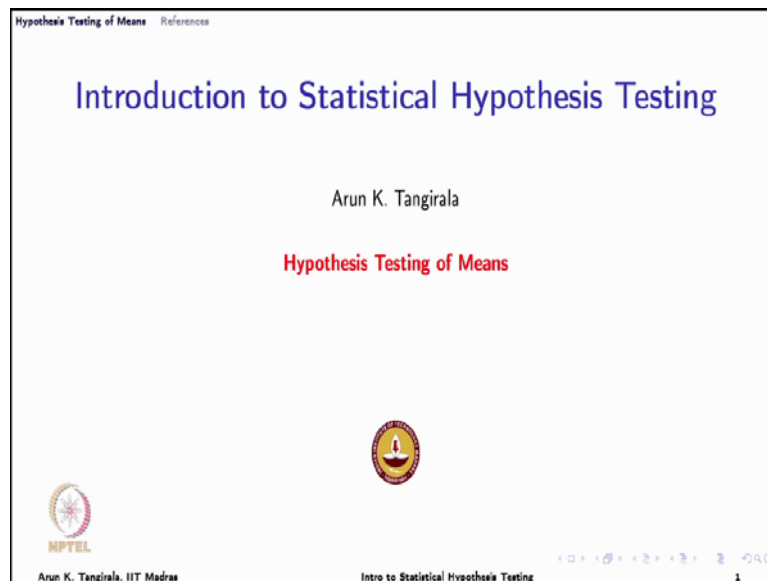


**Introduction to Statistical Hypothesis Testing**  
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**Indian Institute of Technology, Madras**

**Lecture – 11**  
**Hypothesis Testing of Means**

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So, welcome to the lectures on Introduction to Statistical Hypothesis Testing. In the previous lecture we learnt somewhat in detail the basics of hypothesis testing, the terminology and more importantly the interpretations of the statistical terms and the quantities that we encounter in hypothesis testing namely type I error, type II error, significance level and so on.

What we are going to do in this and the next lecture is essentially learn, how to set up the hypothesis test and run the test using all the theory that we have learnt earlier and the procedure for hypothesis testing that we learnt in a previous lecture.


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Hypothesis Testing of Means References

## Learning objectives

- ▶ One-sample  $z$ -test and  $t$ -test
- ▶ Two sample tests
- ▶ Paired test

for the **mean** with illustrations in R.

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In this lecture particularly, we will look at the tests on mean alone because there are a variety of situations associated with mean testing compare to that of variance and proportions, this is an exclusive lecture on the testing of means. To be more specific what we are to going to look at so called One-sample meaning One-population,  $z$ -test again here,  $z$ -test means that we are going to look at either large sample size or data coming from a Gaussian population and then the  $t$ -test which will be concerned with testing of a mean of a single population but with unknown variance, in the  $z$ -test the variance is also known that is important. But, the  $z$  test can also come into play when the variance is unknown, but you have a large sample size. The  $t$ -test comes into picture when you have small sample size and the variance being unknown and the data comes from a Gaussian population.

Then we have the two-sample tests, meaning we are going to look at 2 populations and compare the means where you have again unknown variance, the equal variance and the unequal variances followed by the Paired test. If you recall the Paired test corresponds to the tests for situations where you have a before after type situations, either after treatment or after a program that the individuals go through before and after you want to check the effectiveness of the program and so on. And importantly, we will also learn how to conduct this hypothesis test in  $r$  in this lecture. Let us begin with the so called one sample  $z$ -test for mean.

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Hypothesis Testing of Means    References

## One sample $z$ -test for mean

**Goal:** Test for the mean of a single population  
**Assumption:** Population standard deviation is known, large sample size

1. **Null:**  $H_0: \mu = \mu_0$   
**Alternate:**  $H_a: \mu <, >, \neq \mu_0$  (lower-, upper-, Two-tailed)
2. **Test statistic  $Q_T$ :**  $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim \mathcal{N}(0, 1)$
3. **Critical region  $R_c$ :**  $z < -z_{\alpha}, z > z_{\alpha}, \{z < -z_{\alpha/2} \text{ or } z > z_{\alpha/2}\}$ .

Use `qnorm` to compute critical values in R

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So the parameter that we are testing is mean of a single population, assumptions of course I will not state this repeatedly it is assumed implicitly that the observations are drawn randomly which means the sample constitutes a random sample. Apart from that we are we assume that the population standard deviation is known and that we have a large sample size of course, this large sample size is not a required assumption if the sample is being drawn from a Gaussian population we have repeatedly learned this. Again, we have this is a generic statement of null hypothesis and alternate hypothesis the actual alternate hypothesis depends on the problem in hand, and then as usually we have learnt many a times that the test statistic for this is a standardized sample mean, which follows a standardized Gaussian distribution and the critical region once again depends on the alternate hypothesis and the significance level that we specify. If you recall from the previous lecture in order to compute the critical value for a given alpha we can use `qnorm` in R.

Alright, so let us look at an example here.

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Hypothesis Testing of Means    References


### One sample $z$ -test for mean: Example

**Example: Propellant burning rate**

To test: Average propellant burning rate is  $\mu_0 = 50$  cm/s given,

$$\bar{x} = 51.3 \text{ cm/s}; \sigma = 2 \text{ cm/s}; n = 25$$

**Solution:**  $H_a: \mu \neq \mu_0$ ,  $z = (\bar{x} - \mu_0)/(\sigma/\sqrt{n}) = 3.25$ .  
Critical value at  $\alpha = 0.05$  is  $z_c = 1.96$ .  
Reject  $H_0$  at  $\alpha = 0.05$ .



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Again, this is our familiar example of the Propellant burning rate from the book by Montgomery et al, the goal here is to test whether the average propellant burning rate of the population is 50 centimeters per second. For this we draw a random sample of size 25 and estimate the sample mean to be 51.3 centimeters per second and the standard deviation is known let say it is known to be 2 centimeters per second. Clearly we have discussed this a many times before the alternate hypothesis is that the true population is not equal to the postulated value which is 50 centimeters per second. Compute the  $z$  statistic which is  $\bar{x} - \mu_0$  by  $\sigma$  over root  $n$  and it works out to be 3.25. And then the critical value as we have computed in the previous lecture is 1.96, this is a standard thing for a Gaussian distribution at alpha 0.05 for a two-tailed test the critical value is  $z_c$  is 1.96 it is nothing, but  $z_{\alpha/2}$ . Since, the observed statistic is lying to the right of the critical value we reject the null hypothesis at this significance level. Once again you can use the P-value method also to determine the outcome of this hypothesis test, in fact this is nothing, but a recap of what we did in the previous lecture right. Therefore, we will not discuss much,

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Hypothesis Testing of Means References

## One-sample $t$ -test for mean

**Goal:** Test for the mean of a single population  
**Assumption:** Population standard deviation is **unknown**, small sample size.

