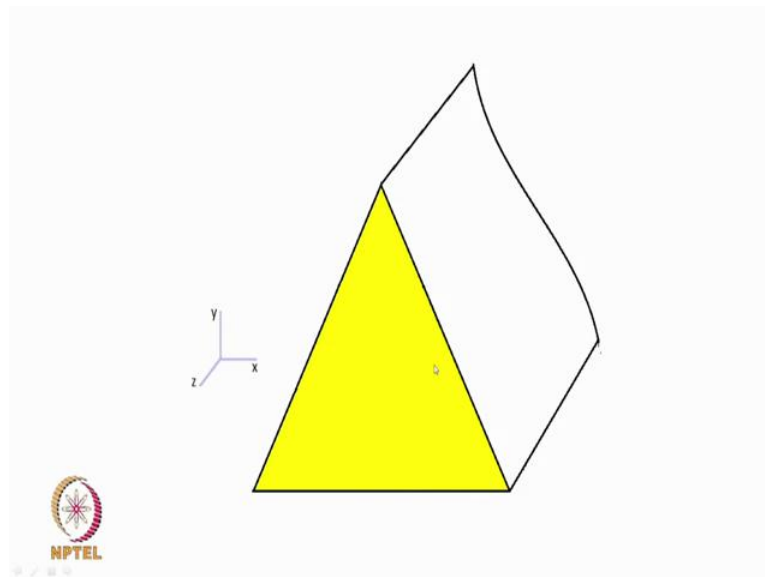


Computational Fluid Dynamics
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Lecture – 09
Description of FV method and solution using G-S method

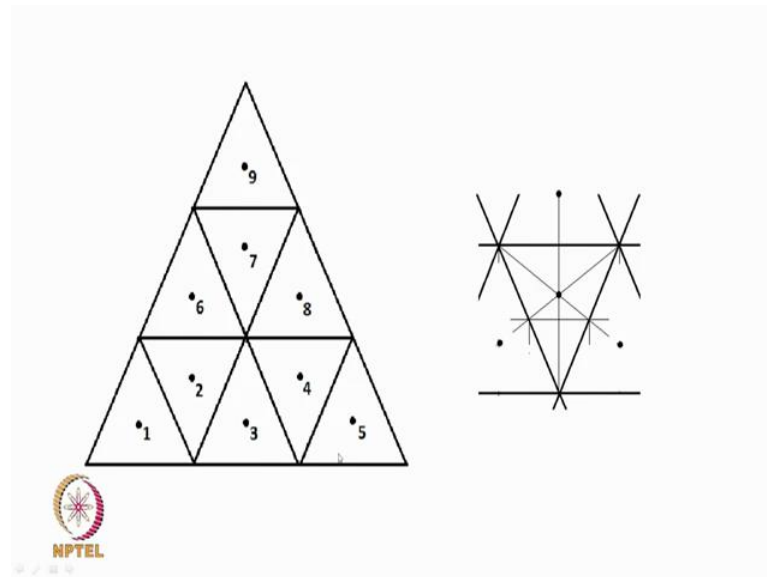
In the last tutorial lecture, we have seen the basic idea of how to convert a partial differential equation into a set of algebraic equations by discretizing it at specific points for the case of triangular duct. We saw that for two cells - cells one and cells two. Now in today's lecture, we will do it systematically for the entire set of nine cells and then look at what kind of discretized equation we get and how we can solve this.

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So, we are looking at the triangular flow through a duct of triangular cross section the flow is fully developed which enables us to look at just the triangular cross section.

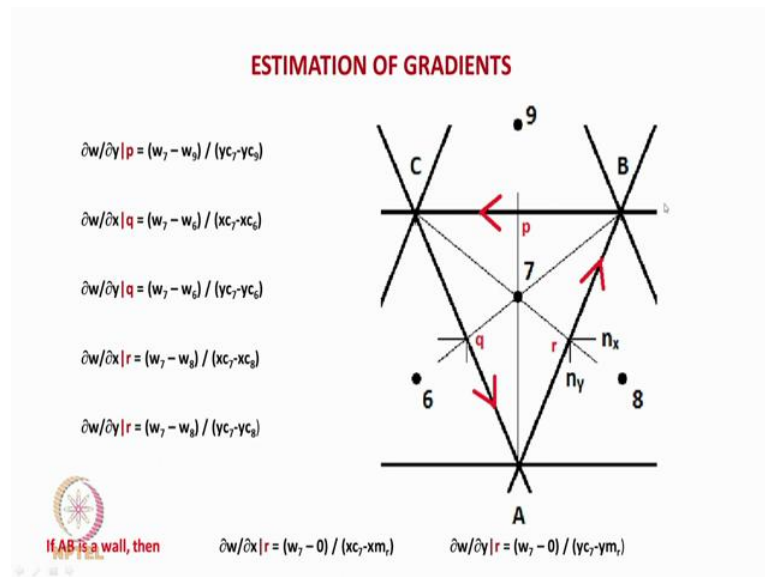
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And we have divided this into nine cells equilateral cells each of side point one meters and as part of the discretization we took a specific cell which has a structure like this for example, this is cell 7. And we identify the centroid of the cell, and then the midpoints of the three sides and then the three neighboring points. And at the mid points, we have the direction cosines the n_x and n_y at this point, and at this point, and this point. For example, on this space, the outward normal vector is like this. So, n_x is positive and n_y is negative. Based on the geometry that this is 60 degrees, we can say that n_x is cosine 30 and n_y is minus sin 30. And although we put it as 30 here, the angle is actually to be counted from here, so it has to be like this. So, in that sense, we can see that this is cosine 30 and here we have n_x as negative and n_y as negative. And here we have n_y as positive and n_x as 0.

