Computational Fluid Dynamics
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Lecture – 62
More generic formulation and summary

We have come to the last lecture of this course. And we are looking for a closure of the turbulence problem something which is in which we can have a generic model for turbulent viscosity or eddy viscosity and not something that would require us to specify the mixing length which we do not really know for the general case.

For example, if I have a room and inside which I have some people sitting, and then I have some flow of air from the fan going round, then how do I specify the mixing length for and how do I account for the three-dimensional bodies of people who are staying in the in the room. So, in such those kind of three-dimensional problems, it is difficult to come up with a mixing length.

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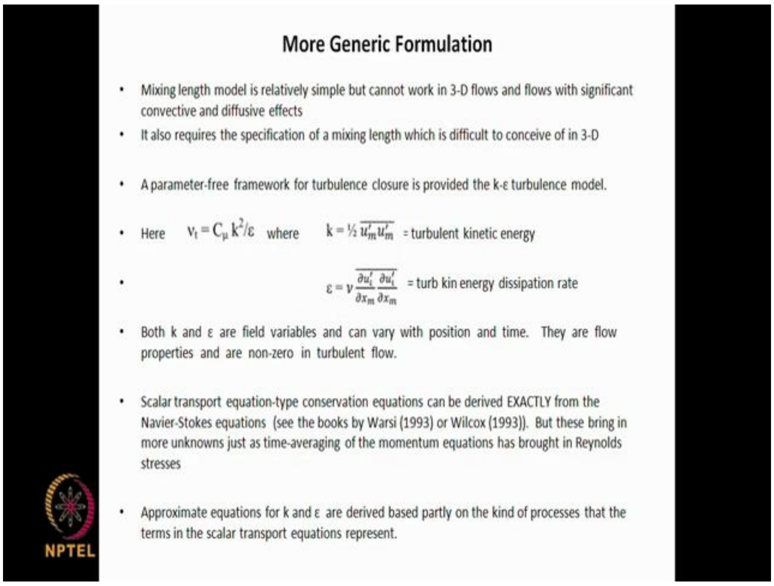

Another big disadvantage of mixing length model is that since whatever turbulence is there, it is because of the local velocity gradients.

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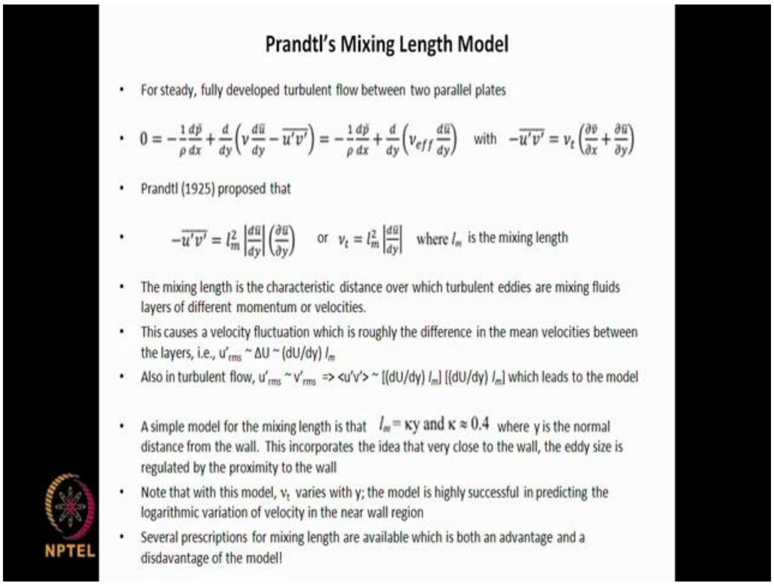

So, for example, here we have another disadvantage with a mixing length model is that it attributes the presence of this Reynolds stresses to a local velocity gradient. And we can see that here in this expression for u prime v prime bar as this is the distance from the wall, and there is dU by dy and dU by dy. So, if there is no du by dy - velocity gradient, there is no stress; and if there is a stress there is there must be a velocity gradient. So, this is a limitation which is not necessarily true in all cases.