1. **Null:**  $H_0: \mu = \mu_0$   
**Alternate:**  $H_a: \mu <, >, \neq \mu_0$  (lower-, upper-, Two-tailed)
2. **Test statistic**  $Q_T: T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1)$
3. **Critical region**  $R_c: t < -t_\alpha(n-1), t > t_\alpha(n-1), \{t < -t_{\alpha/2}(n-1) \text{ or } t > t_{\alpha/2}(n-1)\}$ .

R: Use `t.test` with `conf.level = alpha`

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let us move on to the second example or a second scenario where we are looking at testing mean of a single population with a unknown variance and assume that we have small samples. If we have large samples then we can actually go back to the z-test itself. So, assumption is that the population standard deviation is unknown and that we have a small sample size. The null as usual is as before and the alternate could be one of the triplet. Test statistic we have now t-statistic in place of z-statistic which is more or less like the z-statistic, but with the theoretical standard deviation replace by the estimated  $S$ . This test statistic follows a t distribution with  $n$  minus 1 degree of freedom. We have already discussed you may refer to previous lectures, why this test statistic follows a t distribution particularly with  $n$  minus 1 degrees of freedom and a critical region is as usual we do not have to elaborate much on it. Now, when you want to conduct these kinds of test in R you can use they built in command from the stats package in R, which is the `t.test` routine with the `conf.level`, this `conf.level` is but your significance level sometimes it is called a Confidential level and other times it is called Significance level, but remember they are one and the same. So you specify at whatever value of alpha you want to work with.

Let us look an example here.

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Hypothesis Testing of Means References

### One sample $t$ -test for mean: Example


**Example: Training Method 'A'**

A training method 'A' is considered effective if the mean score is at least 75 on a test conducted post-training.

Null: Mean score for a trainees is at least  $\mu_0 = 75$  given,

$$\bar{x}_A = 70, s_A = 3.366, n_A = 10$$

**Solution:**  $H_a : \mu < \mu_0, t = (\bar{x} - \mu_0)/(s/\sqrt{n}) = -3.91$ .  
Critical value at  $\alpha = 0.05$  is  $-t_{0.05}(9) = -1.8333$ .  
Reject  $H_0$  at  $\alpha = 0.05$ .



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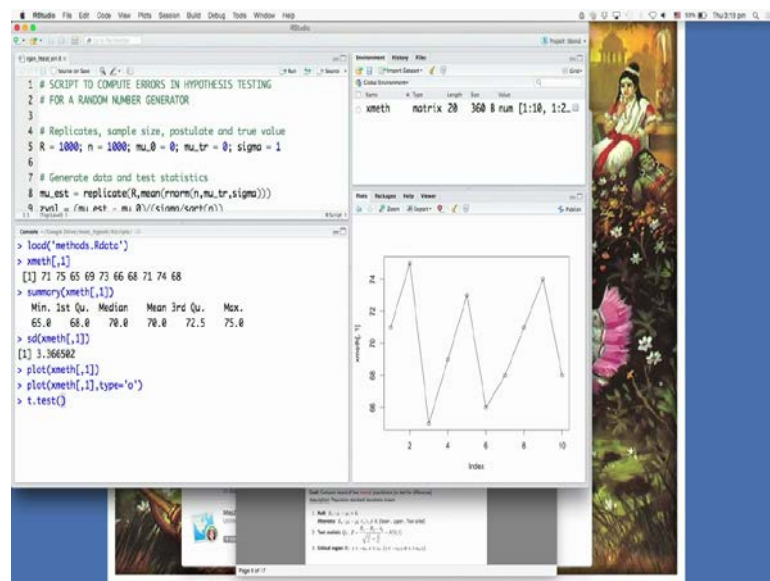
Training method A, which we have discussed in the motivation lecture when we were taking of comparison of means; so training method A is considered effective if the mean score is at least 75 on a test that is conducted after the training session. The null here, is that the mean score for a trainee is at least 75, right and given of course some data, let us talk about the alternate hypothesis. Alternate hypothesis is it is n, so together they span the entire set of possibilities. So, what we are searching for here is evidence to reject this null hypothesis that the training method is effective. Of course, you can also have another kind of situation depending on the null; here the null is that the main score is at least 75. If you think that the default, that means in the absence of data if you are willing to accept that the method is effective this is the null. But if you say no in absence of data I would say that the method A is not effective that is what is your null, then you would say that the null is mean score is at most 75.

And then, the alternate becomes the compliment of it. So here, we are assuming that by default will accept this training method. So, essentially where giving benefit of drought to the person who has come up with this training method, but that need not be. Sample of 10 individuals are drawn at random and the average scores turns out to be 70 with a standard deviation estimated to be 3.366. It fix in perfectly with the scenario of one-sample t-test and we compute the t-statistic turns out to be minus 3.91 and this is a one sided test therefore, we seek the corresponding critical value from the t distribution with 9 degrees of freedom because n is 10 here and that turns out to be minus 1.833. Since the

observed statistic is more extreme than what is tolerable we reject the null hypothesis at this alpha. I have this data set loaded and in fact we will upload this in R, this example is from the book by Ogunnaike.

Let me show you how to load the data set and run the t-test on your machine. If you want to fire up R on your laptop get ready pause a video, alright.

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So, let us see here you will be supplied this file called methods dot Rdata and use the load command to load it and you will see in the environment a variable called xmeth loaded. It has 2 columns in it, scores for training method A training method B because this is an example that will visit again later on when we compare means. We are interested at this moment in the first column, it is a small size. So, we can look at the first column you have 10 observations that is a data, although the data is printed as a row it still a column data please remember that. In fact, we can ask for a summary these are called summary statistics, it gives you mean median and also min and max which gives you an idea of the range.