So, the basic information of the geometry in terms of what the vertices are, what the x and y coordinates of the vertices are, and what are the coordinates of the centroid, and what are the coordinates of the neighboring points, and what are the coordinates of the midpoints of each space, and what are the direction cosines needs to be ascertained for each cell.

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So, once you have this, then you need to go into the estimation of gradients. So, for the estimation of gradients, we are following for the sake of convenience especially in terms of writing a program, we are following this particular convention that any cell we go in the counterclockwise direction. It is a triangular thing, so we can go keep on going round we need to have a starting point, so we are taking for this example the starting point as the horizontal line which is there for every cell, every cell has a horizontal side.

So, we can take start with this go in the counterclockwise direction like this, and from here like this, and then from here like this. So, at this point, for cell 7, for example, the midpoint, I am starting and calling the midpoint of the horizontal cell as P the midpoint of the second cell as q, second side as q, and the midpoint of the third side of as r. So, we are going like this, like this, like this, so this is small p, this is small q and small r. It is at this small p, small q, small r, we need to evaluate the directional cosines, and we also need to evaluate the gradients.

So, when we look at the gradient, we need to have for example, $\frac{\partial w}{\partial x}$ at point p, so $\frac{\partial w}{\partial y}$ at point p, we do not need the $\frac{\partial w}{\partial x}$ at point p because n_x is 0. So, you have only n_y is nonzero and it is equals to minus 1. So, we need to evaluate $\frac{\partial w}{\partial y}$ at point p. And this can be evaluated in terms of the

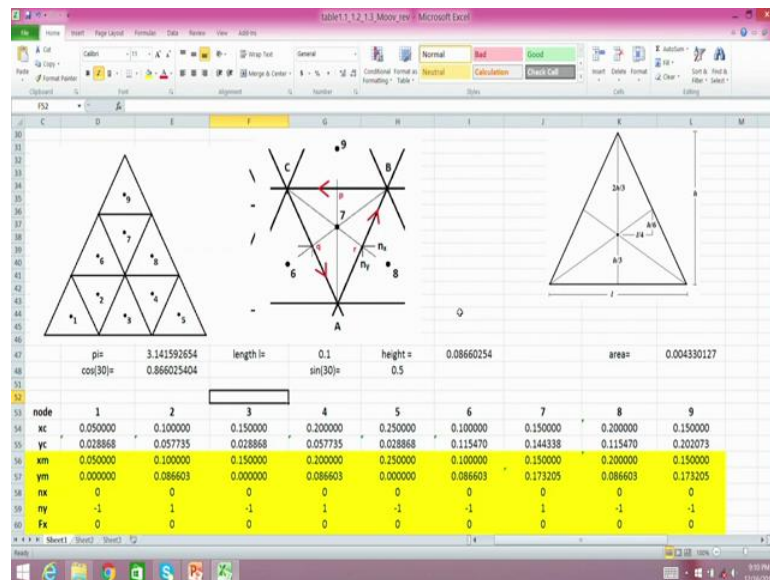
immediate neighboring points at which we know the velocity. So, here we have w_7 is one of the points where we are written the velocity and w_9 is also there. So, we can write this as w_7 minus w_9 divided by y_{c7} minus y_{c9} . where y_{c7} is the y coordinate of the centroid of cell 7 and y_{c9} is the centroid y component of the centroid of the cell 9 like this.

Now, when we come to the second phase, and the midpoint small q here given in red here. Here, you have both n_x and n_y , so we need to evaluate both the $\text{d}w/\text{d}x$ and $\text{d}w/\text{d}y$. So, here again we can write this as because there is a neighboring point 6 here, we can write this $\text{d}w/\text{d}x$ at q as w_7 minus w_6 divided by x_{c7} minus x_{c6} . And $\text{d}w/\text{d}y$ as again w_7 minus w_6 divided by y_{c7} minus y_{c6} . Now if you look at the way that we are doing here in order to evaluate the gradient $\text{d}w/\text{d}y$, we are substituting we are writing this as $\Delta w/\Delta y$. And how are we defining delta, it is the difference between two points in which way we can write this as w_7 minus w_9 divided by y_{c7} minus y_{c9} , or we can write it as w_9 minus w_7 divide by y_{c9} minus y_{c7} .

