Applied Time-Series Analysis
Prof. Arun K. Tangirala
Department of Chemical Engineering
Indian Institute of Technology, Madras

Lecture - 13
Lecture 06C - Probability and Statistics Review (Part 2)-3

Alright, there is another notion; I mean that is built on this conditional expectation, it is a concept called iterative expectation which is again useful for theoretical analysis more than anything and this is the idea of iterative expectation.

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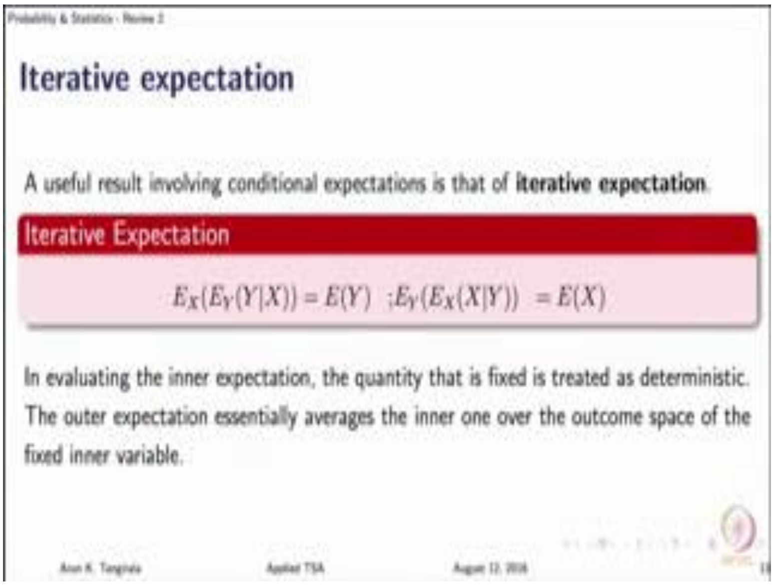

So, if you look at the conditional expectation, what we are doing is we are anchoring X on to some value or within the vicinity of some value and evaluating the expectation.

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So, if you go back to this two dimensional plane that we have here X and Y, let us say for the purpose of illustration we have anchored X here and we are evaluating the expectation along this direction here. Now many a times I am interested in the at least the I would say unconditional expectation, the strict word would be the marginal expectation, but let us assume that X is only dependent on Y at the most or Y is dependent only on X is no other factor. So, we will use the term unconditional expectation.

So, suppose I want to find out the unconditional expectation of Y; how do I do that? Well from the joint density I evaluate the marginal density of Y and then calculate the unconditional expectation that is a standard root but suppose I know the conditional expectation of Y given X, can I use that to calculate the unconditional expectation and the result that you see there are the so called expression of the identity that you see there can be intuitively understood this way. You have two layers of expectations there; one expectation which is the inner one is being evaluated in the outcome space of Y correct so; that means, I have walked along this direction; at a given value of X what I need is the unconditional expectation of I which means regardless of the values that X takes which means that I have to be able to evaluate along the entire 2 d plane; in this case I have only walked along one dimension but located at x.

So, now the next step is to walk along the other dimension and that is what the outer expectation is doing. It is being evaluated in the spay outcome space of X, you can also look at it this way; the inner expectation is a function of X. So, you are going to evaluate expectation of g of X; that is what it is and eventually you will get the unconditional expectation of Y and likewise unconditional expectation of X. So, this iterative expectation is actually used in improving many results; we do not use it in practice in any data analysis. So, this result is something that is of purely of theoretical interest I should say alright, we will move on; we will see the use of this result later on.

Now let us understand this notion of independence it is also very timely we have Independence Day around the corner. So and it is perfect timing all the time because time series analysis offered in the odd semester this notion of independence always comes up before the independence day. So, we talked about this concept earlier in fact, some of you have answer this correctly also that when the conditional density works out to be the same as the marginal density then we say that the two events are independent that is one way of defining.

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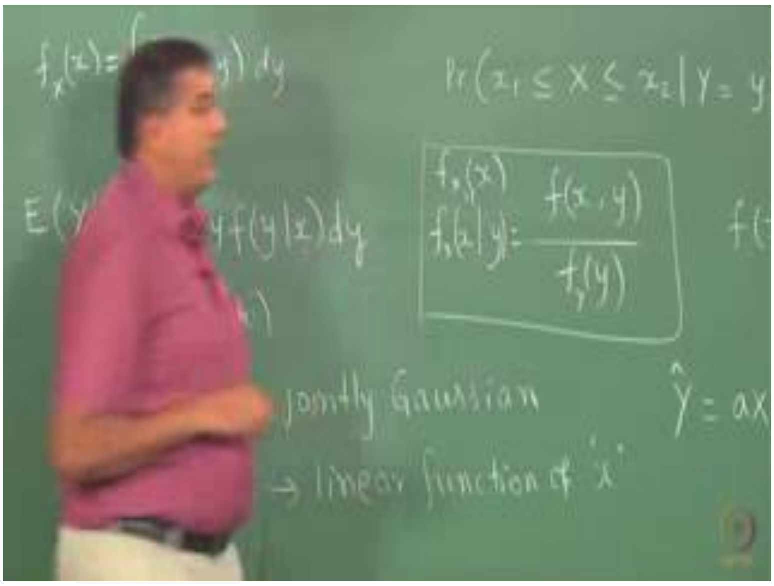

So, you look at it this way here fx of x given y is the same as fx of x. So, then when this is equal we say they are independent; that means, does not matter what has occurred in the y domain, the marginal density remains unperturbed and you can write this result in many; at least in two different ways one way is what you see on the slide that the joint density now can be factorized into the product of the individual marginal densities and that is clear even from this definition or from here you can actually replace this with the marginal density and you get the result.