You can also estimate now the standard deviation which is what I have reported for you in the example. Let us also plot as I always say it is good habit to visualize your data and let us do that, this how it looks like maybe it is better to plotted as a line over laid on marks. We can ask for over laid plot. So you can see here, this is how the data looks like for the method A.

Now, we can also draw box plot, play around with it let us get to the point here, run the t dot test look up the help on t dot test before you do this, there are a number of options. You can bring up the help on d dot test.

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```

1 # SCRIPT TO COMPUTE ERRORS IN HYPOTHESIS TESTING
2 # FOR A RANDOM NUMBER GENERATOR
3
4 # Replicates, sample size, postulate and true value
5 R = 1000; n = 1000; mu_0 = 0; mu_tr = 0; sigma = 1
6
7 # Generate data and test statistics
8 mu_est = replicate(R, mean(rnorm(n, mu_tr, sigma)))
9 zval = (mu_est - mu_0)/(sigma/sqrt(n))
10
11 # Distribution of statistic
12 hist(zval, probabilitv = T, col="grey", main="Histogram of test statistic")
13
> load("methods.Rdata")
> xmeth[,1]
[1] 71 75 65 69 73 66 68 71 74 68
> summary(xmeth[,1])
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  65.0   68.0   70.0   70.0   72.5   75.0
> sd(xmeth[,1])
[1] 3.366582
> plot(xmeth[,1])
> plot(xmeth[,1], type="o")
> ?t.test
> |

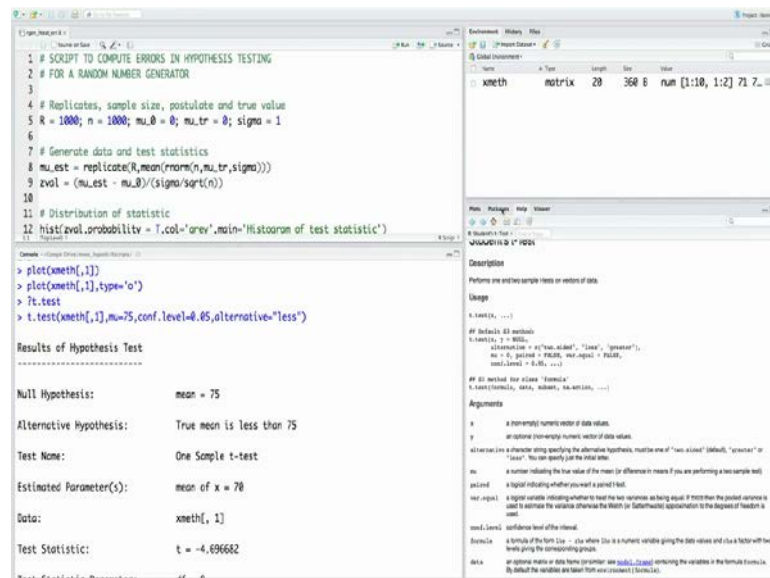
```

You can see that the t dot test is suited for both seeing one-sample and two-sample, single population and two populations.

And then, there is an option to specify the alternative has two sided or less and greater means lower or upper tail test, then you specify the confidence dot level which is nothing but your significance level. Then later on we will have an option called where dot equal that is for a comparison of means of 2 populations with variances unknown, do we assume that the variance of 2 populations are equal or unequal. By default it says it is falls; that means it assumes that the variability is different. And then, there are a few other things but will not a worry about that, alright. So, what is the default version of t dot test that it assumes it is a one-sample t-test and also there is a mu equals 0 forgot to mention that, that mu is nothing but your postulated value. In this example the postulated value is 75; we need to specify that otherwise the default value is 0.



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```
1 # SCRIPT TO COMPUTE ERRORS IN HYPOTHESIS TESTING
2 # FOR A RANDOM NUMBER GENERATOR
3
4 # Replicates, sample size, postulate and true value
5 R = 1000; n = 1000; mu_0 = 0; mu_tr = 0; sigma = 1
6
7 # Generate data and test statistics
8 mu_est = replicate(R, mean(rnorm(n, mu_tr, sigma)))
9 zval = (mu_est - mu_0)/(sigma/sqrt(n))
10
11 # Distribution of statistic
12 hist(zval, probabilitv = T, col='grey', main='Histogram of test statistic')
```

```
> plot(xmeth[,1])
> plot(xmeth[,1], type='o')
> Tt.test
> t.test(xmeth[,1], mu=75, conf.level=0.85, alternative='less')
```

Results of Hypothesis Test

Null Hypothesis: mean = 75

Alternative Hypothesis: True mean is less than 75

Test Name: One Sample t-test

Estimated Parameter(s): mean of x = 70

Data: xmeth[, 1]

Test Statistic: t = -4.696682

Let us do that. We will ask for t dot test, the first variable method A and we specify mu to be 75 and conf dot level by default is 0.05, let us be sure to specify that, right, but alternative that is very important, our alternative here is that the mu is less than 75. So we say alternative is less and we run the test. Here is what we would get here as t dot test. Let me actually make a small change here it looks like t dot test has been taken from another package that I have loaded.

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```
1 # SCRIPT TO COMPUTE ERRORS IN HYPOTHESIS TESTING
2 # FOR A RANDOM NUMBER GENERATOR
3
4 # Replicates, sample size, postulate and true value
5 R = 1000; n = 1000; mu_0 = 0; mu_tr = 0; sigma = 1
6
7 # Generate data and test statistics
8 mu_est = replicate(R, mean(rnorm(n, mu_tr, sigma)))
9 zval = (mu_est - mu_0)/(sigma/sqrt(n))
10
11 # Distribution of statistic
12 hist(zval, probabilitv = T, col='grey', main='Histogram of test statistic')
```

Null Hypothesis: mean = 75  
Alternative Hypothesis: True mean is less than 75  
Test Name: One Sample t-test  
Estimated Parameter(s): mean of x = 70  
Data: xmeth[, 1]  
Test Statistic: t = -4.696682  
Test Statistic Parameter: df = 9  
P-value: 0.000562709  
5% Confidence Interval: LCL = -Inf, UCL = 68.0485