The numerical value does not change, but when we add up all the contribution of the fluxes which are coming from all the sides here it is better to have a consistent way of evaluating this gradient, and which is why I have put here, always you start with the cell node value as the point where you subtract from this the neighboring value. So, you have here w_7 minus w_9 , and here you have w_7 minus w_6 . And you change these things in such a way that you maintain the same order. Similarly, for the third side at midpoint r here which is denoted in a red color. Again, we need to have $\text{d}w/\text{d}x$ at r, here we are writing as w_7 minus w_8 divided by x_{c7} minus x_{c8} . So, we do this then if we say what is the contribution to the coefficient of w_7 in the discretized equation we can just sum up all these things and the net contribution for 6, 8 and 9 will be negative, and they will be coming because of this. If you put it, if you evaluate for this point as w_7 minus w_9 , and for this point you put w_6 minus w_7 then you have to add this here and then subtract this here. So, to avoid that kind of confusion, we are defining estimating the gradient as the value of the node minus the value of the neighboring node, value of the node minus value of the neighboring node. Whether it is to the left side or the right side, it does not matter, it is always w_7 minus w_8 divide by x_{c7} minus x_{c8} and so on.

If for example, we have a wall, suppose a b is a wall at that point we do not have a neighboring point like 8, but we have the boundary condition that at the midpoint r equal to 0. So, in such a case, in order to get the gradient $\frac{dw}{dy}$ at r, we write this as $w_7 - w_{wall}$ which is 0 divided by $x_c - x_m$, where x_m is the x coordinate of the midpoint of the rth side. So, I could have put it as x_r it is the same thing here. Similarly, $\frac{dw}{dx}$ at this point here at r, where this is a wall can be estimated as $w_7 - w_{wall}$ divided by $y_c - y_m$. So, in this way, we can have a consistent way of estimating the gradients. And we also know from the geometry what the direction cosines on each space are.

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And then we can go into making entire program for all the cells. So, here we have to be systematic, and I am illustrating the process in excel, so that we have a clear understanding of what information is being obtained from where. And once you have a clear understanding, then you can go into writing a computer program, so it is excel, we have to do calculations for 9 sets of a points. So, something like excel will actually eliminate the mistakes in the hand calculations that we might make when we are doing for so many cells, so that is the only thing; otherwise the preferred option would be to do hand calculations. So, here we are starting with this triangular cell here, we need information of pi, so I put this here. So, the cell length is 0.1 meters and height is sine 60

times this, so that is that we get as 0.0866. And from the height, I can look at the centroid as $2h$ by 3, and h by 3 distances; and I can look at also the midpoint and all these things become known here. And we can also find the area of this triangular cell as half height into lengths, so that is what we are doing here.

So, we have the area of the cell and we have the basic information. And from this, we first of all we get all the coordinates; and here we have nine nodes 1, 2, 3 up to 9. And in this line, we are looking at the x-coordinates of each of these nodes here. For example, if you consider node one, this is at half distance of this cell length, which is 0.1, so it is 0.5. Similarly, this is at a distance from the coordinate from the origin, which is fixed at this corner here. The x distance is equal to 0.1 - one-cell length, this is 1.5 cell lengths, 2 cell lengths, 2.5 cell lengths, so that is how it is increasing here. And if you consider C, it is exactly above 2 cell-2, so it has the same x distance as cell 2 and 0.7 and 9 have the same x distance as 0.3, so 0.1, 0.5, 0.15, 0.15. And 8 have the same distance as 4.

So we can we can do this we can do it in a more systematic way from the coordinates and we will see that at later stage. Similarly, we can fill the y c of the centroid, we can see that this is at one-third of the total height and so that is how it is given as this the height divided by 3. And if you consider this, this is two-thirds of the centroid height, so this is given two-thirds of the height of the cell. And the heights of 3, 4, 5 are also similar we can get this. And if you consider 0.6 here, this is one cell height plus one-third, so it is 4 by 3 of cell height, and this will be 5 by 3 of cell height, this is 7 by 3 of cell height, so that is how we can get all the coordinates of the centroid in this two-dimensional case.

Now, we have for each cell we would like to have the midpoint of the phases, and we already said that we will always start with the horizontal side, and we go with the counter current direction. So, the horizontal midpoint is defined as point p. So, for the point p here for cell 1, which is at this point, we can put the x, y coordinates. For example, the midpoint is at half cell distance so that is 0.05, and it is at y equal to 0, and it has an outward normal vector which is vertically downwards, so it has n y of minus 1 and n x of 0, so this information for point p of this cell. And similarly, if it consider this cell two this is point p, and there again you have n x equal to 0. In fact, since we are considering

horizontal phase as point p for all of them n_x is 0, n_y will change and n_x will change as per the actual case, so n_y is minus 1.

