example, here I said instead of e, you could have w where w is the stationary process, let us say you discovered by a single degree of differencing that you have a stationary process for example, in this figure that you see the top comes out of an integrating process, you would not know whether it is a pure integrating process or an arima kind of process a single degree of differencing gives you a stationary series the one that you see in a bottom here. Now obviously by looking at this, you would not be able to say if it is a white noise, if it is a realization of a white noise process, can you? It is not possible.

And that is going to be the subject of discussion very soon where will study auto covariance functions which will tell us whether this is a realization of a white noise process or not, correct. It would be great if you can look at the series and determine straight away there it is white noise then the entire theory would come, life is not so easy.

Student: Why is it particularly a difference? Is there, can we have submission or of anything else? (Refer Time: 22:23).

No, no, the origin is this, or some people argue that the origin is this; it is a good equation the difference equation comes because of the random work kind of phenomena. So, I am at this state, this is x k minus 1, what would be x k? A random component adding on to the present position, present state, whether I move to the left or right remember e k can take both positive and negative values in general and there is a probability associated with that. So, all we are saying is that the next position is the current position plus some random component and that is why you can begin with this equation if you like and if the probability that is the phenomenon that is determining might next position is a white noise kind of thing then you have a pure integrating process.

If the phenomenon that is going to govern my next position that is where I move from k minus 1 itself comes out of some stationary process which is also probabilistic process, then we have a more general arima kind of process. So, the starting point for an integrating process can be can be different in the sense I could start with this is what we are started with claim in there is integrating process, but someone else could say no, no I will assume that my starting point is this equation because this allows might explain the random work phenomena and then show that this is an alternative description.

There are several ways of describing the same process this is something that you should slowly get use to any process in general will have different form of forms of descriptions or representations they all mean the same it is says that the mathematically equation looks different and which mathematically equation do we want to work with in other words which model or representation, we want to work with entirely depends on the application that we are looking at and the context that we are looking at. In this case we began with this and then this kind of summation equation and went on to the difference equation, I might have as well began with this and gone to this form as well.

So to answer the question, the difference equation form comes about by virtue of the nature of random work its current position plus some random content. So, let us actually now close the discussion with the small cautionary note on differencing. This differencing is a very attractive option to get rid of the integrating type non stationarity or to model the integrating type non stationarity, but many a times you may end up differencing when there is no need to do so; that means, your process is not an integrating type and you may still end up differencing the series, maybe in advertantly or axenently or whatever or it is an over site. That can actually cause more harm them could. So, one has to be careful you have to be sure has to be enough conviction to believe in other words to difference series. For example, let me give you a very simple example suppose the given series was e k itself.

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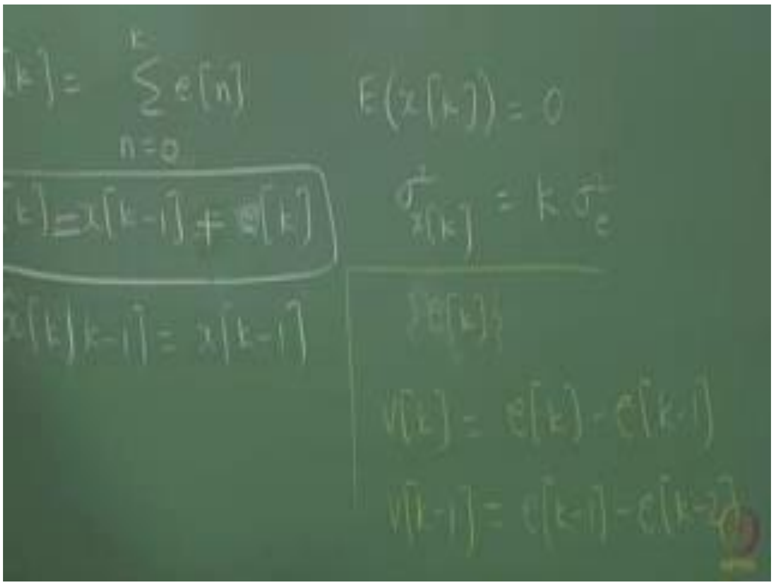

In the simplest case I gave you realization of white noise you would in; let us say bother to look at the series to visually inspect the realization or even conduct the basic tests you simply said OK, now I being talked at the first step is differencing the series and it let us see if it makes a difference. So, you ended up differencing the series and let say you produce a new series let us call at as v which is a result of differencing this white noise.

What have we done here? The original series is coming out of a white noise process right; that means, there is no model to be built, now differencing has actually introduced some kind of a correlation right, you can actually take 2 successive observations of v k and you will find they are correlated, if I have to the expression for v of k minus 1 or even k plus 1 we will see the same thing. So, this is the expression for v of k minus 1 you can see that there is a correlation between v k and v k minus 1, likewise v k and v k plus 1. This is artificially introduced by virtue of differencing, and we will see later on that this can cause problems as is you can see qualitatively you have introduced by accidentally differencing the series as spurious correlation and you say now I have a series that is correlated, I will build a model.

If that is the joy that you want, you can go ahead and do it, even if I give a white noise process you want to build a model you are so desperate then you difference a series and then build a model and say this is the model and then you again go back to your original series.

Ultimately remember you have to forecast this series that has been given to you, you may build a model for v and then come up with a forecast equation all of that is not necessary at all, had you followed a systematic procedure you would not have actually run in to this unnecessary differencing. In system identification which we do not study here, unnecessary differencing can actually amplify the noise effects in the measurement; we will not go into that. So, there are precautionary measures that your take, you have to exercise some restraint when it comes to differencing, just do not keep on differencing, it is like you know beating until the series speaks to you, do not thrash it up. It is going to have its own repercussion. We will come to all of this again when we look into, study arima models.