Or the other way of stating independence is, as we have written here the conditional density is the same as a marginal density whichever way you write it is. What this also implies is that the conditional expectation is the same as unconditional expectations clearly, look at it this here if suppose y and x are independent then the conditional density is the same as marginal density. So, when you replace this with the marginal density you get your unconditional expectation. The pdf is a super boss always then comes your expectation and then comes your notion of covariance in that order and we will understand that in today's lecture and in the lecture on Tuesday as well that this is the order of hierarchy; independence is the strongest inference that you can draw.

If two events are independent, they are uncorrelated, their conditional expectations are identical to the unconditional all the things because whether you look at expectations or whether you look at covariances; there all derived as moments of pdfs, if p d s themselves satisfy a certain relation then you should expect the moments also to satisfy it, I mean at least intuitively the moments also to satisfy or enjoy certain properties not the other way round.

Covariance is an inference on the second order moment only very very often I see not only students but many researchers even in conferences, public forums using independence and uncorrelatedness interchangeably unfortunately that is not correct may be from a dictionary sense you can perhaps, but in a statistical sense independence is a strongest statement that you can make about the relation between x and y. In fact, if x and y are independent there is no model that you can build for y using x; that means, it does not matter whether you give me x or not, the prediction of y is unaltered.

Whereas uncorrelatedness as we will see today in the next lecture only makes statements about the absence of linearity or presence of it that is all. So, independence rules out any relation between y and x even it cannot be like as I always say it cannot be like uncles, cousins, aunts, daughter nothing no relation absolutely these are two different events that are happening; it is a very strong statement whereas, we say no two human beings are related at some at some level they are related; uncorrelatedness rules out immediate relation, maybe they are not in your family or in your next first cousin and so on, that is about it stops at that whereas, independence rules out any kind of relation.

So let us briefly talk about covariance and then we will continue our discussion in the next class do not close your books yet this is just to wake you up sometimes, students are waiting for the class to get over I do not know somehow I mean even as a student I used to be inattentive but the moment I hear somehow that word, it triggers a nice thing in your brains that coming to a close yes then you are alert. So, sometimes we can use that as a weapon to wake up, so as I said independence makes a very strong statement about the relationship between y and x. Now we are interested, we have said already we are going to be linear models; so we would like to know even before we build a linear model whether there exists a linear relationship and covariance is that one tool which gives you a very good assessment of the presence of linear relationship between any two random variables.

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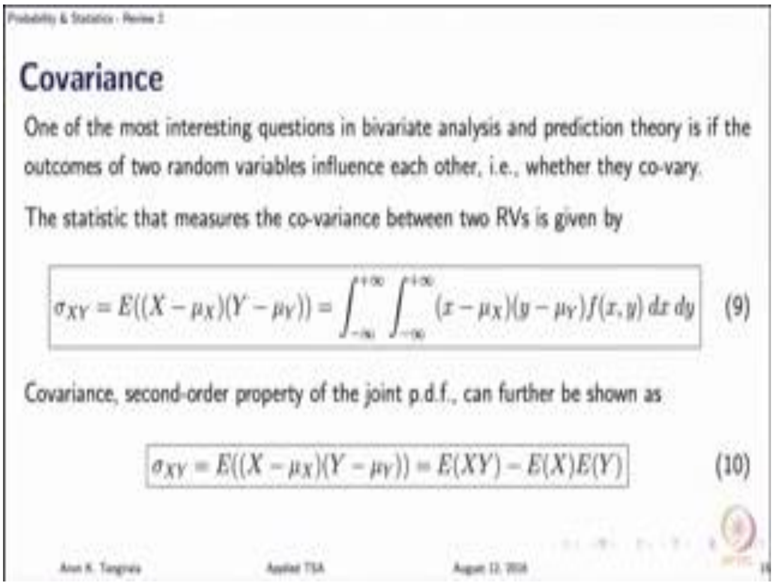

Now, we are doing all of this for the random signals whether particularly this covariance concept and so on or the linear relations on. A similar set of theory exist for deterministic world as well but we will not talk about it at this moment when the time comes we will talk about it. So, the covariance is defined as the second moment like we define variance as a second moment of the individual density functions. Covariance is the second central moment of the joint pdf as you see on the slide and of course you can rewrite the covariance as expectation of X Y which is the second moment; not second, central moment minus the product of the first order moments or first moments you can say and often this other relation that is sigma X Y being expectation of X Y minus the product is quite useful in making certain statements or even in calculations.