And having filled this, we can go to the second point q here which is the second side as we go in the counterclockwise direction. And for cell 2, this is point q; and the outward normal vector is like this, so you have n_x positive and n_y positive, and that is how you have n_x positive and n_y positive, and these are given by cosine 30 and sine 30 here. And we can also find the midpoints of this.

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node	1	2	3	4	5	6	7	8	9
xc	0.050000	0.100000	0.150000	0.200000	0.250000	0.100000	0.150000	0.200000	0.150000
yc	0.028668	0.057335	0.086002	0.057335	0.028668	0.115470	0.144338	0.115470	0.202071
xm	0.050000	0.100000	0.150000	0.200000	0.250000	0.100000	0.150000	0.200000	0.150000
ym	0.000000	0.086603	0.000000	0.086603	0.000000	0.086603	0.173205	0.086603	0.173205
nx	0	0	0	0	0	0	0	0	0
ny	-1	1	-1	1	-1	1	-1	1	-1
Fx	0	0	0	0	0	0	0	0	0
Fy	(w1-0)/(yc1-ym)	(w2-w1)/(yc2-ym)	(w3-0)/(yc3-ym)	(w4-w1)/(yc4-ym)	(w5-0)/(yc5-ym)	(w6-w1)/(yc6-ym)	(w7-w1)/(yc7-ym)	(w8-w1)/(yc8-ym)	(w9-w1)/(yc9-ym)
xm	0.075000	0.075000	0.175000	0.175000	0.275000	0.125000	0.125000	0.225000	0.175000
ym	0.04330	0.04330	0.04330	0.04330	0.04330	0.12990	0.12990	0.12990	0.21651
nx	0.86603	-0.86603	0.86603	-0.86603	0.86603	0.86603	-0.86603	0.86603	0.86603
ny	0.50000	-0.50000	0.50000	-0.50000	0.50000	0.50000	-0.50000	0.50000	0.50000
Fx	(w1-w2)/(xc1-xc2)	(w2-w1)/(xc2-xc1)	(w3-w4)/(xc3-xc4)	(w4-w3)/(xc4-xc3)	(w5-0)/(xc5-xc4)	(w6-w7)/(xc6-xc7)	(w7-w6)/(xc7-xc6)	(w8-0)/(xc8-xc7)	(w9-0)/(xc9-xc8)
ym	0.04330	0.04330	0.04330	0.04330	0.04330	0.12990	0.12990	0.12990	0.21651
nx	-0.86603	0.86603	-0.86603	0.86603	-0.86603	-0.86603	0.86603	-0.86603	-0.86603
ny	0.50000	-0.50000	0.50000	-0.50000	0.50000	0.50000	-0.50000	0.50000	0.50000
Fx	(w1-0)/(xc1-xm)	(w2-w3)/(xc2-xc3)	(w3-w2)/(xc3-xc2)	(w4-w5)/(xc4-xc5)	(w5-w4)/(xc5-xc4)	(w6-0)/(xc6-xm)	(w7-w8)/(xc7-xc8)	(w8-w7)/(xc8-xc7)	(w9-0)/(xc9-xm)
Fy	(w1-0)/(yc1-ym)	(w2-w3)/(yc2-ym)	(w3-w2)/(yc3-ym)	(w4-w5)/(yc4-ym)	(w5-w4)/(yc5-ym)	(w6-0)/(yc6-ym)	(w7-w8)/(yc7-ym)	(w8-w7)/(yc8-ym)	(w9-0)/(yc9-ym)

And similarly for point side r in which case this is it, and here we have outward normal vector like this, so you have negative n_x and positive n_y , and that is what we are seeing here minus cosine 30 and plus sine 30 here. Now, along with this, we also need to have the gradient information, which I am calling as f for flux, not really flux here, it is a gradient. So, the gradient for point p n_x is 0 so we do not need to evaluate the gradient we can put any value, it is simple to put 0 value. And the y gradient $\frac{dw}{dy}$ at point p for cell 1 here is based on the value of w at 0.1 at the centroid minus the value at the wall divided by $y_c1 - y_m$.

So, similarly, the same thing will appear here; here also we put for cell 2, the gradient at this point is and we have said always we will start with the cell value, so w_2 minus w_6 divided by y_{c2} minus y_{c6} . And for this, we need the wall information. And here again we have immediate neighbor 8 here, so we make use of that value. Again, we have the wall; and for 7, we have 0.9 as the immediate neighbor, so it will be w_7 minus w_9 divided by y_{cm} minus y_{m} of that particular thing. So, this y_m is actually available here already filled up. So, we are looking at 7 here, so w_7 minus w_9 lengths, and we can do the same thing for point q, we have n_x x_m r and all that.