For example, one of the simplest examples is what is known as a grid generated turbulence. If you have a mesh here, a very fine mesh, and air is flowing over it; as air flows over these cylinders of this fine mesh of the wire mesh, there will be flow separation and turbulence will be created here. And this turbulence which is created here at the mesh is brought along here as the flow is taking place; and it gets slowly dissipated.

So, here you have turbulence, and here you have turbulence decreasing, and this turbulence is not generated by a velocity gradient here it is generated by a process here. It is just convicted a long and it may also be diffused in this direction. So, here you have turbulence, but that is not generated by the velocity gradients here; and mixing length model tries to create velocity gradients which are in order to support the presence of turbulence stresses. Whereas, far away from the mesh, you may have no velocity gradients; turbulence is there because it is being brought in from some other place, it is being advected - it is being diffused. So, when you have strong advection and diffusion effects then this mixing length model will not work.

So, these are well known disadvantages of a mixing length model. And people are therefore, worked quite a bit to come up with a parameter free framework for turbulence. We do not want to have a parameter called the mixing length to which you want to fit to explain the turbulence and all that. So, we do not have really the time to go into the details or a close look at the development of turbulence modeling that itself would take a full course and maybe much more.

And there have been many, many approaches for turbulent flow, many have not been successful. But there have been some that have been successful, and one of the models which is developed which had its origin since 1940s, early 40s and 1930s is the k epsilon model, which still survives today as the first step model for any turbulent for calculation.

So, if you are looking at a turbulent flow, then immediately one would say k epsilon model. And this k epsilon model has gone through some transformations, and further developments and various kinds of these k epsilon models have been developed. But the basic idea here is that turbulence is characterized by two quantities which is the turbulent kinetic energy and the turbulent kinetic energy dissipation rate, these can be mathematically defined and that is what we have here; k the turbulent kinetic energy is defined as half u prime m u prime m over bar. So, it is a time average of u prime m u prime m, this is a term in which index m is repeating. So, therefore, it is actually a sum of u prime square bar plus u prime square bar plus w prime square bar.

And obviously, if you have turbulence and you have fluctuations, if you have fluctuations you have u prime is nonzero, and therefore, u prime square is nonzero and therefore, turbulent kinetic energy is nonzero. So, if you have turbulence then there is turbulent kinetic energy. This turbulent kinetic energy describes the strength of these fluctuations. So, the other thing is the dissipation rate, so this is known as epsilon, this is the turbulent kinetic energy dissipation rate.

Now, one other feature of turbulent flow is the energy cascade, where turbulence is generated at some eddy size characteristic of the instability mechanisms in the major shear regions, strongly shear regions of the main flow. And that energy is cascaded down into smaller and smaller eddies, and finally, it is thrown into it is converted into heat dissipated all this kinetic energy is dissipated. And the rate of dissipation is also an important parameter. And this rate of dissipation is a characteristic of the smallest eddies because that is where the dissipation occurs; and that rate of dissipation is also a rate of energy production, it is also rate at which energy is cascaded, so all these processes are encapsulated in this turbulent kinetic energy dissipation rate.

Now, this can also be mathematically written as nu by time average of dou u i prime dou u prime i by dou x m times dou u prime i by dou x m, so these are the instantaneous velocity gradients of the fluctuating velocity components. And again you see that i is it is same thing being multiplied by itself, just like you have u prime m times u prime m, so this is always positive and this is always positive or zero, it is zero only in non turbulent flow. And whereas, here this is talking about velocities and here it is talking about velocity gradients. And this is multiplied by the kinematic viscosity of the fluid which brings in this idea of energy dissipation, because viscosity is playing a part, and viscosity is a energy dissipation, it has a energy dissipation role there.

So, this term is positive and the origin for both this and this is in the formulation of a balance equation for turbulent kinetic energy or the Reynolds stress equation, we can derive an equation for k from momentum equations. And both k and epsilon are field variables and they vary with position and time, so that means, it is not a constant value, it changes locally, and it can change with time in a time dependent kind of problem. And but both of them are time average quantities, so they do not exhibit this rapid fluctuations which are characteristic of turbulent flow like those millisecond variations, because these are already time averaged quantities.