```
> stats::t.test(xmeth[,1], mu=75, conf.level=0.05, alternative="less")
```

So this is what happens in R, when you run a certain command there may be a routine with a similar name in another package and what I am trying to show you here is a t dot test that come from a stats package. There is a t dot test that also comes with env stats package that will use in the later lecture. And t dot test that I just ran from the n stats package. So, let us make sure that I show you the t dot test from the stats package, alright; so when we do that.

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```
> t.test(xmeth[,1], mu=75, conf.level=0.05, alternative="less")
```

One Sample t-test

data: xmeth[, 1]  
t = -4.6967, df = 9, p-value = 0.0005627  
alternative hypothesis: true mean is less than 75  
5 percent confidence interval:  
-Inf 68.0485  
sample estimates:  
mean of x  
70

```
> detach("package:EnvStats", unload=TRUE)
> t.test(xmeth[,1], mu=75, conf.level=0.05, alternative="less")
```

Or the other thing is to just unload the package env stats. So this is the output that you would see when you run the t dot test package, here I have forcefully unloaded the other package which had a conflicting t dot test just to make things simple and this is that output that you will see if you are using the t dot test from the a stats package. If you want to go back to the env stats package and use this is the kind of output that you would see that from the env stats package. In fact, it is very nice the output is nicely formatted compare to the output that you see with the stats package. Let us look at this because later on you will have to install the e n v stats package and load that package as well, so we might as well understand what this t dot test look. The output is the same.

It is just the way the output is presented is making a difference and you have this output formatted in a very nice manner. So what you have here, the null hypothesis is mean 75 confirms with what we have given, alternative hypothesis is again is what we want and it has conducted the one-sample t test. The estimated parameter is 70 the sample mean and then you have the test statistic working out to be minus 4.69. Alright, so the actual test statistic is minus 4.69 this is why it is important to run the data for yourself of course, in this example there was an error but I will leave it at that, may be when I pose the notes I will correct in the slides. We have minus 4.69 will add a note to the slide there I will not correct the value. The degree of freedom is 9 and the p-value is reported. So the p-value is way low I mean it is quite low it does not matter how low it is, it is much lower than alpha. What does it tell us it means that the test statistic has fallen to a more extreme value than tolerable; therefore, this null hypothesis has to be rejected.

And also what you see here is confidence interval which I will briefly talk about later on and more so in detail in a lecture later beyond the next lecture. So, the test does not tell you whether you should reject or not, that is what is important to know it reports a p values, it reports the test statistic and of course the critical value you will have to calculate from the qnorm. But, since you know the p-value you might as well avoid calculations of the critical value and straight away stay that the null hypotheses rejected. For your own convenience, I have reported the critical value here. Whereas, the t dot test gives you the P-value either way null hypotheses rejected.

Now let us look at the two-sample z-test for mean.

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Hypothesis Testing of Means    References

## Two sample z-tests for mean

**Goal:** Compare means of two **normal** populations (or test for differences)  
**Assumption:** Population standard deviations known

1. **Null:**  $H_0: \mu_1 - \mu_2 = \delta_0$   
**Alternate:**  $H_a: \mu_1 - \mu_2 <, >, \neq \delta_0$  (lower-, upper-, Two-tailed)
2. **Test statistic**  $Q_T: Z = \frac{\bar{X}_1 - \bar{X}_2 - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim \mathcal{N}(0, 1)$
3. **Critical region**  $R_c: z < -z_{\alpha}, z > z_{\alpha}, \{z < -z_{\alpha/2} \text{ or } z > z_{\alpha/2}\}$ .

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Again, what we mean by z-test is that the population standard deviation are known, the parameters of interest are still mean. If you look at the problem statements fairly generic the null that we want to test is the difference between the means of the populations is a postulated value delta naught and the alternate hypotheses one of the 3 possibilities. Test statistics we are seen earlier the standardized sample difference in sample means follows a standard Gaussian distribution and the critical regions are as before.

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Hypothesis Testing of Means    References

### Two sample $z$ -tests for mean: Example

**Example: Battery life comparison**

To test: Battery A has longer lifetime (in hours) than Battery B.

Given

$$n_A = 40, \bar{x}_A = 647, \sigma_A = 27; \quad n_B = 40, \bar{x}_B = 438, \sigma_B = 31$$

**Solution:**  $H_a: \mu_A - \mu_B > 0, z = 1.38$ . Fail to reject  $H_0$  at  $\alpha = 0.05$ .

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Here there is no built command per say for z-test because it is a one of the simplest things that you can do by yourself, so there is no routine but you can write a small function or a script to do that for you. And let us go through this example the goal here in this example is to test that battery A has longer life time than battery B. So what is the null here, battery B existed before and someone has said that battery A that I manufactured is better, so the null is the battery A is has good as battery B because battery A has been introduced newly. In the alternate hypothesis is that battery A is better than battery B, essentially what we want to test goes and sits in the alternate hypothesis in some sense. But what we want to test also depends on what is null it is better to state both.

We have randomly drawn 40 specimens from both these populations sample means have been calculated and the standard deviations are known and the critical value, once you put in all this data through this generic procedure the delta naught, what is delta naught here in this case? It is very easy to guess it is 0. So, plug in the values of delta naught  $X_1$ ,  $\bar{X}_2$ ,  $\sigma_1$ ,  $\sigma_2$  and  $n_1$  and  $n_2$ , get the value of  $Z$  work it out it turns out to be 1.38. And which falls within a acceptable region why because alpha is 0.05 and we know that the z-statistic follows standard Gaussian therefore, the situation that we have here is, that the critical value it is a one sided test remember it is an upper tailed test, so  $z_{\alpha}$  is going to be 1.65 roughly and 1.38 is less extreme then what I willing to

tolerate and therefore I fail to reject the null hypothesis. In other words, what it means that the null which is battery A is as good as battery B we hold there is not enough evidence to believe that battery A has a longer life time than battery B that is outcome of this result of hypothesis test, alright.