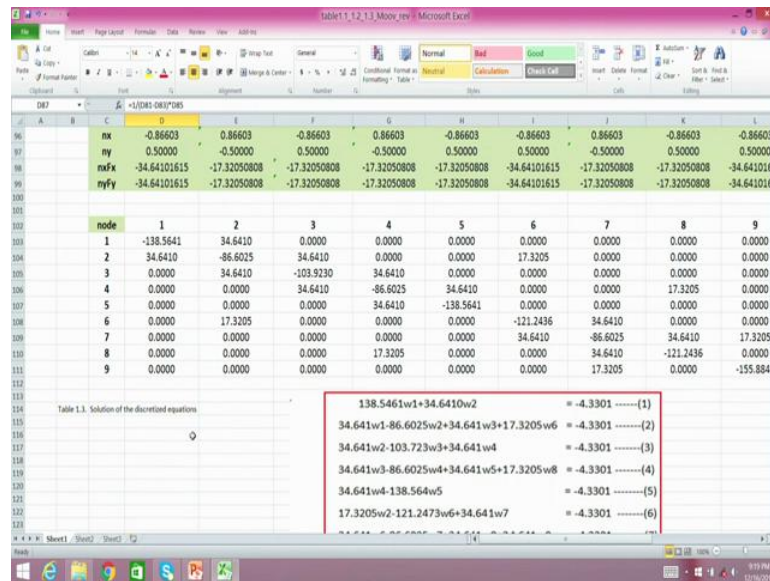
So, for q here corresponds to this, and since there is a neighboring cell here we make use of the neighboring cell to evaluate the gradients here, here, and here. So, consistently we go through systematically through all the phases and evaluate here n_x , n_y , f_x , f_y for each of the phases. Since, all the phases have the same length here, I have not included that, but you can if the length changes then you need to include that also here.

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		D	E	F	G	H	I	J	K	L	
51		cos(30)°	0.866025404		sin(30)°	0.5					
52											
53		node	1	2	3	4	5	6	7	8	9
54		xc	0.050000	0.100000	0.150000	0.200000	0.250000	0.100000	0.150000	0.200000	0.150000
55		yc	0.028868	0.057735	0.028868	0.057735	0.028868	0.115470	0.144338	0.115470	0.202071
56		xm	0.050000	0.100000	0.150000	0.200000	0.250000	0.100000	0.150000	0.200000	0.150000
57		ym	0.000000	0.086603	0.000000	0.086603	0.000000	0.086603	0.173205	0.086603	0.173205
58	point p	nx	0	0	0	0	0	0	0	0	0
59		ny	-1	1	-1	1	-1	-1	1	-1	-1
60		Fx	0	0	0	0	0	0	0	0	0
61		Fy	(w1-0)/(yc-ym)	(w2-w6)/(yc2-yc6)	(w3-0)/(yc3-ym)	(w4-w8)/(yc4-yc8)	(w5-0)/(yc5-ym)	(w6-w2)/(yc6-yc2)	(w7-w9)/(yc7-yc9)	(w8-w4)/(yc8-yc4)	(w9-w7)/(yc9-yc5)
62		xm	0.075000	0.075000	0.175000	0.175000	0.275000	0.125000	0.125000	0.225000	0.175000
63		ym	0.04330	0.04330	0.04330	0.04330	0.04330	0.12990	0.12990	0.12990	0.21651
64		nx	0.86603	-0.86603	0.86603	-0.86603	0.86603	0.86603	-0.86603	0.86603	0.86603
65	point q	ny	0.50000	-0.50000	0.50000	-0.50000	0.50000	0.50000	-0.50000	0.50000	0.50000
66		Fx	(w1-w2)/(xc1-yc2)	(w2-w1)/(xc2-yc1)	(w3-w4)/(xc3-yc4)	(w4-w3)/(xc4-yc3)	(w5-0)/(xc5-xm)	(w6-w7)/(xc6-yc7)	(w7-w6)/(xc7-yc6)	(w8-0)/(xc8-xm)	(w9-0)/(xc9-yc9)
67		Fy	(w1-w2)/(yc1-yc2)	(w2-w1)/(yc2-yc1)	(w3-w4)/(yc3-yc4)	(w4-w3)/(yc4-yc3)	(w5-0)/(yc5-ym)	(w6-w7)/(yc6-yc7)	(w7-w6)/(yc7-yc6)	(w8-0)/(yc8-ym)	(w9-0)/(yc9-yc9)
68		xm	0.02500	0.12500	0.12500	0.22500	0.22500	0.07500	0.17500	0.17500	0.12500
69		ym	0.04330	0.04330	0.04330	0.04330	0.04330	0.12990	0.12990	0.12990	0.21651
70		nx	-0.86603	0.86603	-0.86603	0.86603	-0.86603	-0.86603	0.86603	-0.86603	-0.86603
71	point r	ny	0.50000	-0.50000	0.50000	-0.50000	0.50000	0.50000	-0.50000	0.50000	0.50000
72		Fx	(w1-0)/(xc1-xm)	(w2-w3)/(xc2-yc3)	(w3-w2)/(xc3-yc2)	(w4-w5)/(xc4-yc5)	(w5-w4)/(xc5-yc4)	(w6-0)/(xc6-xm)	(w7-w8)/(xc7-yc8)	(w8-w7)/(xc8-yc7)	(w9-0)/(xc9-yc9)
73		Fy	(w1-0)/(yc1-ym)	(w2-w3)/(yc2-yc3)	(w3-w2)/(yc3-yc2)	(w4-w5)/(yc4-yc5)	(w5-w4)/(yc5-yc4)	(w6-0)/(yc6-ym)	(w7-w8)/(yc7-yc8)	(w8-w7)/(yc8-yc7)	(w9-0)/(yc9-yc9)

So, having gotten this now so this is how we are evaluating and this is actual computation numerical computation. And what I have done here in this cell is to I am trying to assemble the coefficient matrix a.