And they are flow properties and not the properties of the fluid. So, you have fluid that is that is coming here fluid property like viscosity, but essentially this is the one which determines the level of turbulence dissipation rate. So, these are properties of the flow the velocity gradients are created by the flow and fluctuations are also created by the velocity gradients, so all things are properties of the flow and a nonzero in turbulent flow.

So, one can derive scalar transport equation type of conservation equations for both k and epsilon exact equations can be derived from the Navier-Stokes equations using a set of mathematical operations, you first write the instantaneous momentum equation, the ith direction. You subtract from that the time averaged ith momentum equation and that subtraction will give rise to conservation equation for the ith fluctuating velocity component, and then you multiply with u jth component and then you add the two and then you contract the indices, all these steps for derivation of this have been known for number of decades.

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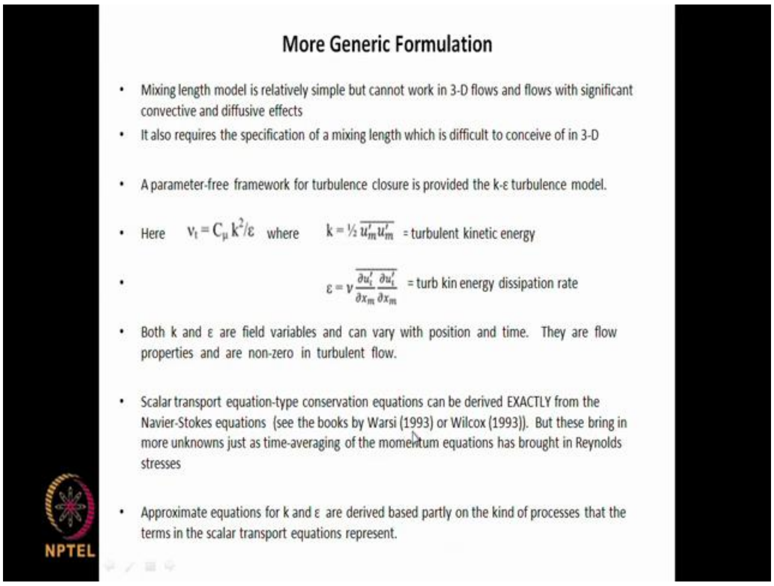

The books modern versions of thus are given in Warsi 1993, Wilcox, these are well known books in turbulence modeling. And these are also there in 1950s book by Hinze is also there where these equations have been derived. And I think 1940s, there is a version of the derivation by Kolmogorov, so these have been the derivation is known, it is tedious it can be done, but it is quite tedious, it is time consuming. And importantly these equations contain large number of new unknowns not just the Reynolds stresses. And this also represents the turbulence closure problem in the sense that you are now defining two new properties k and epsilon, which represent these fluctuations.

And if you want to write, if you want to determine these things using Navier-Stokes equations in the process of deriving, some equations for those, we are introducing new terms which are again not known. And so the number of unknowns' increases as you bring in more and more number of equations and that is where turbulence closure problem, and we cannot solve the exact equations we have to make lot of approximations, and ignore certain terms, and then rewrite certain terms in terms of other variables.

For all these things lot of debate and discussion has taken place. And finally, people have proposed approximate equations for both k and epsilon; based partly on the kind of processes that they represent, for example, we can say that this particular series of terms looks like it is diffusive transport of this k, and this particular set of terms is like production of epsilon type of things. So, if you say production or destruction then that becomes a source term. If you say diffusion that becomes a diffusion term, and they can be advection term, so all these kind of processes are identified among these terms, and some of the terms are just put into this general basket of diffusion type and this type and something are ignored.