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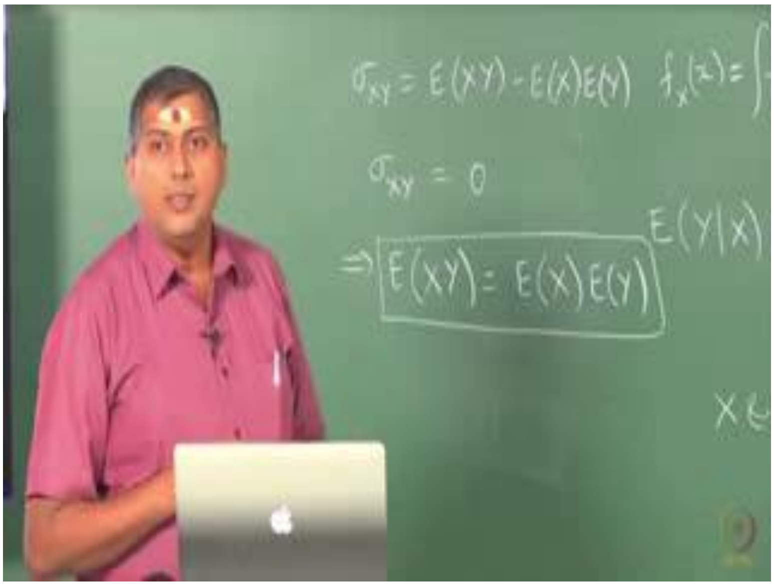

So this is the expression for covariance; once again you may think you know I need to know the joint pdf and so on. Yes for theory you need when we go to estimation, we will give an expression there we will come across several ways of estimating covariance. We will talk about that at that point in time but let us come back to this here; we said independence is when f of x comma y is simply f of x times f of y.

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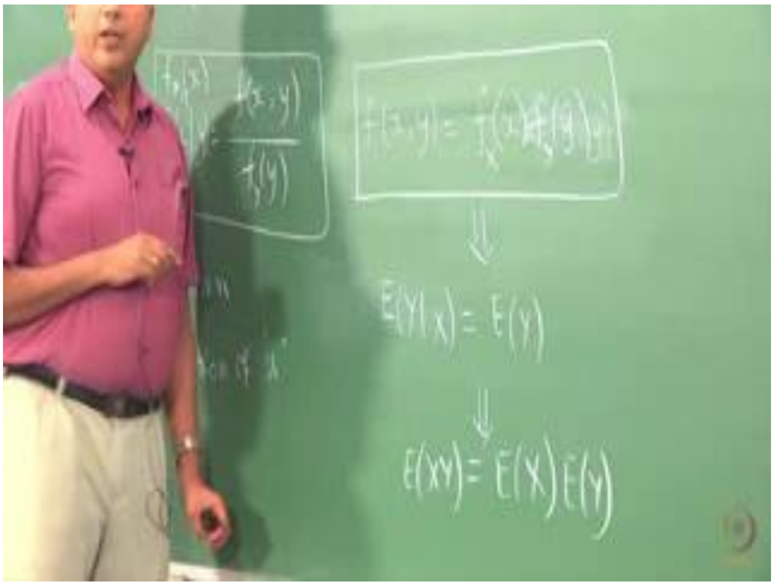

So, we will keep that in mind; now let us look at uncorrelated. So, this is what is independence and as I said strictly speaking, you need to have the marginals there. Suppose x and y are uncorrelated, we have not introduced correlation but you can think of correlation as a standardized measure you must have already sat through the lectures. Two variables are uncorrelated, if the covariance between them is 0; remember covariance implicitly has it named co variance we are measuring whether x and y; the variation in x and y; the spread in x and y are related. If x moves in this way, is it affecting y and vice versa, so two variables are uncorrelated when the covariance is 0. In other words the expectation of the product is the same as a product of expectations. Independence says the joint density is a product of the individual or the marginal densities whereas, uncorrelatedness is essentially saying that the expectation of the product is a product of expectations.

Now, you can see straight away that independence implies uncorrelatedness and not necessarily the other way around because if you look at expectation of X Y; what would be the theoretical definition of expectation of X Y, X times Y it is not double integral X times Y times a joint density d X d Y right you can, if they are independent then you can bring in this relation and factorize a double integral into a product of single integrals and then prove that the expectation of the product is a product of the expectations but the other way need not be true and that is what we mean by uncorrelatedness not implying independence.

So, independence rules over all non-linear relation; any relation of course, that means, linear also whereas, uncorrelatedness actually rules out only linear relations. So we have not proved that, we have not seen that yet as to how lack of correlation or lack of covariance means that there is no linear relation alright that we will have to talk about correlation and then we will also have to talk about partial correlation and so on but this is something to keep in mind and we will leave this class with an important point.

Independence would mean that the conditional expectations, let me write this here for example, you can write this and likewise for X as well and this and then you could also have here uncorrelatedness alright but not the other way round. So, independence implies everything that the conditional expectation the same as unconditional expectation, that the variables are uncorrelated but not the other way round. So, we will begin the next class with just a brief disposition on this; exposition on this.