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And when I put the coefficient matrix eventually, I will have something like this, this is coefficient matrix and this is w. So, the w's do not appear in the coefficient matrix; it is just the product of this n x's and the coordinate information like the distances, only those appear here, so that information is what is evaluated here. So, this matrix here just contains numerical values for these things except here the numerator the w 1, w 2 are not included here. So, with this, we can evaluate for example, this one is equal to 1 by d 81 minus d 83 and d 81 is y c minus d 83 is y c y m p, and that is what we have here, y c 1 minus y m and so times d 85 which is n y.

And if you look at this one here, this is d 91, since we are looking at n y times F y d 91 is this divided by d 81 minus e 81. And if you look at the formula for this, this is side q here, so you have y c 1 minus y c 2. And d 81 is this; d 81 minus e 81 is the distance between the vertical distances between the two centroids. So, in that way we can evaluate the n x, f x and n y, f y in terms of the geometric parameters into this.

So, now we have this one. And so from these we need to sum up all the contributions from this, this, this, this, this, this times the corresponding n x's. So, there needs to be summed and for example, when you look at the contribution to w 1 from node 1, you see that there is a contribution coming from point p which is equal to n y times minus 1

divided by $y_c - 1 - y_m$ here. And then there is a contribution coming from this and this so that is there is a contribution coming from, there is a contribution coming from this and there is a contribution coming from this, and there is a contribution coming from this and again from this. So, the locations the sides in which w_1 figures is the one that needs to be added up here in order to get this whole thing.

So when you look at this value here, this is sum of $d_{86}, d_{87}, d_{92}, d_{93}, d_{98}, d_{99}$; d_{98}, d_{99} are these two, and d_{92}, d_{93} are these two, and d_{86}, d_{87} are these two. Similarly, if you go to this, this is center, this is w_2 coefficient from the second equation, this is also again the sum of these two plus sum of these two plus sum of these two, so that is how we can evaluate. And here again it is the sum of this is the fifth cell, so the corresponding sum of these two so that is d_{86}, d_{87} , and then d_{92}, d_{93} these two here, and then d_{97}, d_{98}, d_{99} . So, in that way, we can evaluate the diagonal coefficients.

And the other terms here, this is essentially the first equation the coefficients in the first equation. And cell one interacts with cell two and we can see that in the formula here. So, in the cell one equation, we have contribution from w_2 and that contribution can be estimated it is contributing in point two here, so it is the sum of these two and that is what we have here. So, it is $-d_{92} - d_{93}$, because it is coming out as negative valid here. So, in that way, we can fill all the coefficients here from this table. Once you put that all the coefficients, this becomes the coefficient matrix a .

(Refer Slide Time: 23:24)

	1	2	3	4	5	6	7	8	9
myf	-34.64101615	-17.32050808	-34.64101615	-17.32050808	-34.64101615	-17.32050808	-17.32050808	-17.32050808	-17.32050808
xmq	0.07500	0.07500	0.17500	0.17500	0.27500	0.12500	0.12500	0.22500	0.17500
ymq	0.04330	0.04330	0.04330	0.04330	0.04330	0.12990	0.12990	0.12990	0.21651
nx	0.86603	-0.86603	0.86603	-0.86603	0.86603	0.86603	-0.86603	0.86603	0.86603
ny	0.50000	-0.50000	0.50000	-0.50000	0.50000	0.50000	-0.50000	0.50000	0.50000
mfx	-17.32050808	-17.32050808	-17.32050808	-17.32050808	-34.64101615	-17.32050808	-17.32050808	-34.64101615	-34.64101615
myf	-17.32050808	-17.32050808	-17.32050808	-17.32050808	-34.64101615	-17.32050808	-17.32050808	-34.64101615	-34.64101615
xmr	0.02500	0.12500	0.12500	0.22500	0.22500	0.07500	0.17500	0.17500	0.12500
ymr	0.04330	0.04330	0.04330	0.04330	0.04330	0.12990	0.12990	0.12990	0.21651
nx	-0.86603	0.86603	-0.86603	0.86603	-0.86603	-0.86603	0.86603	-0.86603	-0.86603
ny	0.50000	-0.50000	0.50000	-0.50000	0.50000	0.50000	-0.50000	0.50000	0.50000
mfx	-34.64101615	-17.32050808	-17.32050808	-17.32050808	-17.32050808	-34.64101615	-17.32050808	-17.32050808	-34.64101615
myf	-34.64101615	-17.32050808	-17.32050808	-17.32050808	-17.32050808	-34.64101615	-17.32050808	-17.32050808	-34.64101615

And these are the variable matrix.