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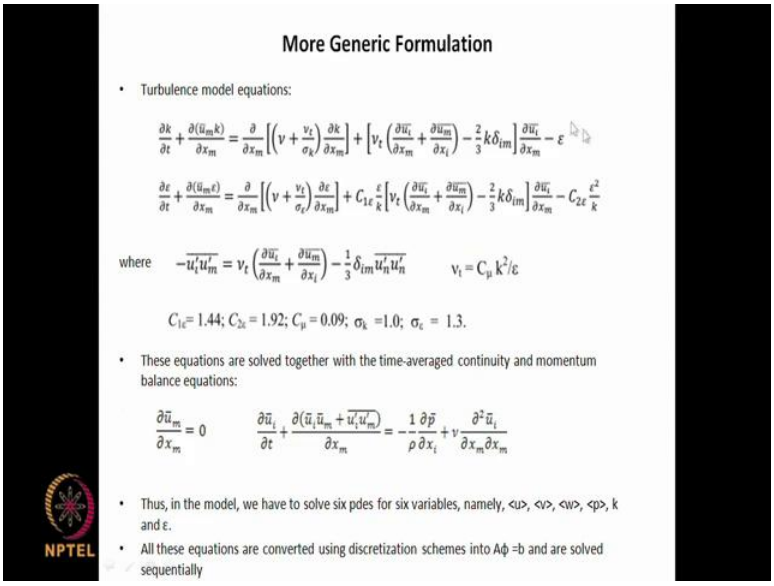

And ultimately we have scalar transport equations and those equations are actually given here. In using the Einstein notation, we have dou k by dou t plus dou by dou x m of u bar m k, so this is our typical time derivative, temporal derivative, and this is advection. So, this is representing conservation of k rate of change of k in the control value is what is a net advected out term plus dou by dou x m of some diffusivity here, viscosity times dou k by dou x m this is the gradient of k. So, diffusivity times gradient represents a diffusion. So, this is the net turbulent diffusion of k. And then you have this term here and this is Reynolds stresses term and one can show that for typical flows this is a net positive thing so that means, that if all the other terms are not there dou k by dou t will increase. Because of this, so this is known as a production of k and this involves mean velocity gradients.

You know that turbulence is produced primarily by the energy containing eddies are produced by strong mean velocity gradients, and so this is a production term of k. And this is the epsilon which we have seen in the definition of which is given here. So, this epsilon definition comes from writing deriving an equation for k; and this comes out as a negative minus epsilon and we have seen that epsilon itself is positive. So, that means, that if all the other terms are not there then k will decrease with time if epsilon is constant so that is why we can say that this is rate of dissipation of turbulent kinetic energy or turbulent kinetic energy dissipation rate, so you have this.

And we can also write a similar kind of equation for epsilon and this is slightly much more arbitrary than the k equation, but again we have advection diffusion and some kind of production and some kind of destruction here. Because again you have C 2 which is a constant a positive constant, epsilon square is positive k is positive, so this describes the a sink term for epsilon, destruction or the rate of destruction of epsilon type of thing. Now, within this you have some constants like an eddy viscosity here, and eddy viscosity is put in this particular form and it is now expressed not in terms of mixing length, but constant times k square by epsilon is how eddy viscosity is defined.

So, if you know k by solving this equation, if you can evaluate epsilon by solving this equation at a particular grid point, then we can evaluate nu t using this expression at that grid point. And then you can substitute here and then get this u i prime u m prime bar. And you can now put this into this here, and then you can have a momentum equation which is all in terms of time average quantities, time average velocities and pressures and all that.

So, now you can solve this using for example, the simple method. And so you can get from this the u bar v bar w bar, and then those things are also coming here and they you can use this discretized this using our FTCS and FTBSCS type of thing can be used for something like this. But you have a source term which is quite active and it also couples all the equations, but we have laid down the general principles by which we can solve a scalar transport equation. So, the equations describing turbulence are also put in the same from so that we need to solve this for k and then we need to solve this for epsilon and then evaluate nu t plug it into this, and plug this term into this, and then again solve this and then we have to solve all these equations.

So, in this model, we have to solve 6 PDEs for 6 variables; and the 6 variables are u bar, v bar, w bar, p bar, this are time average velocities and pressure. And in addition to this, k and epsilon all these things are define defined throughout the fluid at every grid point; and for all these things we need boundary conditions and initial conditions in case of initial value problem. And each of this is described by a scalar transport equation type of equation. And so all these equations are converted using discretization schemes into A phi equal to b type and solved sequentially; and as a result of solution then you can get all these quantities. So, one can use an extended form of the simple method for the solution of this for typical internal flow type of applications.