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**Hypothesis Testing of Means** Reference

### Two sample $t$ -tests for mean

**Goal:** Compare means of two **normal** populations (or test for differences)

**Assumption:** Population std. dev. unknown, but  $\sigma_1 = \sigma_2$ ; small  $n_1, n_2$

- Hypothesis:** Same as in two sample  $z$ -test
- Test statistic:** (based on **pooled variance**)
 
$$T = \frac{\bar{X}_1 - \bar{X}_2 - \delta_0}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} \sim t(n_1 + n_2 - 2); \quad S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$
- Critical region:**  $t < -t_\alpha(\nu)$  (lower),  $t > t_\alpha(\nu)$  (upper),  
 $\{t < -t_{\alpha/2}(\nu) \text{ or } t > t_{\alpha/2}(\nu)\}$  (two-tailed);  $\nu = n_1 + n_2 - 2$ .

**R:** Use **t.test** with var.equal=T option

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So let now let us move on to the case, where the standard deviations are unknown we know there are 2 possibilities variances are equal, but unknown and the other possibilities variances are unequal and unknown of course. We assume also that the populations are Gaussian then only we can use this results strictly speaking otherwise it is going to be only an approximation. The hypothesis is the same as in the two-sample  $z$ -test that we went through, test statistic is different now based on that the fact that we are going to use pooled variance also, but even otherwise even in the variances were unknown the test statistic would be still this  $t$ -statistic which follows a  $t$  distribution with  $n_1 + n_2 - 2$  degrees of freedom.

Again, may be as we go through each of the scenarios you may want to keep the lectures on sampling distribution by your sight to see at what point and to go to the point where we discuss a sampling distribution of each of this statistics. The pooled variance is calculated using the given formula the critical regions are as usual based on the  $t$  distribution and a specified value of alpha. Here, we use again the  $t$  dot test with

where dot equal being set to true that is the case. So, let us look at an example now. We are going to compare training methods.

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Hypothesis Testing of Means References

### Two sample $t$ -tests for mean: Example

**Example: Comparing training methods**

To test: Method B is more effective than Method A. From data,

Given  $n_A = 10$ ,  $\bar{x}_A = 70$ ,  $s_B = 3.3665$ ;  $n_B = 10$ ,  $\bar{x}_B = 74$ ,  $s_B = 5.3955$

**Sol:**  $H_0 : \mu_A - \mu_B < 0$ ,  $t = -1.989$ ,  $t_{0.05}(18) = -1.73$ .

**Reject  $H_0$  at  $\alpha = 0.05$ .**

R: Use `t.test` with `var.equal = T` and `alternative="less"` options

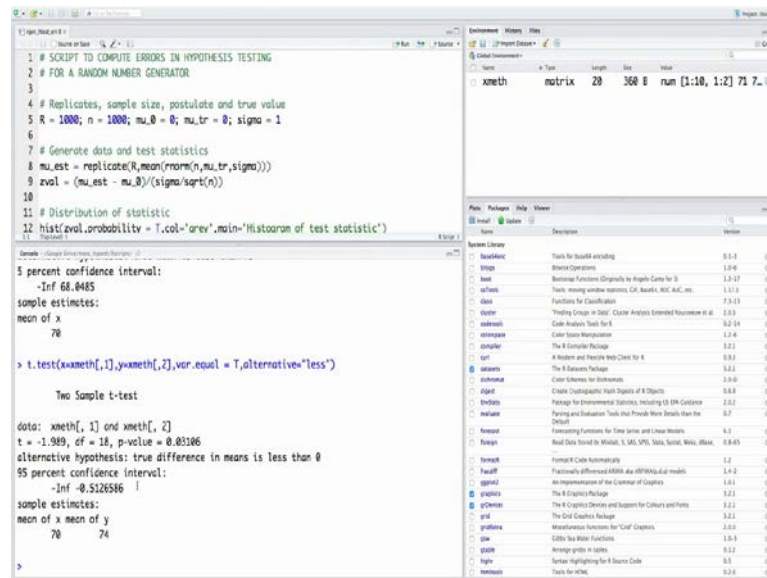
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Earlier we tested the effectiveness of method A, now method B has been introduced and we want to test if this method B is more effective than method A. So, the default is method B is as good as method A right like we did in the battery case, of course here B and A are reverse; there B existed was introduced now A exists and B is being introduced and being claimed as more effective. So, again we draw random samples from both these populations of size 10 and the data is also already preloaded for us.

We will go back in to R and test this, but let us see whatever numbers tell us. The numbers are reported alternate hypothesis is clearly  $\mu_A - \mu_B < 0$  and the test statistic turns out to be minus 1.989 when you plug in all the values here that is what the `t.test` routine essentially does for you. And the critical values minus 1.73 since minus 1.989 is more extreme then what I am tolerating remember this is a lower tailed test. My observed statistic falls to the left of the critical value and therefore, I reject the null hypothesis which means in deed method B is more effective than method A. Let us look at this in R. How to do it in R?