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	1	2	3	4	5	6	7	8	9
3	-138.5641	34.6410	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	34.6410	-86.6025	34.6410	0.0000	0.0000	17.3205	0.0000	0.0000	0.0000
5	0.0000	34.6410	-103.9230	34.6410	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	34.6410	-86.6025	34.6410	0.0000	0.0000	17.3205	0.0000
7	0.0000	0.0000	0.0000	34.6410	-138.5641	0.0000	0.0000	0.0000	0.0000
8	0.0000	17.3205	0.0000	0.0000	0.0000	-121.2436	34.6410	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	34.6410	-86.6025	34.6410	17.3205

And this is the b, the right hand side here.

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	3	4	5	6	7	8	9	variable
102	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	w1
103	34.6410	0.0000	0.0000	17.3205	0.0000	0.0000	0.0000	w2
104	-103.9230	34.6410	0.0000	0.0000	0.0000	0.0000	0.0000	w3
105	34.6410	-86.6025	34.6410	0.0000	0.0000	17.3205	0.0000	w4
106	0.0000	34.6410	-138.5641	0.0000	0.0000	0.0000	0.0000	w5
107	0.0000	0.0000	0.0000	-121.2436	34.6410	0.0000	0.0000	w6
108	0.0000	0.0000	0.0000	34.6410	-86.6025	34.6410	17.3205	w7
109	0.0000	17.3205	0.0000	0.0000	34.6410	-121.2436	0.0000	w8
110	0.0000	0.0000	0.0000	0.0000	17.3205	0.0000	-155.8846	w9

112	138.5461w1+34.6410w2 = -4.3301 -----(1)							
113	34.641w1-86.6025w2+34.641w3+17.3205w6 = -4.3301 -----(2)							
114	34.641w2-103.723w3+34.641w4 = -4.3301 -----(3)							
115	34.641w3-86.6025w4+34.641w5+17.3205w8 = -4.3301 -----(4)							
116	34.641w4-138.564w5 = -4.3301 -----(5)							
117	17.3205w2-121.2473w6+34.641w7 = -4.3301 -----(6)							
118	34.641w6-86.6025w7+34.641w8+34.641w9 = -4.3301 -----(7)							
119	17.3205w4+34.641w7-121.2473w8 = -4.3301 -----(8)							
120	17.3205w7-155.0846w9 = -4.3301 -----(9)							

And if you write in plain equations then the corresponding equations will be like this, we have nine equations for the nine cells. So, these are the nine equations which can also be put in a matrix form like this. Now, so we have using this very detailed procedure, systematic procedure, and only after this systematic procedure, we have been able to convert the partial differential equation into a set of algebraic equations that we see. And so in that sense finite volume method is much more tedious and more difficult to program than finite difference method, in terms of converting finding the coefficients of a and that is the simplicity of finite differences.

But finite volume method can be used for any cell not necessarily composed of topologically quadrilateral kind of thing with grid points at intersection of coordinates. So, this can be used for any number of sides, it can be used for a 6 sided hexagon or any polygon, it can also be used this can be used for any of those things and so that is advantage, but because that advantage is offset to some extent by the need to do so much of bookkeeping and accounting and all that.

At the end of that, you have a set of equations, which share features which are similar to what we have got from the finite difference method. If you look at these equations here, this is the coefficient matrix. Now when we want to solve this the question of whether

we can apply Gauss-Seidel method comes in; and for that we have the diagonal dominance condition, which means that the diagonal element along any row, the diagonal element must be greater in magnitude then the sum of the other off diagonal elements in the same row. So that is satisfied for first equation. It is also satisfied with an equality because 34, 34 and 17, all this will add up exactly to this. So, it is satisfied with equality here. And here it is less than again equality less than equality. Now this is again less than and equality, and less than, and less than.

So, that less than and equality are with reference to this. For this one, we estimated the fluxes using the three neighbors there is no wall. So, when you have something estimated using the three neighbors and no wall information is coming, then the diagonal dimension condition is satisfied with an equality side, so that is true for cell 2, because there are three neighbors here; cell 4, there are three neighbors here; cell 7, there are three neighbors. But for cell 3, you have wall here; cell 1, you have two walls here; and cell 6, you have wall here; and 9, you have wall, for these things you do not have you get a greater than sign for this type of problems.