So, in this model, there are no arbitrary terms, you have equations, you have all these equations and everything about these equations is specified here in this slide there are number of constant sigma k is a constant, sigma epsilon is a constant, and the values are given here. And c nu is a something that is coming here as a constant, and its value is given as 0.09. And then the other two constants are c 1 epsilon and c 2, epsilon which are coming here and those values are also fixed.

These values are fixed by trying to optimize this value for a range of test problems in which measurements are there; and based on that people have come out with this. There can be slight variations from researcher to researcher in terms of what these values are, but together all these things will give you a set of equations which are self consistent within among themselves. To the extent that if you solve them you can get a velocity profile, which matches with experimental values to in many cases or in some cases to certain extent

This is the k epsilon model this is the simplest closure model that we can think of for a turbulent flows in which you do not have to specify anything else everything is contained in the equations. But there are deficiencies of this model and the number of variants of this k epsilon type of models has been there, there is one for low Reynolds symbol k epsilon model you have some RNG k epsilon model which is supposed to be good for swirling flows. These swirling flows are encountered, for example, in burners in furnaces.

And so when you have swirl stabilized flames then the corresponding flow and temperature and species burning and all those things are described by equations, and there you have a strong effect of turbulence and you have some extended k epsilon type of models that are there for those type of applications. And then you also have two layer models and those types of things are also there. And there is also another class of turbulence models in which you do not solve just for these two parameters k and epsilon.

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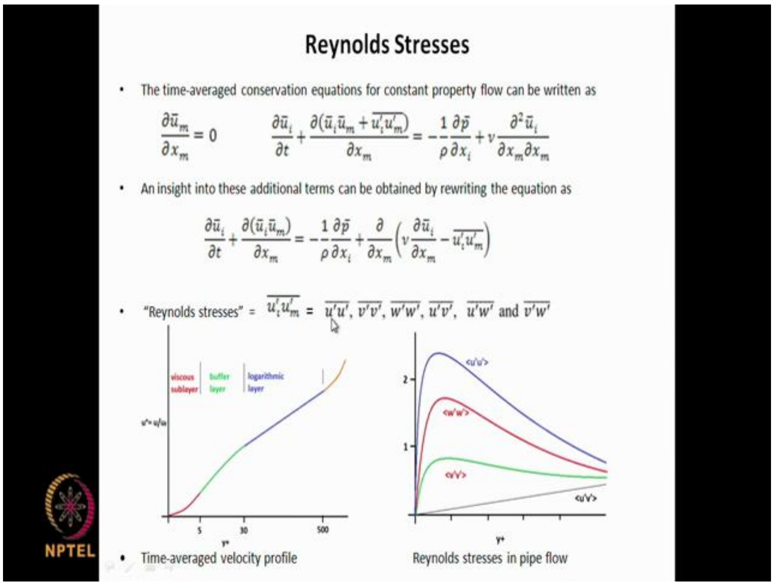

because what we have in the actual equations are the 6 Reynolds stresses, 6 stresses here. So, instead of solving only for the sum of these three the first three which is k and then not solving for anything else and then using another variable epsilon is considered not to be accurate enough for certain cases.

Especially, when you have strong curvature effect then you can have the curvature is known to have a significant effect of on turbulence stresses. And if you have stabilization by favorable gravitational head then again you can have an effect on turbulence. And you can have so there are number of specialized effects in which, which are not captured properly by the k epsilon type of models because you are not solving for all the 6 stresses.

There is another modeling approach which is called Reynolds stress model in which write down cancellation equations for each of the 6 and solve for them separately, not like put the first three normal stresses together and then define k so that is a better approach but you have to solve 6 extra equations not two equations. In fact, in it are 7 equations for that. And the more the number of coupled equations the more difficult it is to solve.

We have seen that we have a single scalar transport equation it is relatively easy, but if we want to solve the Navier-Stokes equations then we have to have special methods. Now you want have k and epsilon as extra set of equations, we can see that you have to solve for k, you have solve for epsilon, and then you have to get the viscosity, and then put the Reynolds stresses and then put them back into momentum equations resolve for all these things, so that becomes a bigger loop and that makes coupling more difficult.