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```
1 # SCRIPT TO COMPUTE ERRORS IN HYPOTHESIS TESTING
2 # FOR A RANDOM NUMBER GENERATOR
3
4 # Replicates, sample size, postulate and true value
5 R = 1000; n = 1000; mu_0 = 0; mu_tr = 0; sigma = 1
6
7 # Generate data and test statistics
8 mu_est = replicate(R, mean(rnorm(n, mu_tr, sigma)))
9 zval = (mu_est - mu_0)/(sigma/sqrt(n))
10
11 # Distribution of statistic
12 hist(zval, probabilitv = T, col = "green", main = "Histogram of test statistic")
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```
5 percent confidence interval:
 -Inf 68.8485
sample estimates:
mean of x
 70
> t.test(x=xmeth[,1], y=xmeth[,2], var.equal = T, alternative="less")
Two Sample t-test
data: xmeth[, 1] and xmeth[, 2]
t = -1.989, df = 18, p-value = 0.03106
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
 -Inf -0.5126586
sample estimates:
mean of x mean of y
 70 74
```

Let us get back to R here and now perform the t dot test in the stats package and we say here x equals x method 1 that is our first column and y is x method is second column and we want to set where dot equal to true and the rest of, of course, mu is 0 by default we do not have to specify, the rest of the defaults hold in place. So, one thing that we will have to do is alternative should be less. So, that the critical value or the p-value is calculated correctly that is very important, right. That is what we had. If you look at the alternate it is lower tailed test and that is what we are doing, alright.

So, this is the outcome we could of course, use the env starts package see how the things comes out to be. As I said earlier, the output is the same it is just a formatting or the weights printed on the screen that makes a difference alright. So, the test statistic works out be minus 1.989 as I had reported in the slide for you, and the degrees of freedom is 18, because it is n 1 plus n 2 minus 220 minus 2, the p-value is given and the p-value is less than the specified value of significance level which is 0.5. That means what I have here is more extreme than what I am willing to tolerate, what I am willing to tolerate is p-value should have been 0.05 in other words. And, alternative hypothesis also reported for you and once again this t dot test reports a confidence interval as I said will come to that part of the output a bit later. That is it. So, this is how you implement things in R now let us get back to and move on to the next kind of situation.



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Hypothesis Testing of Means References

## Two sample $t$ -tests for mean: Unequal variances

**Goal:** Compare means of two **normal** populations (or test for differences)  
**Assumption:** Population std. dev. unknown, and  $\sigma_1 \neq \sigma_2$ ; small  $n_1, n_2$

- Hypothesis:** Same as before
- Test statistic:**
$$\tilde{T} = \frac{\bar{X}_1 - \bar{X}_2 - \delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t(\tilde{n}_{12} - 2); \quad \tilde{n}_{12} = \left\lfloor \frac{(S_1^2/n_1 + S_2^2/n_2)^2}{\frac{(S_1^2/n_1)^2}{n_1+1} + \frac{(S_2^2/n_2)^2}{n_2+1}} \right\rfloor$$
- Critical region:** Same as before.

R: Use **t.test** with **var.equal=F** option

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Where we have unequal variance, this story is more or less similar for as in the equal variance case except that the degrees of freedom calculation is a lot more involved and the way we estimate the variances are different.

Earlier, we used a pooled estimate obviously, because we assume the population variances to be equal, we exploited that fact and used naught 10 from A and 10 from B, we pooled it together to estimate the variance. Here we cannot afford to do that, variances are different therefore, we have to use a respective random samples to estimate the standard deviations and rest of the story is a same, except that now you use variance dot equals; equal falls in the t dot test which is the default.

(Refer Slide Time: 27:00)

Hypothesis Testing of Means References

## Two sample $t$ -tests for mean: Unequal variances

**Example: Comparing yield data**


It is desired to test if two reactors A and B have the same yield.

From data,  $n_1 = 50$ ,  $\bar{y}_1 = 75.52$ ,  $s_1 = 1.4314$ ;  $n_2 = 50$ ,  $\bar{y}_2 = 72.47$ ,  $\sigma_2 = 2.764$

**Sol:**  $H_0 : \mu_1 - \mu_2 \neq 0$ ,  $t = 6.92$ ,  $\bar{n}_{12} = 73$ , C.I.:(2.169, 3.924).

**Reject  $H_0$  at  $\alpha = 0.05$ .**

**Q:** Suppose we wish to test  $\text{Yield}(A) - \text{Yield}(B) > 2$ . Then, the result?



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Now, the example is different here, it is desired to test if 2 reactors A and B have the same yield, alright and from data we have  $n_1$  equals 50 and  $n_2$  equals 50; we have the same the sample size, but of course, a sample means are going to be different, the standard deviations are different. Because the goal is to test if 2 reactors A and B have the same yield, alternate is naturally that they are not, one could have more or less than the other. And for this data we have here the  $t$  test statistic worked out as reported here 6.92, that degrees of freedom after all this calculations works out to be 73, this is much more complicated than in the equal variance case. Now, I report the confidence interval for you, which is 2.169 and 3.924 and we know that we can use here either the critical value; you can go and use the  $q_{norm}$  for example, sorry,  $q_t$ ; for this example at alpha equals 0.05. In fact, because it is a two-sided test we should be looking at  $q_t$  with a  $q_{norm}$  of  $1 - \alpha / 2$ ; that is how  $q_t$  works, with 73 degrees of freedom.

(Refer Slide Time: 28:33)

```
R Script:
1 # SCRIPT TO COMPUTE ERRORS IN HYPOTHESIS TESTING
2 # FOR A RANDOM NUMBER GENERATOR
3
4 # Replicates, sample size, postulate and true value
5 R = 1000; n = 1000; mu_0 = 0; mu_tr = 0; sigma = 1
6
7 # Generate data and test statistics
8 mu_est = replicate(R, mean(rnorm(n, mu_tr, sigma)))
9 zval = (mu_est - mu_0)/(sigma/sqrt(n))
10
11 # Distribution of statistic
12 hist(zval, probabilitv = T, col='grey', main='Histogram of test statistic')
13 }

sample estimates:
mean of x
78

> t.test(x=xmeth[,1], y=xmeth[,2], var.equal = T, alternative='less')

Two Sample t-test

data: xmeth[, 1] and xmeth[, 2]
t = -1.949, df = 18, p-value = 0.03106
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
 -Inf -0.5126586
sample estimates:
mean of x mean of y
 78      74

> qt(0.975, df=73)
[1] 1.992997
> load("yielddata.Rdata")
```

So, we can actually go and check in R, what are the critical values here let us ask; what is a critical value for 0.975 and degrees of freedom 73? It is fairly high it should be more or less like a Gaussian thing and you see that, it is almost close to what you would see with a Gaussian value, right. Now, there is this yield data that I have for you, you can also do it in R. So, whatever we did in the slides we can do it in R, let me clear this screen so that it is not jumbled up.

(Refer Slide Time: 29:15)

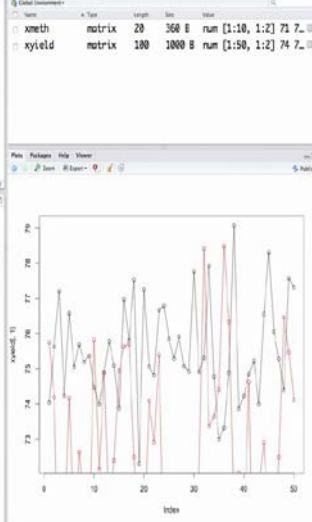
```
R Script:
1 # SCRIPT TO COMPUTE ERRORS IN HYPOTHESIS TESTING
2 # FOR A RANDOM NUMBER GENERATOR
3
4 # Replicates, sample size, postulate and true value
5 R = 1000; n = 1000; mu_0 = 0; mu_tr = 0; sigma = 1
6
7 # Generate data and test statistics
8 mu_est = replicate(R, mean(rnorm(n, mu_tr, sigma)))
9 zval = (mu_est - mu_0)/(sigma/sqrt(n))
10
11 # Distribution of statistic
12 hist(zval, probabilitv = T, col='grey', main='Histogram of test statistic')
13 }