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node	1	2	3	4	5	6	7	8	9	variable
1	-138.5641	34.6410	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	w1
2	34.6410	-86.6025	34.6410	0.0000	0.0000	17.3205	0.0000	0.0000	0.0000	w2
3	0.0000	34.6410	-103.9130	34.6410	0.0000	0.0000	0.0000	0.0000	0.0000	w3
4	0.0000	0.0000	34.6410	-86.6025	34.6410	0.0000	0.0000	17.3205	0.0000	w4
5	0.0000	0.0000	0.0000	34.6410	-138.5641	0.0000	0.0000	0.0000	0.0000	w5
6	0.0000	17.3205	0.0000	0.0000	0.0000	-121.2436	34.6410	0.0000	0.0000	w6
7	0.0000	0.0000	0.0000	0.0000	0.0000	34.6410	-86.6025	34.6410	17.3205	w7
8	0.0000	0.0000	0.0000	17.3205	0.0000	0.0000	34.6410	-121.2436	0.0000	w8
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	17.3205	0.0000	-155.8846	w9

138.5461w1+34.6410w2	= -4.3301 -----(1)
34.641w1-86.6025w2+34.641w3+17.3205w6	= -4.3301 -----(2)
34.641w2-103.723w3+34.641w4	= -4.3301 -----(3)
34.641w3-86.6025w4+34.641w5+17.3205w8	= -4.3301 -----(4)
34.641w4-138.564w5	= -4.3301 -----(5)
17.3205w2-121.2473w6+34.641w7	= -4.3301 -----(6)
34.641w6-86.6025w7+34.641w8+34.641w9	= -4.3301 -----(7)
17.3205w4+34.641w7-121.2473w8	= -4.3301 -----(8)
17.3205w7-155.0846w9	= -4.3301 -----(9)

So, with this thing we can use the Gauss-Seidel method. So, we have already seen how to convert this into an equivalent Gauss-Seidel method. In the first equation, we keep this to

the left hand side - w_1 to the left hand side, and we take all the other terms to the right hand side. In the second equation, we keep this largest value here as the on the left hand side, and we take everything else to the right hand side. The right hand side terms are evaluated at k th level and the left hand sides are the value actually at $k + 1$ th iteration level and that is how we can do this.

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The screenshot shows a Microsoft Excel spreadsheet with the following content:

Equations (rows 120-124):

- 34.641w4-138.564w5 = -4.3301 (5)
- 17.3205w2-121.2473w6+34.641w7 = -4.3301 (6)
- 34.641w6-86.6025w7+34.641w8+34.641w9 = -4.3301 (7)
- 17.3205w4+34.641w7-121.2473w8 = -4.3301 (8)
- 17.3205w7-155.0846w9 = -4.3301 (9)

Solution of the discretized equations (rows 125-133):

Iter #	w1	w2	w3	w4	w5	w6	w7	w8	w9
0	0	0	0	0	0	0	0	0	0
1	0.031250	0.062500	0.062500	0.075000	0.050000	0.044643	0.067857	0.065816	0.035317
2	0.046875	0.102679	0.100893	0.123520	0.062130	0.069770	0.111298	0.085160	0.040144
3	0.056920	0.127079	0.125200	0.141964	0.066741	0.085668	0.126360	0.092098	0.041818
4	0.063020	0.142421	0.136462	0.149701	0.068675	0.092163	0.132068	0.094834	0.042452
5	0.066855	0.149759	0.141487	0.153031	0.069508	0.094842	0.134361	0.095965	0.042707
6	0.068690	0.153039	0.143690	0.154472	0.069868	0.095966	0.135314	0.096443	0.042813
7	0.069510	0.154473	0.144648	0.155095	0.070024	0.096443	0.135717	0.096647	0.042857
8	0.069868	0.155095	0.145063	0.155364	0.070091	0.096647	0.135889	0.096735	0.042877
9	0.070024	0.155364	0.145243	0.155481	0.070120	0.096735	0.135963	0.096772	0.042885
10	0.070091	0.155481	0.145320	0.155531	0.070133	0.096772	0.135995	0.096789	0.042888
11	0.070120	0.155531	0.145354	0.155552	0.070138	0.096789	0.136009	0.096796	0.042890
12	0.070133	0.155552	0.145368	0.155562	0.070140	0.096796	0.136014	0.096799	0.042890
13	0.070138	0.155562	0.145374	0.155566	0.070141	0.096799	0.136017	0.096800	0.042891

So, this is starting with again 0, 0, 0 values for C value of 100 here. We see that we are getting converge solution soon pretty soon within about 10 iterations, 10, 15 iterations, we have essentially got a converge solution and so the velocities are changing. So, 2 and 6 are symmetric, so we get the same value; 2 and 4 are symmetric, and so that kind of symmetric 6 and 8 are symmetric, so we get this; 1 and 5 are symmetric, so we get that that kind off.

So, in this way, we can get a solution which is evaluation of the w velocities at the centroid of each of the cells. If you increase the number of points then the accuracy of the whole calculation proves, but that will involve more number of discretization and evaluations, but it can be done it is done and that is what CFD is all about it is about how to do this algorithmically, so that we can get a computer solution.