And if you are solving for 6 extra equations, it becomes more and more difficult. So, the more you define the more you refine your model the more accurate the model can be, but the more difficult it may be to get a solution. So, there are different gradations of turbulence models, but one would and it would take too much of our time to talk about that.

So, we would like to stop at this level, because I think this itself is good progresses made by us in terms of having a set of equations which are adequately describe turbulent flow for a large number of turbulent flow cases. And in most industrial simulations of turbulent flows, the k epsilon model is the one which is most preferred, partly because it is robust it works better than many others in terms of generating a solution and that itself is considered as a big achievement.

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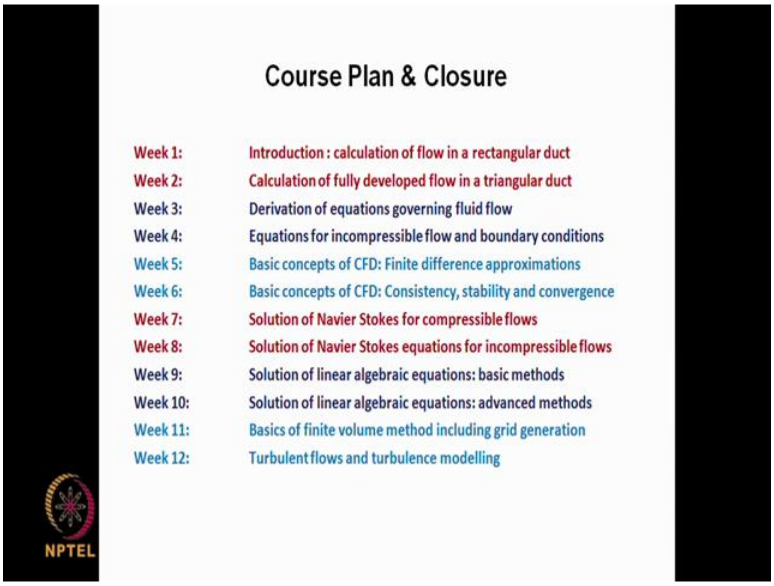

So, where are we now, we have come to the end of this course. And I would like to close this by recollecting what we have been trying to do over the past 12 weeks. And I am going back to the course plan here, and so we started with some two introductory examples of calculation of flow in a rectangular duct, in a triangular duct. The first one using finite difference method and the second one using finite volume method, we introduce the basic concept of a CFD solution, how you take a partial differential equation, substitutes an approximations, converted into a set of equations, and solve them together to get a solution.

How we also have to do produce the Gauss-Seidel method there itself to show that we need to have some specialized method for the solution of these linear algebraic equations. Without revealing why and all that we just did that so as to get a flavor for what a CFD solution is and how it is different from a regular solution of a numerical methods and regular numerical solution of a partial differential equation or how the getting a solution in CFD is different from getting a solution analytically.

We also said that at that stage that we have a flavor, but we do not know the real difficulties, because in order to solve we need to have governing equations. And we spent the second module in the third and fourth weeks to discuss what are the equations which govern fluid flow, and what is the form of these equations, what type of information is needed in terms of boundary conditions initial conditions all these is what we discussed in the third and fourth weeks. By which time, we had a fairly good idea of how to formulate the CFD problem in terms of saying that these are the equations we want to solve, this is the computational domain and these are the boundary conditions

Then in the third module, we looked at trying to define the approximations that need to be made to the derivatives. So, that we could convert these differential equations into set of algebraic equations so that this is where we brought in the kind of good CFD practices which lead us to consistent stable convergent accurate schemes which is what we did in the fifth week as the first part of module three. And we also looked at methods of verifying that these good practices are there in the choice of the scheme that we want to use for a given partial differential equations.

So, module three discuss the basic concepts that going to the choice of a proper discretization scheme. In module four, we looked at how to apply this particular template for the solution of the coupled equations that are actually required for fluid flow problems. And there we distinguished clearly between the compressible flow type of fluid flow problems, and the incompressible flow type of fluid flow problems. And we said that the compressible flow types of problems are linked strongly by the density and pressure.