> load("yielddata.Rdata")
> plot(xyield[,1], type='e')
> points(xyield[,2], col='red')
> lines(xyield[,2], col='red')
> t.test(xyield[,1], xyield[,2])

Welch Two Sample t-test

data: xyield[, 1] and xyield[, 2]
t = 6.5211, df = 73.519, p-value = 1.416e-09
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 2.169488 3.923792
sample estimates:
mean of x mean of y
75.5188 72.4722

> |
```



In any case, going back to before we jump into the R session, the critical value is 1.96

and the observed statistic is 6.92; it is quite extreme. In fact, it does not matter as I said how far it is to the right, it is to the right of the critical value which is roughly about 1.9394 and therefore, we reject the null hypothesis. What this means is, that indeed this 2 reactors have different yields, alright. So, let us actually see if we can run this in R on the data that we have, alright. So, this actually again loads a variable called x yield, we will make this data available to you. This example is from the book by Ogunnaike and what we want to run is the t dot test. In fact, it may be a good idea to plot the yield data both A of B and A. So, we can here plot the first column. So, this is the one that you see for A and we could also plot here lines or may be points for B and here change the colour, let us say to red, so that b is in red. So, you can see here that b is in red.

Maybe we can also do lines here along with points, there you go. So, of course, this is the problem with R, that it does not adjust the access automatically. We could have plotted B first and then plotted A, anyway you may want to actually plot A and B individually. There are ways of overlaying this, I am just doing a very preliminary thing here; just to get a feel of the variability that you are seeing in this data how different things are with respect to variability and averages and so on. You could do a box plot and so on.

But, let us focus on the task in hand which is to run the t dot test and here we have x yield, the first column and then yield the second column and variance naught equal false mu is 0 and it is alternative is two sided. So, all the defaults hold and we do this. So, let us see what we get here? Degrees of freedom is roughly 73, I mean it is 73 point something, but degrees of freedom has to be an integer because it has got to do with a number of sources of variability and that number is an integer. However, with the expression that we had for the calculation of degrees of freedom, you are not guarantee to get an integer. Therefore, in practice we round it off to the lowest value so that is the 73, to be conservative we round it off to the lowest one not to the higher one.

And then, the p-value is reported here extremely small, what does this tell us? It is very low, does not matter how low, it is much lower than alpha by 2 sorry, alpha. And therefore, the null hypothesis must go. The confidence interval that I have reported here is also displayed here. Now, let us understand very quickly what is conference interval here? The technical details of which will be discussed in later a lecture. The confidence interval offers another way of conducting the hypothesis test, it is essentially the region

in which we believe the true value is residing alright, but again that is not with 100 percent confidence. Because, alpha here is 0.05 we say that this is a 95 percent confidence interval; that means, there are 95 percent chances that the truth is contained in this interval. It might as well be outside this interval you may end of with the realization that will not give you an estimate which is consistent with this confidence region, but let me put it the other way, that there is a 5 percent chance that the truth is outside the interval; that is a way of understanding.

We will illustrate those points later on the technical discussion. Right now, let us quickly understand how to use this confidence region? If the postulated truth is contained in a confidence region then the null hypothesis will not be rejected, may not be rejected. Now, the postulated value for the difference in means is 0 and 0 is not contained in this confidence region that means, most likely the fact is the truth is not 0. There is a 5 percent chance that is a type one error, there is a 5 percent chance that truth is 0 and I am not able to capture that in this confidence interval. But, I am a kind of 95 percent sure that the truth is in this region, based on that assertion we reject the null hypothesis. Now, this is for a two-sided test, for a one sided test the confidence interval if you recall, looked different.

For example, for a lower-tailed test it was anywhere between minus infinity and some value or for an upper-tailed test it was some value the confidence region was some finite value and infinity. There also we can conduct hypothesis test using confidence region, the principle is the same, postulated null value should be in the confidence region, if I cannot find it in the confident region; that means, it is not one of the possible truth then we reject the null hypothesis and that is the case in this example as well, alright.

So, that is a scenario here you can see the postulated value is not. But, suppose we wish to test, now that we have realized that A and B are reactors with different yields and now suppose I wish to test that the difference now; that means, I know for sure A and B are different. Now, I want to test whether the yield of A is greater than yield of B by 2 percentage let us say, then what would be the result since you have the data. Now since you know the alternate hypothesis you can actually conduct the t dot test or t test and let us know the answer alright, on the forum or you know you can send an email privately; it is just a home work problem. You know what now the mu should be set to, do not forget to set the mu in t dot test to what the value, guess what that value is and then run the t dot

test accordingly with a right alternative, you will get the results, you will also get the confidence interval. See if you can use both the p-value and the confidence interval to arrive at the answer, very good.

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Hypothesis Testing of Means    References

## Paired test

**Goal:** Test for differences of means  $\delta = \mu_X - \mu_Y$ ;  
 $(X, Y)$  is an **ordered pair** from **normal** population.

**Main point:** The variables in the pair  $(X, Y)$  are **not** independent.

1. **Hypothesis:**  $H_0 : \delta = \delta_0, H_a : \delta <, >, \neq \delta_0$
2. **Test Statistic:**  $T = \frac{\bar{D} - \delta_0}{S_D / \sqrt{n}} \sim t(n-1); \bar{D} = \frac{1}{n} \sum_{i=1}^n D_i; S_D^2 = \frac{1}{n-1} \sum_{i=1}^n (D_i - \bar{D})^2$
3. **Critical region:**  $t < -t_\alpha(\nu), t > t_\alpha(\nu)$  (upper),  
 $\{t < -t_{\alpha/2}(\nu) \text{ or } t > t_{\alpha/2}(\nu)\}$  (two-tailed);  $\nu = n - 1$ .



R: Use **t.test** with paired=T option

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Now, the last in discussion is the Paired test. As you remember even in the case of sampling distribution of mean, this was the last scenario where we are still testing for difference in means, but with populations that are not independent. And, we had said at that time there are many scenarios, that many situations which corresponds to these scenarios. So, here the samples that I am, samples that I am drawing; form an ordered pair, that mean it is a same individual which goes through 2 different programs or may be an object that is before, let us say I am testing a detergent for its cleanliness index or effectiveness in terms of cleaning I have a cloth, dirt levels and then I put it through the cleaning agent or the detergent and then comes out, I have another cleanliness index that also falls into this scenario and so on.

So, the main point is that the variables in the pair are not independent unlike in all the previous comparison of means cases. The null hypothesis is now that the difference is the postulated value, here we use delta we do not say  $\mu_1 - \mu_2$ , but we simply test for the difference straight away,  $\delta = \delta_0$  and the alternate hypothesis is 1 of the 3 possibilities. Now, the test statistic is the t standardized t statistic,  $\delta_0$  is the postulated value, SD is estimated standard deviation from that given data and we

have gone through all of this and critical region is as usual. Now, in the t dot test as I said there are several options including that for paired, which is usually set to false now we set it to true and here we have an example of a weight-loss program.

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Hypothesis Testing of Means References

### Paired test: Example

**Example: Test a weight-loss program**

Suppose we wish to test in a weight-loss program if weight "before" is different from weight "after". From data,

$n = 20$ ,  $\bar{D} = 8.5$ ,  $S_D = 3.5615$

**Sol:**  $H_0 : \delta \neq 0$ , C.I.: (6.833, 10.167). **Reject  $H_0$**  at  $\alpha = 0.05$ .

**Q.:** Suppose a two sample t-test is used inadvertently. Then,?

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Where we wish to test if the weight loss program has caused any difference in the weight of course, the natural and a intuitive thing is to check if the weight has decreased, but it is possible that the program has also caused a harm, many a times people go through some weight loss programs and a after while immediately gain weight. So, we want to see if also that has happened and therefore, initially we will ask if there is a difference between the weights before and after.

Once again here I have the data for you that you can play around within R there are 20 points, the calculated difference and the standard deviation are reported here, alternative is delta not equal to 0 because we have only want to see if there is a difference and the confidence interval is reported here. Based on the guideline that I have given you earlier the postulated value 0 is not within the confidence interval, this 95 percent confidence interval and therefore, we reject the null hypothesis. So, what does it mean? That the weight loss program has indeed cost at difference in the weight. We have not tested however whether it has decreased the weight or increased the weight, that you can do. But let me, since you have the data, right. So, we will come to that. Let us actually see how we do this in the R.

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So, let me load the data here. So, the data is contained in weight loss dot R data will provide this data for you. And now, we have here x w t as the variable and once again we run the d dot test, x w t contains 2 columns. But anyway, before doing that let us actually plot x w t, I am sorry this is one against the other. So, let us plot.

Alright, so, this is the weight before the weight loss program of this 20 individuals and now, we can actually draw. So, you can see here. In fact, we can now perhaps join them by lines as well, like we did in the previous case. You can see they look pretty similar they better be because they are the same individuals and their weights before and after cannot be different; they can be different, but they should if it has been effective and for an individual you expect that it should be effective for a lot of other individuals sets well. But the point there are 2 points to observe, one that there is a strong correlation between the weights, which means they are not independent, remember we talked about independence and correlation. In fact, you can compute the correlation coefficient for this and also run a test of correlation whether it is 0 or not.

And Secondly, you can see because of the heavy correlation, the variability within the population is much higher than the variability across population that means, within group variability is much larger than across group variability. That calls for a paired test. If it is not a case then you can actually treat it as different.

Alright, so, let us look at this here. I have a t dot test, let us quickly do this here and we








set paired equals true and the rest of the defaults are the same;  $\mu$  is 0, the significance level is 0.05, it is a two-sided test therefore, I do not have to be worried about anything else and that is it. So, here we have the values. Once again you see that the numbers displayed here tally with what we had, especially the t statistic value and the confidence region you can also see the p-value being reported as usual which have not reported on the slide, it is much it is low than alpha, right. I do not want to say it is much low, it is low than alpha lower than alpha and therefore, the null hypothesis is rejected.


Now, what I would like you to do as a home work is; try and run a two-sample t test that means, treat this population as independent and see what the result comes out to be. I leave that as a matter of surprise for you. What happens when you turned the paired option of and maybe you can say variability is the same variance naught equals may be true and try to run the test and variance dot equals being false, try to run the test and see if you had inadvertently treated this 2 populations as being different, whether the result is contradictory to what you get from the paired test or in line with what you see from the pair test. And, the other thing that you could do is also once you are convince that the paired test is a better way of doing, it go and run now with a different alternate hypothesis that after the weight loss program the weights are lower than before, right. So, you have to frame the alternate hypothesis, but still it will be a paired test and see if the null hypothesis that they are equal is rejected in favor of the alternate hypothesis, that weight after the loss program is the lower than weight before the weight loss program, alright. So, make sure that you work out things in R.

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### Bibliography I

-  Bendat, J. S. and A. G. Piersol (2010). *Random Data: Analysis and Measurement Procedures*. 4<sup>th</sup> edition. New York, USA: John Wiley & Sons, Inc.
-  Johnson, R. A. (2011). *Miller and Freund's: Probability and Statistics for Engineers*. Upper Saddle River, NJ, USA: Prentice Hall.
-  Montgomery, D. C. and G. C. Runger (2011). *Applied Statistics and Probability for Engineers*. 5<sup>th</sup> edition. New York, USA: John Wiley & Sons, Inc.
-  Ogunnaike, B. A. (2010). *Random Phenomena: Fundamentals of Probability and Statistics for Engineers*. Boca Raton, FL, USA: CRC Press, Taylor & Francis Group.
-  Tangirala, A. K. (2014). *Principles of System Identification: Theory and Practice*. CRC Press, Taylor & Francis Group.

  
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We will have all data sets available for you, play around with them, plot, do go through the summary statistics, draw the box plots, put everything together in prospective and then hopefully you will enjoy the hypothesis testing a lot more than just simply running the data through this t dot test.

In the next lecture, we look at testing for variances and proportions. Alright then, until then have a joy our session.

Bye for now.