The density variation provides a strong linkage between the continuity equation and the momentum equations. And we also encounter the problems of non-linearity and coupling when we are looking at Navier-Stokes equations. We solve methods specialized methods which take account of this for example, the Mac-Cormack scheme as an explicit method we looked at it, and then we also looked at beam warming method as an implicit method which is able to take better account of non-linearity in them.

And then in the second week of that particular fourth module, we looked at the specialized methods that have been developed for incompressible flows, where the linkage between the continuity equation and the momentum equations is broken because density is not varying. So, that means, that possibility of evaluating pressure through the density which is evaluated from the continuity equation is no longer there.

So, then we looked at specialized methods for how to deal with pressure, how to get pressure which is required in the momentum equations and we looked at the artificial compressibility method steam function vorticity method, and the pressure equation method, pressure correction method all different types of philosophies approaching the same objective of recovering pressure from continuity equations somehow. So, that we can get that pressure put that into the momentum equations and then get a solution.

So, at the end of module four we knew how to solve these equations, but solving these equations itself is not sufficient, because we have to solve them in many at many, many grid points as many grid points as possible, so that we have good accuracy. Because we know that we are making gross approximations; if you say first order accurate or second order accurate, we are retaining only the first two terms or first three terms of the Taylor's expansion and that is not good enough. So, you have to make sure that you put the points very close to each other and that means, solution of large number of equations. And if you have large number of equations the matrix size becomes very large and if the matrix size becomes large then the amount of time taken for the solution is large.

So, that is why we looked at in module five at specialized methods at the range of methods that are available and often used for CFD type of problems. We looked at some direct methods we looked at basic iterative methods and established that these Gauss-Seidel types of methods are better than the Gauss-Seidel elimination or Lu decomposition type of methods. In the context of having CFD type of problems, where the matrix a is passed and it has certain special properties like diagonal dominance.

Then we also looked at the second week of the module 5 at specialized method which accelerates the rate of convergence of this iterative method more than what is possible with the Gauss-Seidel method. And we also looked at the basic ideas of multi grid approach which actually makes the solution much, much faster than what we can achieve with Gauss-Seidel method

In the last module, we looked at the first part of the last module is what we where we looked at how to take these ideas into practical flow computations involving irregular geometry. So, we looked at how to do the discretization how to do grid generation for we took the case of a two d thing and then we looked at special grid generation methods for dividing the computation domain in two sets of triangles.

And then we also looked at how from the vertices we can derive all the information that is necessary to discretize the governing equations over each of these cells using the finite volume method. So, these things put together will give us an A phi equal to b type of solutions for this irregular geometry. And we have already seen in module how to solve these equations, so that was the first part of the module.

And in the second part of the module the 6th module is what we have just now finished and it is on how to deal with turbulent flow. Turbulent flow we said is characterized by very rapid and highly localized fluctuations. These are so small that these are not really of interest to us, but if we neglect then we are neglecting all the beneficial and special effects of turbulent flow like high diffusivity, high heat transfer coefficient, and mass transfer coefficient. So, then we said these are so rapid that we can try to smoothen them out by time averaging then we got into the turbulence closure model.

The turbulence closure problem that as a result of time averaging, we have more number of unknowns than the number of equations that are available; then we looked at two different simple approaches one is the Boussinesq hypothesis combined with the Planck's mixing length layer, which will work for wall dominated flows in the near wall region. And then we looked at the generic formulation of the two equation model or the k epsilon model in which we solve two extra partial differential equations to describe the two properties of turbulence the k and epsilon, the turbulent kinetic energy and this dissipation rate to define the turbulent or eddy viscosity in the Boussinesq hypothesis using which we can get an expression for estimation for the Reynolds stresses which can then we put into the momentum equation.

So, with this thing, with all these 12 weeks of things, we have a basic understanding of CFD as can be applied for turbulent flows, this is only a beginning, it is not the end. There is lot more there have been lot more developments of this in many different ways, but I hope that this coverage has given you a good understanding of the issues in computational fluid dynamics. I wish you all the best to each of you, and hopefully we will take this as a starting point as a springboard for further explorations of CFD and in all it is forms.

Thank you very much.